BEST SPEED STORY

1938-1950

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Introduction

This study has been carried out with the desire to establish the truth of historical facts. We sincerely hope that the information contained in it will enlighten any glider pilot who cares to read it. We would like to apologize to those persons who are allergic to mathematics for the somewhat technical content of the study. We hope, however, that others will appreciate this aspect. The events are covered in chronological order of the known existing written documents.

Early History

When the American John Montgomery (Figs. 1 and 2) in 1884, the German Otto Lilienthal (Figs. 3 and 4) in 1892, and his British disciple Percy-Sinclair Pilcher (Figs. 5 and 6) in 1895, made their very first 'gliding' flights, no-one could have imagined the incredible developments that gliding was to make in the next century, thanks to their first attempts. It is tragic that all three pilots were killed – victims of their invention.

It was in 1909 that gliding was really to start as a sporting activity. A small group of enthusiastic pilots, members of the Aeronautical Association of Darmstadt, and led by **Hans Gutermuth (Fig. 7)**, succeeded in making several interesting glider flights on the slopes of the Rhön, in the heart of Germany (**Fig. 8**). They did not wait for the interdiction of flying an engined aircraft imposed on Germany eight years later by the treaty of Versailles, to invent gliding. Tragically, the First World War put a stop to these superb efforts for five years, during which time most of these pilots perished in the conflict. Those who survived started gliding again in 1920, resulting in the construction of the first modern gliders, and the mastery of slope soaring.

Let us also mention two famous scientific precursors : 1.) The Englishmen **Gordon-England**, who in 1909 climbed up to an altitude of 12 metres above his starting point, flying a glider made by another Englishman, **José Weiss**, born and bred in France. 2.) The American **Orville Wright**, the youngest of the brothers, who in 1911, trying to realise a dynamic flight, flew for almost 10 minutes, leaving from the dune of Kill Devil Hill in North Carolina and making in fact a magnificent slopesoaring flight, that was unfortunately not followed by any gliding development.

At this time, despite the genius of scientists such as the Frenchmen Louis Mouillard (Fig. 9) and especially Pierre Idrac (Fig. 10), who, in 1922, using irrefutable scientific methods, described the phenomena of thermals and how birds used these, nobody believed that thermals could be used for gliding. This was only understood in 1928 by the German Dr. Alexander Lippisch, the brilliant glider constructor, who found the key to success when he equipped his very recent prototype glider, the Professor, with a variometer. The new piece of equipment, derived from statoscopes that were used on balloons, was manufactured by the Etablissements Badin in Paris. Statoscopes were a sort of very precise altimeter, which operated over areas of 200 meters altitude. Some gliders had already been equipped with these, in particular the Thomas I (Fig. 11) in 1923, which had been piloted by the Frenchman Jean Hemerdinger (Fig. 12), who was tragically killed in his glider before being able to experiment with the new equipment. It was the professional Austrian pilot Robert Kronfeld (Fig 12a), in 1928, who carried out the first real thermal flight, on board the Professor. He flew from the summit of the Rhön to a cumulus, where he succeeded, thanks to his variometer, in gaining enough altitude to return to his point of departure. Thermal soaring had been discovered.

The use of the variometer in gliding was, unfortunately, kept secret until 1931. After that time, its usage became rapidly widespread, allowing enormous progress in gliding. Long distance flights became more and more common, and pilots were ever seeking to glide further and further, which also meant gliding faster and faster. But how could they do this?

A Bit of Technical Information

The well-informed reader may wish to skip this chapter.

What exactly is the question in hand? A glider, for which we know the speed polar, cruises through sinking air straight to the next thermal, in which, with a constant climb rate, he climbs back up to the altitude from which he had started. At which speed should he have cruised to achieve the best average cross-country speed? A second question follows from this first one: what means can be made available to the glider pilot that enable him to cruise at this speed?

In answer to the first question, the most elegant method is a graphic one (Fig. 13). Taking the speed polar into consideration, one draws a second horizontal axis for which the ordinate is equal to the air mass sink rate in which the glider is cruising. The estimated climb rate in the next thermal is plotted on the ordinate axis in relation to this second horizontal axis. From this point, one then draws the tangent to the polar. The point of contact determines the best speed-to-fly required to achieve the maximum average crosscountry speed, this being equal to the abscissa of the point of intersection of this tangent with the second horizontal axis. This graphic construction can be justified in several ways:

- Through algebra, by calculating the relation determining the best speed-to-fly, and by deducing from this the graphic construction.
- 2) Geometrically, using the properties of similar triangles. This method is universally used nowadays.
- 3) Vectorially, the most elegant method, described hereafter :

Three vectors are represented on the plane of the polar curve, having their common origin at the intersection of the vertical axis with the second horizontal axis defined above (**Fig. 14**). The first vector, *V1*, having its extremity on the polar curve, represents the flying speed of the glider in cruise,

sum of the air mass sink rate and the relative speed of the glider. The second, V2, drawn along the vertical axis, represents the estimated vertical speed of the glider in the next thermal, difference between the rising air speed and the relative sink speed of the glider. The third vector Va, representing the average cross-country speed of the glider, is related to the first two by:

$$(t1 + t2)\overline{Va} = t1\overline{V1} + t2\overline{V2}$$

Va is the average of the vectors *V1* and *V2*, balanced by the corresponding flying times,

$$\overline{Va} = [t1/(t1 + t2)] \overline{V1} + [t2/(t1+t2)] \overline{V2}$$

This can be graphically determined as follows :

First result: the extremities of these three vectors are aligned on a line that we call *L*. The demonstration of this can be made vectorially by splitting each vector *V1* and *V2* in two vectors, one of them being the average speed vector *Va*, and the second one being the complementary vector. It can then be deduced that the two complementary vectors are necessarily aligned. Another demonstration consists in considering two oblique coordinates axis based on the vectors *V1* and *V2* working as units, in which the coordinates of *Va* are x = t1/(t1 + t2) and y = t2/(t1 + t2). The coordinates *x* and *y* are bound by the relation x + y = 1, which is nothing else than the equation of the straight line *L*.

