A SIMPLE APPROXIMATION OF THE BEST-SPEED-TO-FLY THEORY

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Presented at the XX OSTIV Congress, Benalla, Australia (1987)

Summary

If we know the speed at the minimum rate of sink and at the sink rate of 2 m/s we can easily calculate the MacCready function with good approximation. The speed error, in the useful speed range, is less than 10 km/h for standard class gliders.

The MacCready function can be calculated as

$W_{MC} = k\,V\,(V - V_{min})$

where

V is the actual speed of the glider

V_{min} is the speed at minimum rate of sink

k is a constant

$$k = \frac{5}{V_2\,(V_2 - V_{min})}$$ for all gliders of the old generation up to ASW19, etc.

$$k = \frac{5.5}{V_2\,(V_2 - V_{min})}$$ for all modern standard class gliders like LS4, Discus, etc.

V_2 is the speed at a sink rate of 2 m/s

The best-speed-to-fly V_{mc} can be calculated as

$$V_{MC} = \sqrt{\frac{1}{k}(4 + W_{st})}$$

where

W_{st} is the MacCready setting or the rate of climb in the next thermal.

1. Introduction

The traditional way to obtain the MacCready curve is to draw tangents to the speed polar, i.e. a graphical solution. A more rational way of obtaining the MacCready curve is to describe the speed polar numerically. Reichmann (1), among others, suggests the use of a polynomial of the second order

$$W_s = aV^2 + bV + c \qquad (1)$$

where

W_s is the rate of sink

V is the speed

a, b and c are constants

To solve a, b and c we need three equations, i.e. three known points on the speed polar. These points should, according to Reichmann, be chosen at minimum sink, high speed and at the middle of the speed range.

The MacCready function is the sum, W_{MC}, of the glider's rate of sink, the sinking/climbing speed of the airmass and the rate of climb in the next thermal, at a certain speed. The tangent to the speed polar at this speed gives us W_{MC}. It can be formulated as

$$W_{MC} = \frac{dW_s}{dV}\,V = 2aV^2 + bv \qquad (2)$$

This is a very simple way of calculating the MacCready curve, provided we know three points of the speed polar. The resulting error is very small for most gliders.

The trend in the standard class is towards optimizing the glider in the speed range between 100 and 150 km/h (without water ballast). The Discus and the new ship from Schleicher, ASW24, are examples of this new philosophy. At slow speeds the polar is almost horizontal and at around 160 km/h there is a characteristic bend. At higher speeds, the polar is almost a straight line. It is quite obvious that a polynomial of the second order is not a very good approximation of this type of speed polar. We must use polynomials of a higher order, which means more constants to solve. In Figure 1 the speed polar of the Discus (as measured by DFVLR) is shown together with two approximations according to eq. (1).

