INSTATIONARY STOCHASTIC MODELING OF THERMALS

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Summary

Evaluation methods of today do not give a fully satisfactory picture of the fine structure of atmospheric turbulence. A possible way of improvement might be to:

- employ a running mean for assessment;
- generalize the Kármán spectrum for smoothing of the raw spectra;
- introduce regular-instationary modeling for the treatment of transient turbulence.

Natural parameters of turbulence calculated this way will be much more consistent and reproducible than those obtained using bulk assessment for whole flight records. Updraft/turbulence measurements done by three powered sailplanes on a single-cell thermal can be recommended for the trial and development of the process.

1. Spectrum Formulae

Atmospheric turbulence theory has - as regards spectrum processing - a flying start from the general turbulence research. First of all, it has inherited the concept of the integral scale of turbulence as:

$$L = \lim_{\xi_1 \to \infty} \frac{1}{\sigma_w^2} \int_0^{\xi_1} R_w(\xi)\, d\xi \tag{1}$$

and two spectrum formulae. The first one of them (Dryden, 1937) reads:

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1 + 3L^2\Omega^2}{(1 + L^2\Omega^2)^2} \tag{2}$$

giving $\lim_{\Omega \to \infty} \frac{dG_w}{d\Omega} = -2.$

The exponent of the Dryden spectrum being a little on the high side, Kármán (1948) revised it to read:

$$G_w(\Omega) = \sigma_w^2 \frac{2L}{\pi} \frac{1 + \frac{8}{3}(1.339\, L\Omega)^2}{\left[1 + (1.339\, L\Omega)^2\right]^{11/6}} \tag{3}$$

with $\lim_{\Omega \to \infty} \frac{dG_w}{d\Omega} = -\frac{5}{3}.$

Later, for high-speed low-altitude work Lappe (1966) recommended:

$$G_w(\Omega) = \sigma_w^2 \frac{L}{(1 + L\Omega)^2} \tag{4}$$

modified soon at Lockheed-Georgia (Firebaugh, 1967) to read:

$$G_w(\Omega) = \sigma_w^2 \frac{0.8L}{(1 + 0.8L\Omega)^{1.8}} \qquad (5)$$

Both formulae (4) and (5) can be shown not to fulfill the Kovásznay criterion ([4] pp. 91-94.):

$$\lim_{\Omega \to 0} G_w(\Omega) = \frac{2}{\pi}\sigma_w^2 L \qquad (6)$$

so the author proposed for them [6]:

$$G_w(\Omega) = \frac{2}{\pi}\sigma_w^2 \frac{L}{(1 + \frac{12}{5\pi}L\Omega)^{11/6}} \qquad (7)$$

Checking and ranking of the analytical PSD formulae can be made-by using them for smoothing measured turbulence spectra as shown on Figure 1. First step in the smoothing process should be the correct determination of the mean value.

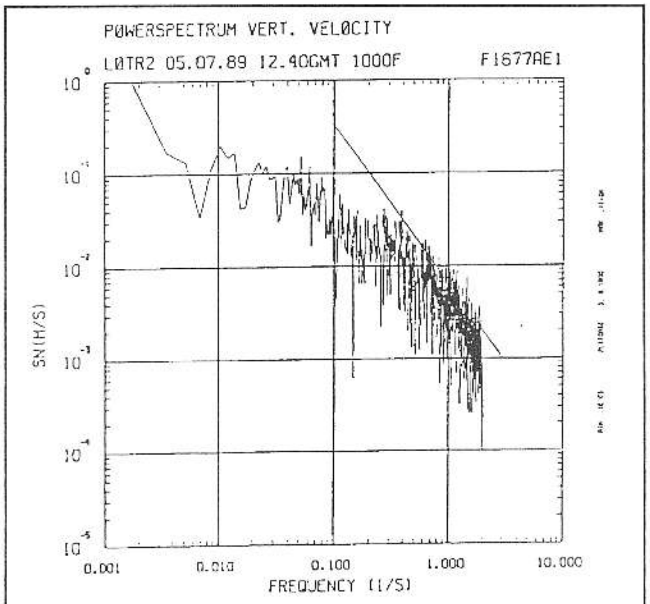

FIGURE 1. Atmospheric turbulence spectrum (Enstrasser, 1991).