Second result: the average speed Va is horizontal, i.e. positioned on the second horizontal axis, because departure and arrival of the glider trajectory are at the same altitude. Conclusion: the extremity of Va is at the intersection of the line L with the second horizontal axis. This point determines the average cross-country speed, which is maximized when this line becomes the tangent to the polar curve, with the point of contact determining the corresponding best speed-to-fly. This demonstration was already known in the nineteen fifties. It is also clearly explained in the article: "Why does the best-speed-to-fly construction work?" published by Prof. Anthony W.F. Edwards in *Sailplane and Gliding*, June-July 1980.

The second problem, the piloting method, was solved successively by using tables, or specially adapted slide rules, these being rapidly replaced by the **MacCready** ring, which rotates around the dial of a total energy variometer having a linear climb/sink scale. An optimum speed scale is engraved on the ring, corresponding to the type of glider, and to the altitude at which the glider is flying. The pivoting ring is positioned depending on the anticipated climb rate in the next thermal. Piloting is done by successive iterations.

The Years 1937-38. The Partial Results Achieved by Szukiewicz-Szwarc and Späte, and Fox's Stroke of Genius

When the use of variometers had become widespread and pilots had begun flying long distances, the time had come to elaborate techniques that made it possible to cover the longest distances possible, which also implied covering these distances as quickly as possible.

This created the technical problem of how best to proceed. In fact, there were two problems: a theoretical problem, which involved establishing the best speed-to-fly depending on the different parameters involved, and a practical problem of how to provide the pilot with the means to fly at this speed.

Pilots had sensed for a long time that if they flew a little faster between thermals, they would save more time than they would lose recovering in the next thermal the lost altitude due to the decreased glide ratio. Furthermore, when flying through strong sinking air, they were aware that it was necessary to increase their speed in order to spend as little time as possible in the unfavourable area, and that the time required to regain the altitude lost due to the lesser glide ratio would be inconsequential compared to the time saved from having spent less time in the sinking air. In both cases, the question remained – to what extent should they increase their speed ?

It was the Polish pilot, **Romuald Szukiewicz** (Figs. 15 and 16), who had taken part in the international Rhön competition in July 1937, associated with Leszek Szwarc, both of whom were Dipl. Engineers, who had an excellent article published in the magazine *Skrzydlata Polska* in April 1938, under the title

"Osiagi szybowcow wyczynowych i ich wykorzystanie w przelotach." (Glider Performances and Operating Tehniques). Szukiewicz was to be one of the test pilots involved in choosing the Olympic glider in 1939 in Rome. During the war, he was test pilot for RAF transport gliders, and after the war worked for de Haviland as a Dipl. Engineer. Szwarc worked for Handley Page during the war. These authors described a result already presented by the German aeronautical engineer A. Lippisch at the beginning of the nineteen thirties: how to determine, using the tangent to the speed polar, the best gliding angle in still air, or in rising or sinking air, without wind, or with a tail or headwind. The authors go on to describe how extremely sensitive the performance of gliders (of that period) was when confronted with headwinds or sinking air, or even worse, with a combination of the two.

Szukiewicz and Szwarc also determined the speed required for the glider to achieve its best glide ratio depending on the sink rate of the air mass it flew through, and therefore depending on the corresponding variometer readings that represents the sum of the sink of air and the relative sink of the glider flying at this best glide ratio speed. Thus, they came up with the idea of placing a second scale next to the airspeed indicator's speed scale that gives the value of the variometer reading corresponding to the maximum glide ratio speed. During flight, the speed for the best glide ratio is obtained when the sink rate indicated by the variometer, and that on the additional airspeed indicator scale are the same. They published a picture of an airspeed indicator graduated in this way for the Polish glider PWS 101 (Fig. 17), that included a slight mistake.

In addition, assuming that the air in cruise is still, Szukiewicz and Szwarc elaborated a formula that determined the glider's average cross-country speed, which depended on the cruising speed and on the climb rate in the next thermal. This meant that for any given glider, they were able to plot the curves, which show, for different climb rates in the next thermal, how average cross-country speeds vary depending on the gliding speed chosen (Fig. 18). Each curve shows a maximum value that corresponds to the best speed-to-fly. The authors illustrated, for different types of Polish gliders, how the best speed-to-fly and the average crosscountry speed increase with the climb rate in the next thermal. The problem of the influence of sinking air in cruise, however, was not dealt with.

Finally, Romuald Szukiewicz pointed out that German pilots who had participated in the international Rhön competition in 1937 had already had the feeling that there existed a best speed-to-fly, in particular Heinrich Dittmar, whom he had observed piloting the glider Sao-Paolo.

Following the Rhön competition in 1938, around the month of August, Wolfgang Späte (Fig. 19) from the DFS (German Research Institute for Gliding) in Darmstadt, who was one of the best known glider pilots of that period, and who had also taken part in the international Rhön competition in 1937, published his "Flight Report," NSFK limited edition (Fig. 20). In this he explained:"Diese Ausarbeitung sollte ursprünglich bereits Anfang d. J. zur Veröffentlichung gelangen. Ich entschloss mich jedoch, erst einmal selbst die Brauchbarkeit meiner Theorie bei Gelegenheit der diesjährigen Rhönwettbewerbs-Flüge unter Beweis zu stellen." (This report was originally to have been published at the beginning of the year. However, I decided to put my theory to the test during the Rhön competition being held this year). This leads us to believe that the concept had possibly not been quite ready, unless he had wanted to keep it a personal secret. What is possible is that a certain number of German pilots already had some ideas on the subject, but given the competition, each pilot possibly wanted to keep his ideas to himself.