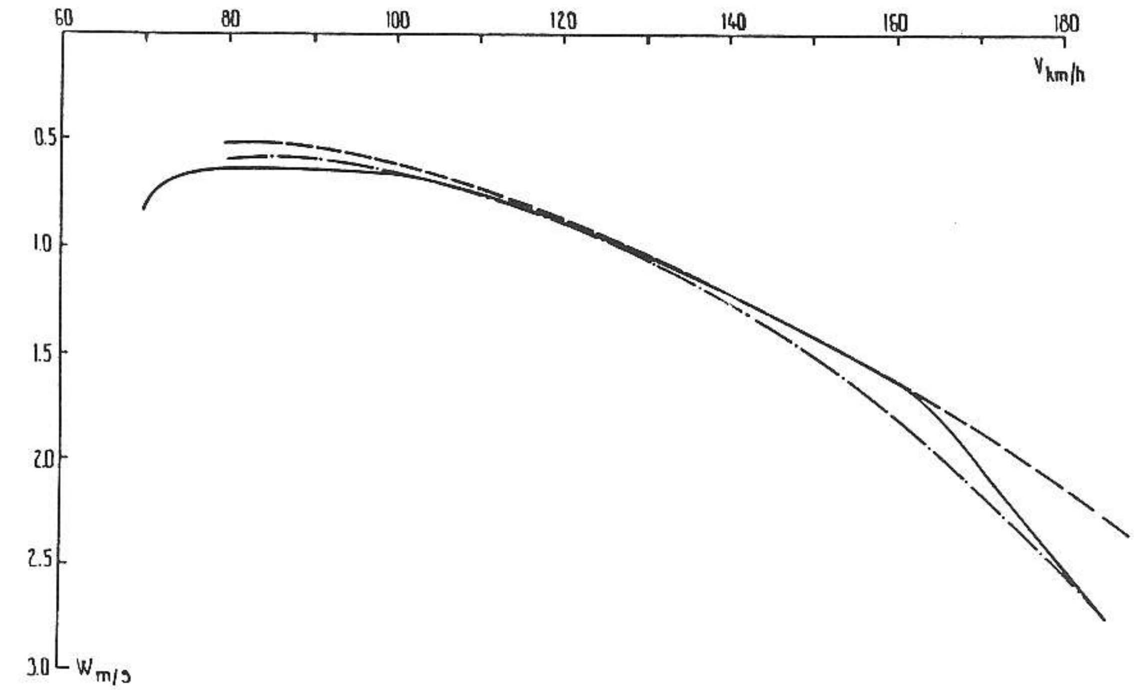

Figure 1. Speed polar of the Discus. The straight line is the polar as measured by DFVLR, the other two are approximations according to eq. (1).

This problem lead the author to try to find a simple approximation of the best-speed-to-fly theory, which would give errors that were negligible for practical use. We will only consider standard class gliders.

2. A simple approximation

Lars T. Johansson (2) has built up a data bank containing the speed polars of a lot of gliders. He has used, mostly, polars as measured by DFVLR and a polynomial of the 10th order to describe the polar. Thus the polynomial used is a very good approximation of the true speed polar. When studying the work of Johansson, it was found that the MacCready speed for an anticipated rate of climb of 3 m/s was very close to the speed corresponding to a rate of sink, W_s, of 2 m/s. Table 1 shows the rate of sink, W_{MC3}, of 20 gliders for the MacCready speed correspondng to an anticipated rate of climb of 3 m/s.

type of glider	W_{MC3} (m/s)
ASW19	1.86
Std Cirrus	2.13
Astir CS	2.17
Std Libelle	2.09
LS1-f	2.03
Phoebus A	2.26
Phoebus B	2.18
Club Libelle	2.08
Salto	2.11
Ka6CR	1.94
Ka6E	2.03
K8B	1.75
L-Spatz	1.79
Pirat	2.15
Zugvogel 4	2.26
Foka 4	2.25
Cobra 15	1.76
Pilatus B4	2.27
SF-27A	2.24

Table 1. W_{MC3} is the gliders rate of sink at a speed corresponding to an anticipated rate of climb in the next thermal of 3 m/s.

The mean value of W_{MC3} is 2.06 m/s, with the coefficient of variation $\delta = 0.082$. The wingloading for the values of W_{MC3} in table 1 corresponds to the empty weight of the glider plus 90 kg (i.e. weight of pilot). This is the normal way the polars by DFVLR are published.

The speed at a sink rate of 2 m/s is denoted V_2 and using eq. (1) we have

$$2 = aV_2^2 + bV_2 + c \tag{3}$$

The slope of the polar is zero at the speed corresponding to the minimum rate of sink, V_{min}, and this gives us

$$\frac{dW_s}{dV} = 2aV_{min} + b = 0 \tag{4}$$

Now, we make use of the values in Table 1, by simply stating that $W_{MC} = 3 + 2 = 5$ corresponds to a speed that equals V_2. Using eq (2) we get

$$W_{MC} = \frac{dW_s}{dV} V = 2aV_2^2 + bV_2 = 5 \tag{5}$$

We have three equations and three unknown constants, a, b and c, which means that we can solve these. The speed polar can be written as

$$W_s = \frac{2.5}{V_2(V_2 - V_{min})} V^2 - \frac{5V_{min}}{V_2(V_2 - V_{min})} V - \frac{2.5V_2 - 5V_{min}}{V_2 - V_{min}} + 2 \tag{6}$$

Our main interest is not the speed polar itself, but the MacCready function. We derivate eq (6) and multiply with V. After rewriting, we arrive at

$$W_{MC} = \frac{5}{V_2(V_2 - V_{min})} (V - V_{min}) V = k V (V - V_{min}) \tag{7}$$

where

$$k = \frac{5}{V_2(V_2 - V_{min})}$$

To calculate the speed ring we only need to know the speed at minimum sink and at a sink rate of 2 m/s!