2. General Circulation

When measuring boundary layer or wind-tunnel turbulence it is not the whole instantaneous local velocity which is treated as such. Only the difference from the mean, i.e.

$$w(\vec{r},t) = v(\vec{r},t) - c(\vec{r},t) = v(\vec{r},t) - \lim_{t_1 \to \infty} \frac{1}{t_1}\int_0^{t_1} v(\vec{r},t)\,dt \qquad (8)$$

is declared and treated as turbulence. Now, Equation (8) is good for tower measurements but in-flight measured updrafts cannot be treated this way, because it is not possible to evaluate the right-side integral. Instead of this we can try with a suitable running mean possibly cross-checked with parallel flying of say three planes.

On the other hand, it seems we have here a new and interesting problem. The, let us say, effective intensity of turbulence σ_e seems to be strongly dependent on the flight speed and on the first eigenfrequency of the plane flying in it. The difference in effective intensity as registered by the crew of a jet fighter or airliner on the one hand, a light primary sailplane on the other hand, can be expressed in the mathematical form:

$$\sigma_w \leqq \sigma_e \leqq \sqrt{\sigma_c^2 + \sigma_w^2} \qquad (9)$$

where σ_c is meant to be the standard deviation of the general circulation as measured along the flying route. Of course, Equation (9) is meant only as a theoretical concept, not as an evaluation formula.

An idealized picture of what to expect from a good running mean calculation for the traverse of a single-cell thermal is shown by the wire-mesh model on Figure 2.

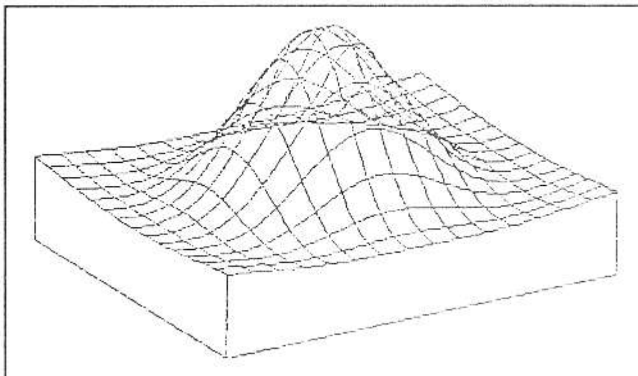

FIGURE 2. Wire-mesh model of a single-cell thermal updraft profile.

3. Generalized Kármán Spectrum

A productive and correct analysis of turbulence spectra from flight records can be made e.g. as follows. After the determination of and conversion to the running mean, the standard deviation σ_w and the integral scale parameter L should be calculated using the respective definition formulae in Table 1, reproduced from (8). These can be then cross-checked by smoothing the raw power spectrum using an appropriate spectrum formula. Now, formulae (2) - (5) and (7) are not ideal for such a purpose, all of them having fixed exponents. The fourth parameter of the Kármán formula, the peak coefficient, has also the fixed value of 8/3.

All this induced the author to look for a more flexible form of the PSO function. It has been found in the form of a generalization of the Kármán formula, originally developed for the treatment of road/terrain and rail spectra (Gedeon, 1992). It reads as follows:

$$G_w(n) = 4\sigma_w^2 L \frac{1 + A(CLn)^2}{\left[1 + (CLn)^2\right]^{\alpha}} \qquad (10)$$

with

$$C = \frac{2}{\Gamma(\alpha)}\left[\sqrt{\pi}\,\Gamma(\alpha - \tfrac{1}{2}) + A\,\Gamma(\tfrac{3}{2})\Gamma(\alpha - \tfrac{3}{2})\right] \qquad (11)$$