The subject of the report by Späte was how to determine the best speed-to-fly in still air, according to the climb rate in the next thermal. It was published five or six months after the article written by the Poles, and was almost identical. Same diagrams, same results. Only the gliders were different. Was Späte aware of the Polish article? It is easy to think that he was not, because otherwise it would not have been consistent for the report to be treated as secret, which was the case (**Fig. 21**).

In order to make himself better understood, Späte illustrated his theory using a graphic diagram that was very easy to understand, showing the trajectory followed by several identical gliders leaving simultaneously one and the same point at different speeds, cruising to the same thermal where they recovered the altitude that they had lost (Fig. 22). From this, the reader could clearly see that an optimum gliding speed exists, which is a great help for understanding the phenomenon.

For piloting, Späte recommended taking a small chart on board, on which the instructions for bestspeeds-to-fly, and the corresponding average crosscountry speeds were registered, this depending on the anticipated climb rate in the next thermal. Like other authors who had written about this problem, he insisted that it was essential to use his theory with caution, and that a pilot should not hesitate to ignore it if the conditions required them to do so.

According to Gerhard Wissmann from former East Germany, author of the book "*Abenteuer in Wind und Wolken*" (Adventure in the Wind and Clouds), Späte may have received advice from Prof. Scheubel from the Hochschule in Darmstadt, while he was studying there. This information has been confirmed by Gerhard Waibel, who had the opportunity to talk to Späte sometime before his death.

It is worth considering the relative anteriority of the work done by Späte and by Szukiewicz-Szwarc. At the beginning of 1938, according to Späte himself, his theory still needed some checking, whereas the report written by Szukiewicz and Szwarc had already been submitted to the publishers in March. As far as the concept itself was concerned, it is probable that it was devised by both parties during the Rhön competition held the year before (in 1937), when the idea had possibly been discussed.

According to Frank Irving, of Imperial College, the Späte result was published in the UK by Philip Wills in 1940 using the pen name "Corunus".

In July of the same year 1938, three months after Szukiewicz and Szwarc, and probably just a little before Späte, the well-known English pilot **John Eliot Sylvanus Fox (Fig. 23)**, who had also taken part in the international Rhön competition in 1937, where he had set the first British two seater duration record with William Murray in a Falcon III (**Fig. 24**), wrote an article in the British magazine *Sailplane* entitled "Variometer Speed Calibration." In this article, he described a numerical calculation method that made it possible, as Szukiewicz and Swartz had already done, to determine the cruise speed corresponding to the best glide ratio when the glider was flying in sinking air. Flying at this speed enabled him to reach the highest possible altitude in the next thermal. At this time, and particularly in England, where thermals are often not very strong, pilots had to do their utmost to save their altitude as far as possible, which is a good explanation for why Fox was so keen to find means of doing so.

In particular and quite remarkably, he also described a method of how to pilot the best glide ratio speed in sinking air. The best glide ratio speed of a glider is a function of the sink rate of the air mass it flies through. Reciprocally the air mass sink rate is a function of the best glide ratio speed of the glider flying through it. The relative sink rate of the glider being also a function of this speed, the sum of those two sink rates, that is the variometer reading, is a function of the best glide ratio speed of the glider. And reciprocally the best glide ratio speed of the glider is a function of the variometer reading. Thus, Fox could then add an extra scale to his variometer (a vertical scale Cobb-Slater), one that was placed against the sink rate scale and indicated the best glide ratio speed of the glider corresponding to the total sink rate indicated by the variometer. If the glider's speed is the same as indicated on the second scale of the variometer, the glide ratio is maximal. If the speed is higher, it must be reduced, and if slower, it must be increased, until reaching the moment where the two speeds coincide, this being swiftly achieved since the system is rapidly convergent. He specifies, "When flying into a downdraught one has to dive to gain speed, but this dive only temporarily upsets the calculation, and it is very soon possible to adjust one's speed to the corresponding variometer calibration." Thus, he described the process of successive iterations well known to pilots using the MacCready ring. This instrument is the equivalent of the MacCready ring positioned and blocked at zero (zero climb rate in the next thermal).

It is worth noting that the double scale process was again used, but in this case on the variometer and not on the airspeed indicator, as Szukiewicz and Szwarc had done. In December of the same year, another Polish pilot, **Witold Kasprzyk** (Fig. 25), a champion who was well known for never taking his hat off, published an article entitled "*Technika osiagania maksymalnych szybkosci przelotowych szybowcow*," (Glider maximum speed technique) in the magazine *Skrzydlata Polska*. This article resumed the previous results, but in addition proved that the optimum average speed is a fairly flat curve and, consequently, it is not too disadvantageous to deviate somewhat from the optimum (Fig. 26). Witold Kasprzyk ended his professional career working for Boeing, where he was known by the name Kasper.

International events were to put an abrupt end to these investigations, and it was only ten years later that they started up again with renewed energy.

The War

Wars boost technical advancement.... at least, for weapons. As far as gliding techniques were concerned, however, it was the opposite that occurred. The First World War had put a stop to all the work being done by the team in Darmstadt, and the majority of the pilots were killed in the conflict. Those who had survived took a certain time to get back to their pre-war level. The war had also prevented any other team from becoming involved in the adventure, and in the aftermath, immediate concerns were of a different nature.

The Second World War was even worse than the first. For glider pilots, the war was a bloodbath. Technical developments in the sphere of gliding were practically stopped, even though the production of training gliders, essential for training military pilots, developed considerably, particularly in Germany. A great many gliders were destroyed. Some of the occupying military troops were anything but understanding with this sort of equipment, and instead of being saved, gliders were destroyed under the label "military equipment".

Gliding was prohibited in Germany until 1950, a country that was traditionally a leader in this discipline. Immediately after the war, gliders from the 1930's were

built again, and it was only somewhat later that technical progress started up again.