3. Evaluation

Eq (7) is very simple to use and the natural question is: how good is it? Figure 2 to 7 shows both the MacCready function

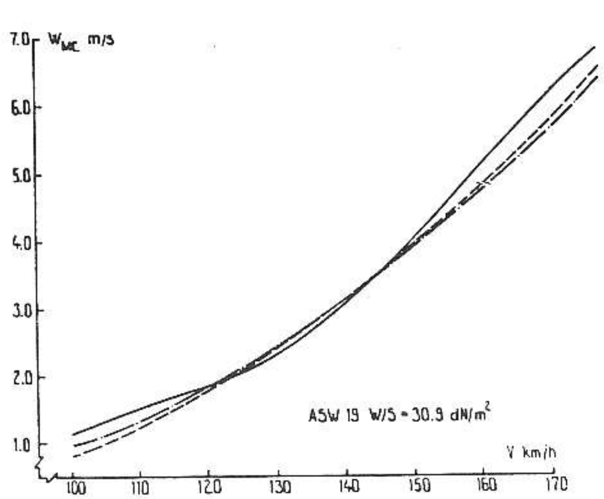

Figure 2.

Figure 2 - 9 ———— Johansson
- - - - - - eq (7)
-·-·-·-·- eq (2)

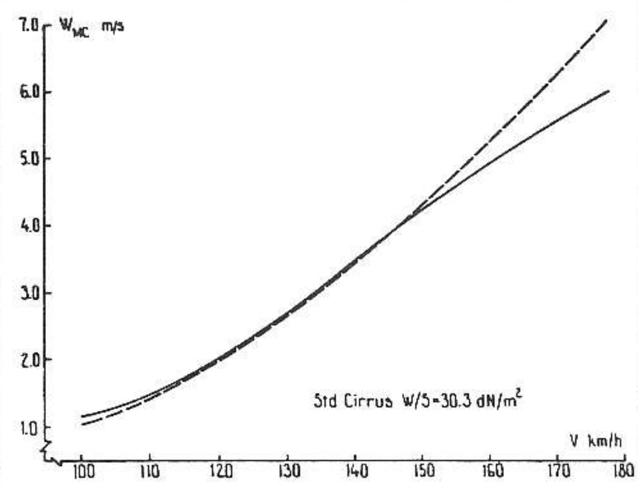

Figure 3.

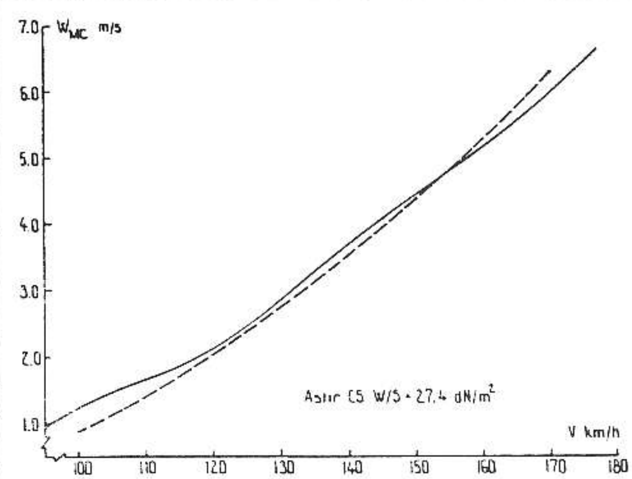

Figure 4.

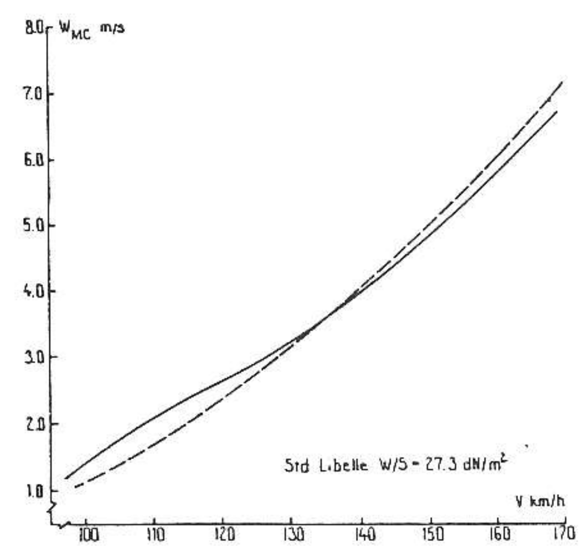

Figure 5.