Parameter:	Calculation formula using $w(t)/\overline{w} = 0/$	autocorrelation func. $R_w(\zeta)$	spectral density func. $G_w(n)$ resp. $G_w(\Omega)$
Standard deviation σ_w	Def.: $\sigma_w = \lim_{S \to \infty} \left[\frac{1}{S} \int_0^S w^2(t)\, dt \right]^{1/2}$	———	$\sigma_w^2 = \int_0^\infty G_w(n)\, dn = \int_0^\infty G_w(\Omega)\, d\Omega$
Scale parameter /integral/ L	———	Def.: $L = \lim_{\zeta_1 \to \infty} \frac{1}{\sigma_w^2} \int_0^{\zeta_1} R_w(\zeta)\, d\zeta$	Regression analysis
Taylor's scale parameter λ	———	Def.: $\lambda = \frac{\sqrt{2}\,\sigma_w}{\left[-\left(\frac{d^2 R_w(\zeta)}{d\zeta^2} \right)_{\zeta = 0} \right]^{1/2}}$	$\lambda = \frac{\sigma_w}{\sqrt{2}\pi} \left[\int_0^\infty n^2 G_w(n)\, dn \right]^{-1/2}$ $\lambda = \sqrt{2}\,\sigma_w \left[\int_0^\infty \Omega^2 G_w(\Omega)\, d\Omega \right]^{-1/2}$
Exponent α	———	———	Regression analysis

$$R_w(0) = \sigma_w^2 \qquad G_w(n): \quad G_w(0) = 4L\sigma_w^2 \qquad n_{max} = \frac{1}{\lambda}$$
$$G_w(\Omega): \quad G_w(0) = \frac{2}{\pi} L\sigma_w^2 \qquad \Omega_{max} = \frac{2\pi}{\lambda}$$

Table 1. Natural parameters of stationary stochastic processes.

The goodness of fit of the smoothing with this formula as well as with the other ones can be checked with the value of the relative error standard deviation:

$$\Delta = \left[\frac{1}{m} \sum_{i=1}^{m} \left(\frac{G_{wi} - G_w(n_i)}{G_w(n_i)} \right)^2 \right]^{1/2} \qquad (12)$$

where G_{wi}- n_i ($i = 1 \div m$) are for the raw spectrum points and $G_w(n_i)$ are values given by the smoothing function for the i-th spectrum point.

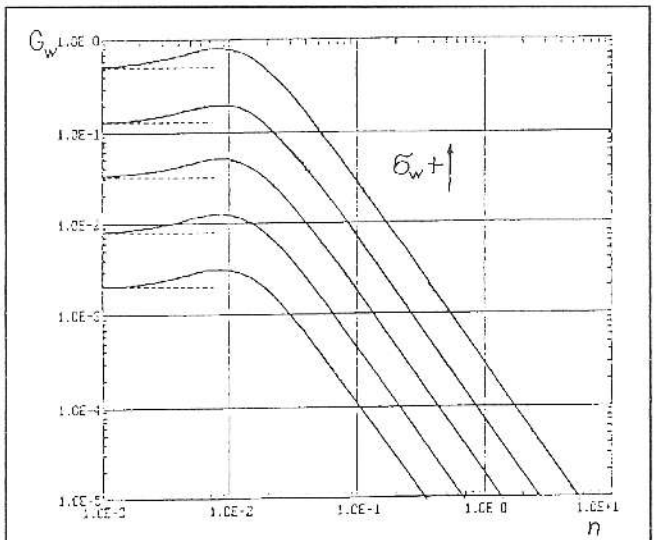

FIGURE 3. Influence of the standard deviation δ_w on the character of the PSD function.

The influence and role of the single parameters can be seen on Figures 3 to 6, drawn on logarithmic scales. Increase of the standard deviation σ_w increases all spectrum values uniformly for all frequencies (Figure 3). Increase of the scale parameter L does not affect the overall intensity of turbulence but lowers the dominant frequency (Figure 4). Augmentation of the exponent α diminishes the intensity on higher frequencies but without much affecting the location of maximal intensity (Figure 5). An increase of the peak coefficient A does what the name implies (Figure 6) resulting in a "narrower band" spectrum with a lower dominant frequency.

A suitable regression algorithm for the new formula, too, has been found.

4. Regular-Instationary Process Evaluation

The standard requirement for the proof of stationarity is the independence between time-of-start and location-of-start statistical parameters. This, while obviously a correct mathematical interpretation, seems to be only a necessary but not a sufficient proof in case of some

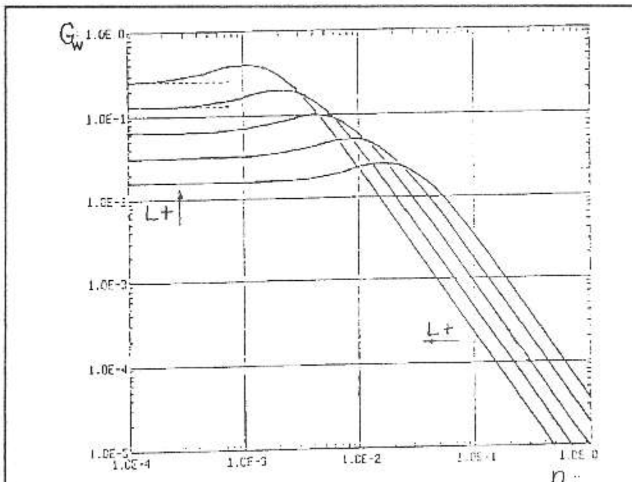

FIGURE 4. Influence of the scale parameter L on the character of the PSD function.