The Period 1947 – 1949. Part 1: The MacCready Ring

After the war, several British pilots wrote articles related to our subject in the quarterly magazine *Sailplane and Glider*. Amongst these, four articles of significant interest were published in 1947 by the following authors: Flight Lieutenant **Neubroch** in January, **G.O. Smith** in March, **George W. Pirie** and **E. Dewing** in June. On October 26th of the same year (1947), the American **P. MacCready** drafted some unpublished notes (**Fig. 27**), in which he developed his theory, made public during a symposium held by the Association of Southern California Soaring, but today it is difficult to lay hands on these. He also spoke at the symposium held by the SSA-IAS of Elmira in July 1949, and some of his notes can still be found.

The contributions of the authors mentioned above must be considered on the understanding that at this point, the graphical solution had not yet been published. Only the algebraic and numerical analyses had been used, which made the problem more delicate.

The contributions of the four British authors mentioned above are as follows :

Flight Lieutenant Neubroch (Fig. 28), of the Royal Air Force, was born in Vienna. He flew at the RAF club of Barntrop, in Germany. He retired as Group Captain. His article "Best Air Speeds" is a copy of what Späte had demonstrated in 1938, but transposed into English units (Fig. 29). Was he aware of the article published by Philip Wills in 1940? What was interesting about this article is that several previous results were recalled to mind, provoking violent criticism by Gerry Smith in the subsequent issue of *Sailplane and Glider*.

Gerald O. Smith, better known by the name of Gerry, was the fourth of the authors mentioned here, who participated in the International Rhön competition in 1937. He worked for Rolls Royce, the engine manufacturer, and probably as a salvatory reaction, he

decided to pilot engine-free aircraft and became a chief gliding instructor. In his article "Best Air Speeds", he rightly criticizes the fact that Neubroch had not taken into consideration the effect of sinking air in cruise, and that the climb rate in the next thermal was not known to the pilot. He revives the two major ideas presented nine years previously by his compatriot John Fox: fly with the best glide ratio in sinking air, without taking the strength of the thermals into account (Fig. 30), and in order to do this, use a scale of the best glide ratio speeds next to the variometer sink rate scale (Fig. 31). He was probably aware of the work that Fox had done, although there is nothing to prove this. In any case, it was good to recall these ideas so that future authors might be informed, and for them to be able to react.

George W. Pirie, who graduated from Cambridge University in 1940 rightly criticized Smith's criticism of Neubroch's article. He confirms the existence, for cruising in still air, of a best speed-to-fly that increases with the strength of thermals, a result he had also obtained through numerical calculation. He proves, in a somewhat curious manner, that the best speed for cruising through sinking air towards a thermal of minimum strength, that is to say, the speed corresponding to the maximum glide ratio given this sink of air, is the same as the best speed-to-fly in still air when cruising in the direction of a thermal in which the climb rate value is the same as the air mass sink rate value of the preceding case. This is exact, and even fundamental, and adds a vital detail that was missing from the overall synthesis: that the effect on the best speed-to-fly is identical when the sink rate of the air mass while crusing at the best glide ratio is the same as the climb rate in the next thermal while cruising in still air. He writes this pertinent phrase, "flying through a downdraught of 10 ft/sec to a thermal of minimum strength should demand the same optimum flying speed as flying through still air to an anticipated thermal of 10 ft/sec." This theoretical result is important, because it makes it possible to combine the climb rate in the next thermal, with the air mass sink rate in cruise. This means one only needs to calculate once for each total value of both values, by just adding them, instead of having to calculate taking them into account separately.

Given the complex nature of the calculation, impossible to carry out while gliding, G. Pirie gave up. He advises pilots to concentrate on careful observation of clouds and, with an Olympia, to fly at at least 55 mph (102 km/h), except in bad weather. The first piece of advice is sensible, the second, somewhat questionable (Fig. 32).

Pirie's last known location was in New Zealand. E.W. Dewing, who was to achieve a master's degree from Cambridge University in 1948, only appeared very briefly on the gliding scene, which is possibly why he did not have enough time or interest to complete his first results. In June of 1947, he published an article (a letter) in Sailplane and Glider that was much more consistent than those written by the previous authors, although very short. In it, he explains that the articles written by Neubroch and by Smith are not incompatible but complementary, which is true. In particular, he establishes the mathematical formula for calculating the average speed of the glider depending on its cruising speed for a given climb rate in the next thermal and a given air mass sink rate in cruise. Through calculating the derivative and then setting it equal to zero, he establishes the conditions for determining the best speed-to-fly. That is:

where

c = climb rate in the next thermal u = air mass sink rate in cruise S = glider sink rate in still air V = best speed-to-flydS/dV = value of derivative for the value V

 $c + \mu + S = V dS/dV$

This relation well reflects the requirements of the Nickel tangent described further on. It proves that in cross-country gliding, the climb rate in thermals cand the air mass sink rate in cruise u have an identical effect on the best speed-to-fly V, just as Pirie had said.

Thus, Dewing obtained a differential equation of a non-algebraic function for which the numerical solution is very complicated and impossible to obtain in flight given that the pilot has no knowledge of the sink rate of the air mass he flies through. In contrast to the MacCready's solution described below, the pilot does not have immediate access to the necessary data. Thus, like Pirie, Dewing gave up and concluded: "The whole business is getting too complicated."

Dewing deserves credit for having been the first author to have published an algebraic relation which determines the best speed-to-fly, taking into account both the air mass sink rate in cruise and the climb rate in the next thermal. This is considerable theoretical progress. However, he was not the first to have established this relation, as will be explained later on. Unfortunately, this relation does not help the pilot in his gliding, and in any event, until the Nickel tangent method was developed, it was obligatory to proceed using fastidious numerical calculations to determine the best speed-to-fly.

Dewing's last known location was in Canada.