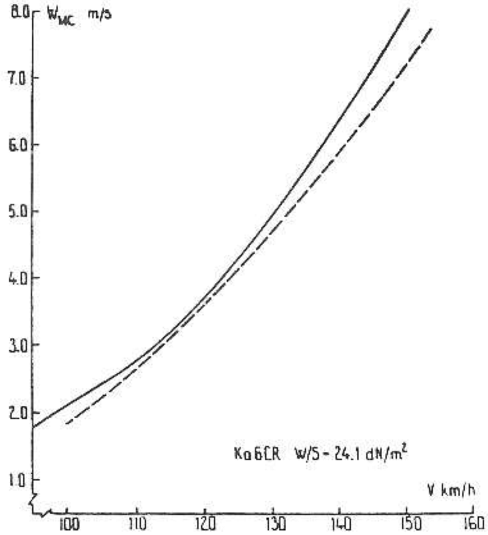

Figure 6.

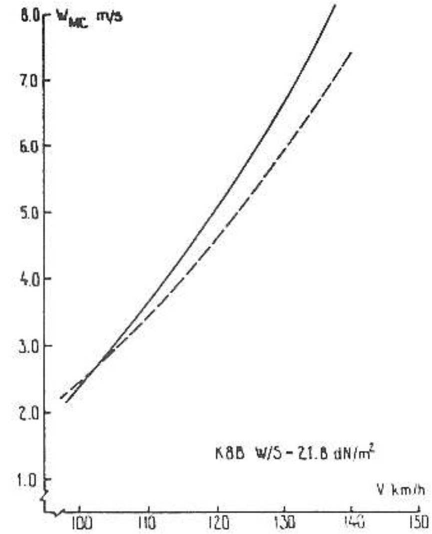

Figure 7.

according to Johansson (2) and eq (7). In the case of Figure 2, ASW19, the MacCready curve obtained when using a polynomial of the second order is also shown. As can be seen eq (7) is a good approximation. The values are valid for unballasted gliders. In this configuration and in European weather one would seldom fly faster than 160 km/h. Below this speed, the speed error resulting from using eq (7) is less than 5 km/h.

Now we turn our attention towards more modern standard class gliders like LS4, DG300 and Pegase. For these gliders (and even for gliders of the 15 m class) eq (7) needs to be modified. Here V_2 corresponds to a W_{MC} of 5.5 (3.5 + 2), and the k in eq (7) becomes

$$k = \frac{5.5}{V_2(V_2 - V_{min})} \tag{8}$$

Figure 8 shows the result for the LS4.

For the Discus (and the ASW24), with its peculiar bend on the speed polar, the MacCready curve also has a peculiar shape. At a certain value of W_{MC} the function becomes a straight line. This is because when the polar itself becomes a straight line, it is not possible to find tangents anymore. This means, as an example, with the speed ring set to zero, at a certain rate of sink we have the same glide ratio independent of the speed.

It is not possible, as shown in Figure 9, to calculate the MacCready function with the help of a polynomial of the second order, as described by eq (1) and (2). We need polynomials of a higher order. Eq (7) with k calculated according to eq (8) is actually a better approximation over a larger speed range, see Figure 9. Below 165 km/h the speed error is less than 7 km/h.