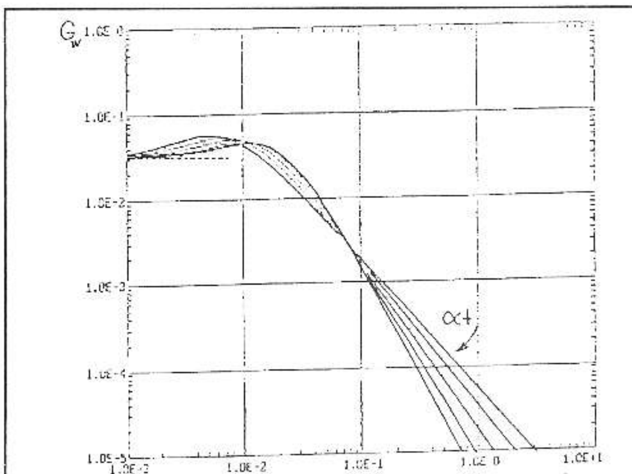

FIGURE 5. Influence of the exponent α on the character of the PSD function.

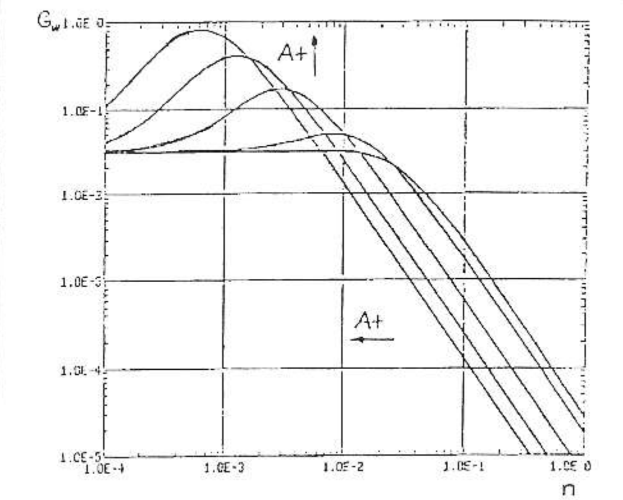

FIGURE 6. Influence of the peak coefficient A on the character of the PSD function.

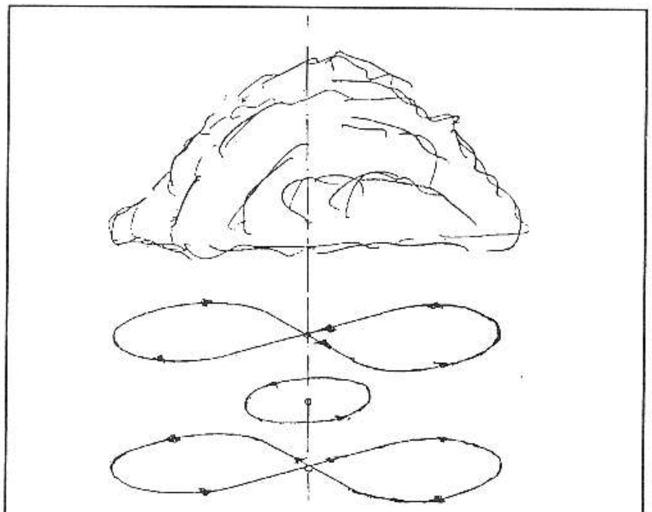

FIGURE 7. Planned flight paths of the planes for the updraft measurement.

technical investigations. So, e.g. the local intensity of thermal as well as of orographic turbulence is not constant along dozens of kilometers but it is of transient character in the surroundings of up- and downdrafts. And the formal proof of stationarity nevertheless succeeds within acceptable tolerances for a sufficiently long flight section.