Dewing was in fact very close to the result obtained by Paul MacCready not long afterwards, that is to say, a method that would provide the pilot with the means to pilot the best speed-to-fly. All that was necessary in the algebraic relation indicated above that determined the best speed-to-fly was to highlight the sum of the air mass sink rate and the relative sink rate of the glider in cruise, (u + S). This sum represents the indication of the variometer that is the only piece of information that the pilot has immediately at his disposal other than his airspeed. What he should have written was:

$$u + S = V \left(\frac{dS}{dV} \right) - c$$

that is

$$v = V \left(\frac{dS}{dV} \right) - c$$

where v is the variometer reading.

This relation proves that when the speed polar for a glider is known, then for a given value crepresenting the climb rate in the next thermal, the variometer reading is a function of the best speed-tofly V. This is the key of the solution.

It would appear that the previously mentioned team of four British pilots, in continuing the work already done by the Poles, Szukiewicz and Szwarc, and the English pilot Fox, made a significant contribution to the development of the overall concept but did not come up with a practical implementation.

This was in fact to be done by the famous inventor, Paul MacCready (Figs. 33, 34 and 35), who had a doctorate, and was to become three times American gliding champion in 1948, 1949 and 1953, and world champion in St-Yan in 1956. His prestigious career is well known to everybody and does not need to be recalled to mind.

It was the articles written by Neubroch and Smith that put him on the right track, as he himself noted. In fact, in his preparatory notes for his presentation of July 1949 for the SSA he wrote. "Several articles have been written on best crosscountry flying speed specifically, ones printed in the British publication Sailplane and Glider. One article tells how to get the best gliding angle thru the air, taking into consideration the downcurrent in which the glider is being flown. This permits the pilot to contact the next thermal up as high as possible, an obvious The second article assumes no advantage. downcurrents between the thermals, but assumes thermals of varying strength The flying speed should be between the speed for best gliding angle and top safe speed. Each of these two articles mentioned describes one of the important effects to be considered. 1st, the downcurrent in which the sailplane is flying, 2nd, the estimated strength of the next upcurrent to be encountered. This paper combines both of these concepts in a readily applied form ... " In order to do this, he re-established the relation already obtained by Dewing, but this time using a slightly different method, that is, by derivation of the time necessary to cover a unit distance, and not by derivation of the average cross-country speed. This makes it look as though he had never read Dewing's article, and consequently that he had never read Pirie's either, given that both were published in the same magazine. He never referred to either. But what may have inspired him most was Fox's idea, luckily recalled by Smith, which was to equip the variometer with an extra scale that enabled the pilot to pilot the best speed-to-fly.

MacCready's calculations led him to the following relation,

$$W + wt = vf'(v)$$

W =variometer reading

where

wt = thermal strength (author's note: climb rate) vf'(v) = some function of velocity

Using different notations, this relation is identical to the one obtained by Dewing. As we have already stated, this relation proves that for a given climb rate in the next thermal, the best speed-to-fly is a function of the variometer reading.

Hence, it is possible to equip the variometer with an extra scale that is situated next to the climb rate scale, and which indicates the best speed-to-fly corresponding to the total sink rate of the glider as indicated by the variometer. If the glider's speed corresponds to the speed shown on the second scale of the variometer, this means that the glider is flying at the best speed. If the glider's speed is, for example, slower than the speed indicated on the variometer, it must be increased, and vice versa. Since the procedure is convergent, the best speed-to-fly is rapidly reached through successive iterations (**Fig. 36**).

The additional speed-to-fly scale depends on the chosen rate of climb in the next thermal. Therefore one must have several specific scales for the different rate of climb. But providing that the climb/sink rate scale of the variometer be linear, the different speedto-fly scales can be deduced one from another by sliding the entire scale. This makes it possible to change the chosen rate of climb by the simple rotation of the ring supporting the scale. This is a point that MacCready had mentioned on his manuscript, "Note: 1 rotatable card will suffice if variometer has linear scale." (Fig. 37). Additionally, knowing that changing speed implies transitory flight regimes, he also mentioned the role of the total energy variometer. MacCready was ahead of his time.

Since his glider, an Orlik (Polish glider dated 1939), was equipped with a non-linear scale circular dial variometer, he had to attach different speed scales for the different climb rates in the next thermal to it. To do this, he made several interchangeable plastic cards from which he could choose the one corresponding to the anticipated rate of climb in the next thermal.

Thus, MacCready had succeeded in extending widely the scope of the Fox/Smith method, which was only effective for the climb rate zero in the next thermal. The success of the MacCready ring went way beyond everyone's expectations.

The Period 1947 – 49. Part 2: The Nickel Tangent and the Zientek Intersection

It looked as though everything had been said, but what followed is also very interesting.

Kalle Temmes (Fig. 38), the Finnish gliding champion (Kalle is a diminutive of Karl) who had been a fighter pilot during the war, published a very interesting article on the same subject in the March-May, 1949 issue of Finnish magazine Ilmailu. This article appeared before that written by Nickel, mentioned below, and before the July 1949 conference of the SSA where MacCready presented his theory. This article "Mikä on edullisin lentonopeus matkalennola" was subsequently translated into English and published in the American magazine Soaring in the January-February 1950 issue, having the title: "Finding the best speed for cross-country soaring." Thus Temmes had worked independently, certainly unaware of the articles that had appeared in the English magazine Soaring and Glider.

From the mathematical point of view, he limits himself to the formula that gives the average crosscountry speed depending on different parameters, this including, obviously, the thermal climb rate but also the air mass sink rate in cruise (Fig. 39). What is significant is that he fully understood the problem posed to the pilot. Having first demonstrated the same curves as Szukiewicz-Szwarc and Späte, which were valid when cruising between thermals in still air, he concluded his presentation by providing, for a given glider, a chart which, for each climb rate in the next thermal, gave several couples of the values for the best speed-to-fly and their corresponding variometer reading, calculated taking the air mass sink rate in cruise into account. It would only require engraving these optimum speeds on a card attached to the variometer for a MacCready ring to be realised. All the results were obtained using numerical calculations.