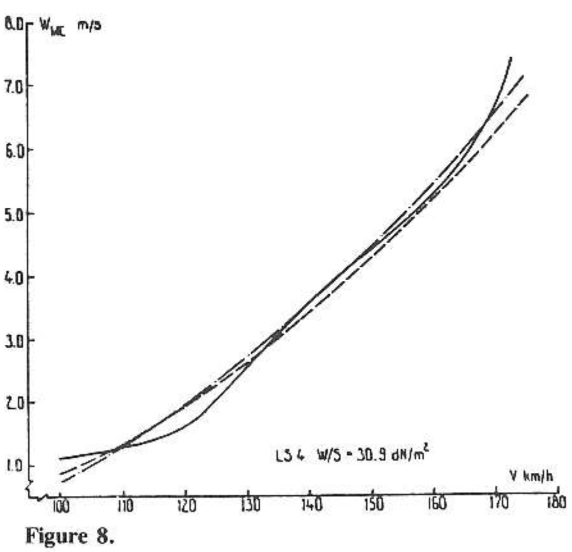

Figure 8.

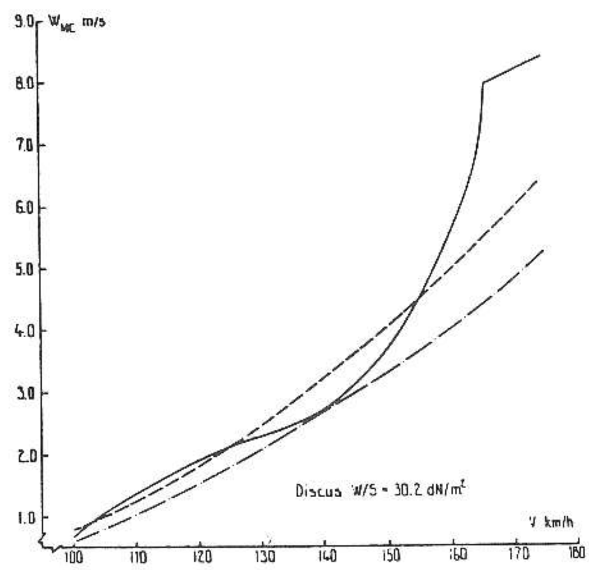

Figure 9.

4. Best-Speed-To-Fly

The best-speed-to-fly, V_{MC}, can be calculated as (see Reichmann (1))

$$V_{MC} = \sqrt{\frac{c - W_{sj} + W_{st}}{a}} \tag{9}$$

where

W_{sj} is the rate of sink of the airmass

W_{st} is the rate of climb in the next thermal

a and c are constants, see eq (1)

We identify a and c in eq (6) and by using k we arrive at

$$V_{MC} = \sqrt{\frac{0.5\,k\,V_2^2 - k\,V_2\,V_{min} + 2 + W_{st} - W_{sj}}{0.5\,k}} \tag{10}$$

which can be written as

$$V_{MC} = \sqrt{V_2^2 - 2V_2\,V_{min} + \frac{1}{k}(4 + W_{st}) - \frac{2W_{sj}}{k}} \tag{11}$$

We are interested in finding the speed in calm air which means that $W_{sj} = 0$. A new approximation is entered, which says that $V_2 = 2V_{min}$. Eq (11) can now be written as

$$V_{MC} = \sqrt{\frac{1}{k}(4 + 2W_{st})} \tag{12}$$

W_{st} is the MacCready setting or ring setting. Table 2 is a comparison between eq (12) and the values calculated by Johansson (2) for three different gliders, Ka6CR, ASW19 and Discus. Eq (12) is a good approximation of V_{MC} for the Ka6CR and the ASW19. It is also good for the Discus as long as we are above the "bend."

W_{st} m/s	V_{MC} km/h					
	Ka6CR		ASW19		Discus	
	Johansson	Eq (12)	Johansson	Eq (12)	Johansson	Eq (12)
0	83	81	86	101	100	102
1	98	99	127	124	118	125
2	119	114	144	143	148	144
3	130	128	157	160	156	161
4	140	140	172	175	161	176
5	150	151	186	189	165	191

Table 2. The speeds are valid for a wingloading corresponding to the empty weight of the glider plus 90 kg.

5. Conclusions

If we know the speed at the minimum rate of sink and at the sink rate of 2 m/s we can easily calculate the MacCready function with good approximation. The speed error, in the useful speed range, is less than 10 km/h.

6. References

(1) Reichmann, H. 1976. Streckensegelflug. Motorbuch-verlag Stuttgart

(2) Johansson, L.T. 1985. Numerical evaluations of a wealth of different gliders. Internal report, Swedish Aviation Sports Federation.