Stationary modeling and bulk treatment of flight records registered over such long distances is formally possible but not really correct. At least part of the inconsistencies in the details of turbulence spectra might be due to this. An answer to the problem can be the introduction of regular-instationary modeling (Gedeon, 1990).

Definition of the regular-instationary process is as follows. Let us suppose the record

$$w = f(x) \text{ and } w = f(t) \tag{13}$$

to be instationary. If there are functions $g(x)$ and $g(t)$ and/or another functions $h(x)$ and $h(t)$ making the transformed record

$$g(x)f(x) = f^*(h(x)) \text{ and } g(t)f(t) = f^*(h(t)) \tag{14}$$

stationary, then and only then can the records $f(x)$ and $f(t)$ be declared to be regular-instationary.

The transformed record is now open to standard frequency analysis procedures. Even reconstruction of the original record is possible from the pseudo-spectrum G^* of the transformed record (14). Re-transformation of the pseudo-standard deviation σ_w^* and of the pseudo-scale L^* and T^* gives the parameter functions

$$\sigma_w(x) = \sigma_w^* g^{-1}(x) \quad \text{and} \quad \sigma_w(t) = \sigma_w^* g^{-1}(t) \tag{15}$$

and also

$$L(x) = L^* h^{-1}(x) \quad \text{and} \quad T(t) = T^* h^{-1}(t) \tag{16}$$

Reconstruction of the original record is now possible using for each point the corresponding actual values of the parameter functions.

A major practical problem will be the development of the procedures for the calculation of the transformation functions g( ) and h( ). For the former, there are at first two possible candidates. The first one is calculation of an approximate "running standard deviation" $\sigma_w(x)$ and $\sigma_w(t)$ giving

$$g(x) = \frac{8}{\sigma_w(x)} \quad \text{and} \quad g(t) = \frac{8}{\sigma_w(t)} \tag{17}$$

So far little can be said of candidate procedures for the transformation functions h( ). Only from practical trials and experience can we hope to choose something. By the way, L may be perhaps stationary, at least for the area of a single cell or for a group of cells.

5. Flight Test Program

In the case of thermal turbulence a flight test program for the development of regular-instationary evaluation methods might be composed as follows.

Updraft measurements are to be made in the flow field of a single cell thermal by three (or more) instrumented powered sailplanes flying at three levels (Figure 7). The middle one is holding position while circling in the thermal core; the others make perpendicular alternate crossings through the core aided in the orientation by the middle one. The method can perhaps be extended by filming, release of little balanced colored balloons, etc. All this will require detailed, careful planning and expert piloting to give coherent results but it seems to be worth the effort.

With good position holding, the record registered by the middle plane can be declared to be nearly stationary. Evaluation will give therefore acceptable values of the turbulence parameters for the circling radius. The two other planes will give transient sections and the calculation of the running parameters can be checked at the circling radius.

Conclusions

Inconsistencies in some details of atmospheric turbu-

Appendix 1: Notation:

c	vertical component of the general circulation	m/s
$f(\ ), g(\ ), h(\ )$	functions	
n	space frequency	1/m
$\vec{r}$	position vector	
t	time	s
v	vertical component of the instantenous local wind velocity	m/s
w	vertical component of turbulence velocity	m/s
x	distance along the flight path	m
A	peak coefficient	
B	constant	
C	constant	
$G(\ )$	(one sided) power spectral density function	
L	integral scale of turbulence	m
$R(\ )$	autocorrelation/autocovariance function	
T	time scale	s
α	exponent	
λ	Taylor's scale parameter	m
σ	standard deviation	
ξ	space lag	m
$\Gamma(\ )$	gamma function	
Δ	relative error standard deviation	
Ω	space circular frequency	rad/m
Subscipts:		
c	of general circulation	
e	effective	
w	of vertical turbulence velocity	

APPENDIX 1. Notation

lence spectra could be reduced or eliminated by introducing new concepts in the evaluation. Turbulent velocity components should be separated from the general circulation by assessment using running means. A four-parameter generalization of the Karman spectrum can be recommended for smoothing of raw spectra. Last but not least, regular-instationary process modeling might be adopted for parts of the records where transient turbulence fields have been crossed. Updraft measurements by three planes on a single-cell thermal are recommended for the trial and development of the method.

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