Probably for reasons of economy of space, since the chart gives results for several climb rates and several vario-speed couples, and at the same time gives the corresponding average cross-country speeds, Temme limited himself to two couples of vario-speed rates for each case, corresponding respectively to gliding in still air and to gliding in sinking air (Fig. 40). It is obvious that he could have listed as many as were necessary. Some of his simplifications, however, are questionable, due to his excessive linearization of certain functions. This does not, however, make his analysis any less inspired or reduce the pertinence of his judgement.

In 1942, Karl L. E. Nickel (Fig. 41) was called up and later posted to work for the Horten brothers. In 1944 Reimar Horten set him to work in particular on the study of best speeds for his flying wings, and on the development of the classic little charts to be taken on board during flight. It was at this time that he first became interested in average speeds. In May or June 1946, while he was still studying mathematics at the University of Göttingen (he completed his studies at the University of Tübingen in 1948), Karl Nickel discovered the graphic solution for determining the best speed-to-fly by constructing the tangent to the speed polar, taking into account the anticipated climb rate in the next thermal as well as the air mass sink rate in cruise.

In 1949, the future Professor Doctor Nickel, a great specialist in flying wings and among other things, in the theory of mathematical intervals, came across an article written earlier by the Swiss Siegbert Maurer (Sigi to his friends) that had been published in the Swiss magazine Aerorevue of November 1948, "Wo liegt die rationellste Geschwindigkeit für den Schnellflug" (What is the best speed-to-fly during a speed flight?), in which the author, having explained the results of his research on the best speed-to-fly through analyzing his barograph records, asked the question of how to determine this speed using theoretical means. In reply to this, Nickel rapidly published an article in the same magazine in June 1949, and also in the German magazine Thermik of October 1949, entitled "Die günstigste Geschwindigkeit im Streckensegelflug" (the best speed-to-fly in distance flight). The article is brilliant, clear, and easy to read, and its chief merit is that for the first time a graphical solution was published for determining the best speed-to-fly by constructing a tangent to the speed polar, taking into account the anticipated climb rate in the next thermal and the air mass sink rate in cruise. This tangent (**Fig. 42**) is drawn from the point on the vertical axis for which the ordinate is equal to the sum of the climb rate in the next thermal, and the air mass sink rate in cruise. The best speed-tofly is determined by the point of contact with the polar.

Unfortunately, Nickel did not publish the demonstration of his theory, probably not wanting to make his presentation even more cumbersome, but it is difficult to imagine that the demonstration had been geometric or using vectors, since he continues to determine the average cross-country speed through calculations and not by reading it on the diagram at the intersection of the tangent and the parallel horizontal axis of the ordinate equal to the air mass sink rate, as the Pole Zientek was to demonstrate only a short time later. It is reasonable to consider that Nickel had first of all established the algebraic relation determining the best speed-to-fly, the same as was independently established a little later on by Dewing and MacCready, and that he had then deduced the graphic solution from this, the scientific value of which being identical. Karl Nickel, in inventing the graphic solution using the tangent, a very elegant and practical method, had made a huge theoretical stride. From now on, everyone was to use this method. Paul MacCready himself was the first to do so, several weeks after the publication of Nickel's article, as we will see below.

Nickel made available to the pilot the following procedure for flying this best speed-to-fly: first, one must calculate the average climb rate in the thermal by dividing the total altitude gained, measured on the altimeter, by the climb time, measured using a chronometer. Then one has to use a slide rule with three fixed horizontal scales and a curser, called the "Thermikschieber" (Fig. 43), and note the rate thus found on the upper scale using the curser. On the middle scale the corresponding best speed-to-fly is indicated, which does not take into account the sink of air, and the pilot must then fly at this speed and observe the corresponding variometer reading. Finally, he must calculate the difference between this last reading, and the relative glider sink rate at this speed, which is indicated on the bottom scale of the rule, this difference being the air mass sink rate, and add it to the climb rate calculated and displayed at the start. He then must move the curser to the newly adjusted rate and fly at the new optimum speed indicated on the middle scale, which is the right one, with both the climb rate in the next thermal and the air mass sink rate in cruise taken into account. A significant amount of work for the pilot.

It should be noted, that in the eighth edition (1963) of the famous gliding handbook of Wolf Hirth, *Handbuch des Segelfliegens* (Handbook for glider pilots), Wolfgang Späte, the winner of the Rhön competition in 1938 already mentioned earlier, was responsible for writing the chapter "Best speed-to-fly." In this he repeats Nickel's text explaining how to use the "Thermikschieber". He also briefly mentions the MacCready ring.

Whereas the method invented by MacCready, a competition pilot, involved using information that the pilot actually possesses on board, in particular the variometer readings, the theoretician Nickel, in order to carry out his calculation, needed to know the air mass sink rate in cruise, which his flight instruments did not provide. He invented a complicated method for calculating this, in order to use it in flight.

How had the American MacCready become aware of the Nickel article so quickly, given that it had been written in German and published in Switzerland? He probably paid a great deal of attention to anything that was written on the subject. Had the graphic solution and the Thermikschieber wounded his pride? Whatever, a certain scientific rivalry became apparent. Their divergence of views was expressed on neutral ground: in Switzerland.

MacCready immediately published an article translated into German in the November issue of the same Swiss *Aerorevue* entitled "*Die beste Streckenfluggeschwindigkeit für Segelflugzeuge*" (the best speed-to-fly for distance flights), in which he developed his theory using the graphic solution revealed by Nickel, showing the chart upside down as was his habit. He provided a clear and precise demonstration of the tangent theory, the first one ever published, but most of all he described how his famous rotating ring really worked. He finished the article with this somewhat humorous phrase, translated for the Swiss magazine as follows : "... und der Pilot muss weder Tabelle noch Rechenschieber gebrauchen..." (and the pilot no longer needs a chart or a slide rule).

This article is the best ever published on the subject. Concise, complete, easy to read, everything is clearly explained, and there is nothing superfluous. MacCready handles the subject perfectly and, in so doing, provides a demonstration of his keen intelligence. It was only in 1954 that he published an article in English on the subject. "Optimum Speed Indicator" appeared in the March-April edition of the American magazine *Soaring*.

Karl Nickel, appreciating MacCready's discovery, was not to rest on his laurels. He had an article published in the German magazine Thermik in April 1950, entitled "Die MacCready'sche Ringskala" (The ring scale of MacCready) in which he explains in detail how the MacCready ring worked, just as MacCready had explained how the Nickel solution using the tangent method worked. At the same time, he made numerous references to his own invention, the Thermikschieber. He pointed out one of the disadvantages of the MacCready ring, that is, that it does not allow the pilot to know the average speed achieved - a parameter that he considered as essential for navigation - and which, it is worth saying, was also one of the drawbacks of his Thermikschieber. For this reason, he then produced a super-Thermikschieber, with five horizontal scales plus two cursers. Four of these scales were fixed, and the last one could slide in relation to the four others. This equipment was presented as being a complement to the MacCready ring for calculating the average cross-country gliding speed. It involves a complicated procedure.

The test pilot Adam Zientek (Fig. 44), who since Samedan was one of the pillars of the Polish Gliding Team, and who was to remain such for a long time, rewrote the results that had been published a year before by Nickel and MacCready in the March and April issues of the Polish magazine *Skrzydlata Polska*, and added the effect of the wind for cases where optimization was in relation to the ground. And in particular he published for the first time the graphical solution, which has since been used a great deal, making it possible not only to determine the best speed-to-fly, but also the average cross-country speed, which is equal to the abscissa of the point where the Nickel tangent crosses the horizontal line for which the ordinate is equal to the air mass sink rate in cruise (Fig. 45). He provided an elementary geometrical demonstration of this. The more elaborate demonstration, as presented in the chapter entitled : "A Bit of Technical Information", was not developed until later. Zientek's article "Zastosowanie biegunowej szybkosci w lotach wyczynowych", which we attempt to translate as "Choosing the best speed-to-fly for performance flights," is very thorough. It includes numerous graphical explanations and, in particular, quantifies the influence of altitude. Unfortunately, Zientek's contribution was not known to the entire gliding world at the time of publication.

Five years later, in 1955, on page 216 of the Swiss magazine Aerorevue, Nickel published the article "Die günstigste Geschwindigkeit im Wellensegelflug" (the best speed-to-fly in wave flights). In this article, he extends the tangent method to wave flights, using the graphic method for determining the average speed. He too, in the meantime, had discovered this solution, independently from Zientek. He wrote: "Für mich war es daher ein besonders und freudiges Erlebnis als ich (...) herausfand, dass man die optimale (mittlere) Reisegeschwindigkeit nach der Tangente-Konstruktion direkt als Schnittpunkt dieser Tangente mit (...).ablesen konnte." (For me it was an exceptional and uplifting experience to discover that using the tangent construction, one could then read directly the average speed at the intersection of this tangent with).

50 years later

Modern electronic calculators provide a visual or acoustic signal during flight, which depends on the difference between the current speed and the best speed-to-fly, making it possible for the pilot to be able to pilot at the best speed by iteration.

Nowadays, it is possible to manufacture high performance electronic equipment that displays the bes speed-to-fly, taking into account the air mass sink rate in cruise and the climb rate in the next thermal (MacCready positioning), as well as the glider's altitude. This display could be incorporated into the airspeed indicator, which, for example, could be equipped with two easily identifiable pointers indicating on the same dial the glider's speed and the best speed-to-fly. Thus, the pilot would be able to assess the situation at a glance and only have to consult one instrument instead of two.

It would be good but much more complicated if this instrument were to also take dynamic effects into account, which are not negligible when the wind blows in gusts with important wind gradients. It should be noted that the present total energy variometers take into account the airspeed of the glider (which depends on gusts), and that for this reason, they are systematically wrong because out of phase when gusts blow. If, for example, the indication of the total energy variometer suddenly increases due to a gust coming from the front, it does not mean that the total energy of the glider has increased. Only the airspeed and consequently the "wrong total energy" variometer indication have increased, and as soon as the gust has disappeared, the apparent gain also disappears, unless in the mean time the pilot has made an appropriate manoeuver creating a dynamic gain. He should have modified the component of his speed in the direction of the gust, so as to oppose his inertia to the gust and move with it. For example, in the above case, he should reduce his speed when flying in straight line or make a turn in order to reduce the component of his speed in the direction of the gust. The development of an exact variometer requires the installation of an inertia platform or a very accurate positioning system.

For those interested, the excellent article of Dr.-Ing **E.D. Dickmanns** presented at the XVII OSTIV Congress in 1981 is suggested, in which the problem of a simple vertical gust is solved (thermal crossing), provided that the atmosphere is stationary. The general problem, taking multiple three-dimensional gusts and a nonstationary atmosphere into account, has not yet been solved.

However, progress will never cease

Conclusion

The best speed-to-fly is determined by the following relation: "the algebraic sum of the climb rate in the next thermal, the air mass sink rate in cruise, and the relative glider sink rate is equal to the result of multiplying the glider's best flying speed-to-fly by the absolute value of the derivative of the speed polars at the optimum speed point"

This fundamental relation had been established independently by three authors (known to date): the German Nickel, who established it in May-June 1946, the Englishman Dewing, who published it in June 1947, and the American MacCready, who included it in his handwritten notes in October 1947.

The true inventors of the speed-to-fly theory are as follows:

Szukiewicz-Szwarc, and Späte, for the best-speed-tofly in still air, depending on the expected climb rate in the next thermal.

Fox, for his method of piloting the best glide ratio speed using an extra speed scale added to the variometer,

Pirie for the equivalent role played on the best-speedto-fly by the climb rate in the next thermal and the sink rate of the air mass in cruise.

Dewing for the relation determining the best-speed-to-fly,

MacCready for the same relation, and for his rotating ring

Nickel, also for the same relation, and for the graphic method using the tangent to the speed polar to determine the best-speed-to-fly,

Zientek and later Nickel for the average cross-country flying speed determined by the intersection of the tangent.

The gliding community as a whole should be grateful to all the authors cited above for their significant contribution to the development of modern gliding.

This study illustrates the important role played by communication technology, as this allowed several persons to take over from each other successively in order to be able to develop and devise a new discovery. MacCready's invention was stimulated by Neubroch and Smith's contributions, which were made known through the distribution of the magazine *Soaring and Glider* in the United States and through the use of a common language.

There is no doubt that nowadays, the current explosion of means of communication will lead to an increase in scientific developments.

Acknowledgements

We would very much like to thank all those who have helped us in our research for documentation, in particular Mr. MacCready in the United States, Mr. Kahn, Prof. Edwards, Mr. Wills in Great Britain, Mr. Sandauer and Mr. Glass in Poland, Prof. Nickel and Mr. Selinger in Germany, as well as the editors of the Polish magazine *Skrzydlata Polska*, the Finnish magazine *Ilmailu*, and the Swiss magazine *Aérorevue*, and also the libraries at the German museum in Munich, and at the Elmira museum in the United States. Particular thanks also go to Helen, Doris and Barbara for their linguistic expertise, and to Prof. Mark D. Maughmer, who supervised the whole text of the paper.

Prof. Edwards of Cambridge University who made a comprehensive study on the same subject was kind enough to make information available for us.

If there are any authors we do not know, who have contributed in an original manner to the development of the above concept, and who have not been cited in this study, we would like to apologise for this oversight, and request that they make themselves known at the following e-mail address: <u>avia40p@aol.com</u>.

We would also like to thank in advance anyone who makes known to us any errors that might have slipped into our text.

The text of this study is also available from the author in German and French.

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Fig. 1. John Montgomery

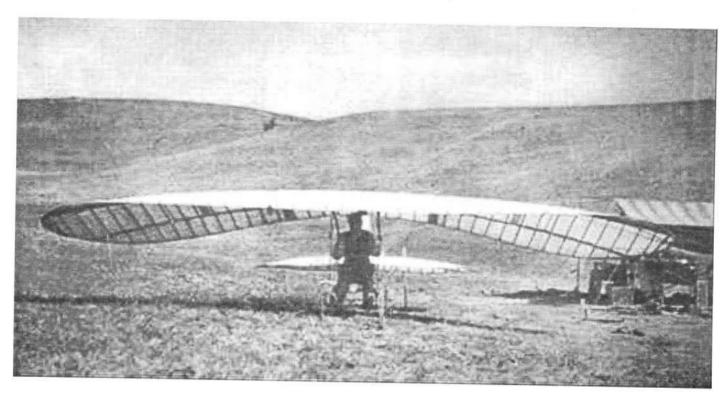


Fig. 2. Montgomery's glider



Fig. 3. Otto Lilienthal

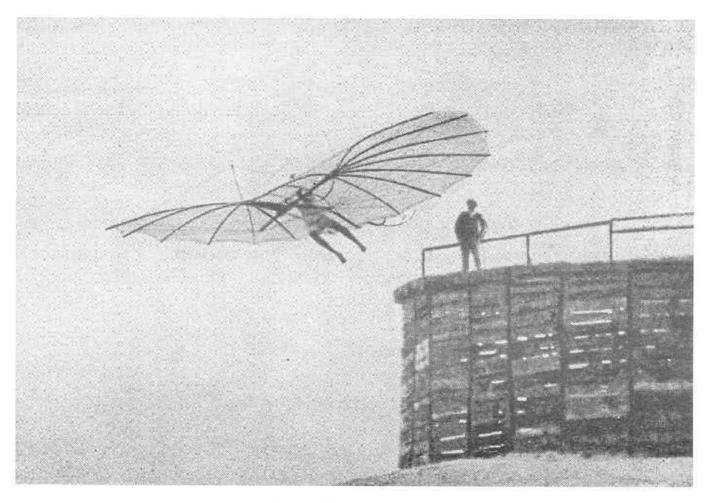


Fig. 4. Lilienthal in flight



Fig. 5. Percy-Sinclair Pilcher

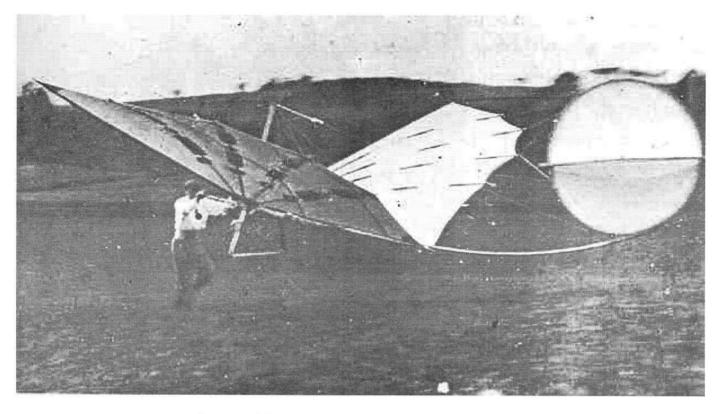


Fig. 6. Pilcher's Bat glider with added stabiliser



Fig. 7. Hans Gutermuth

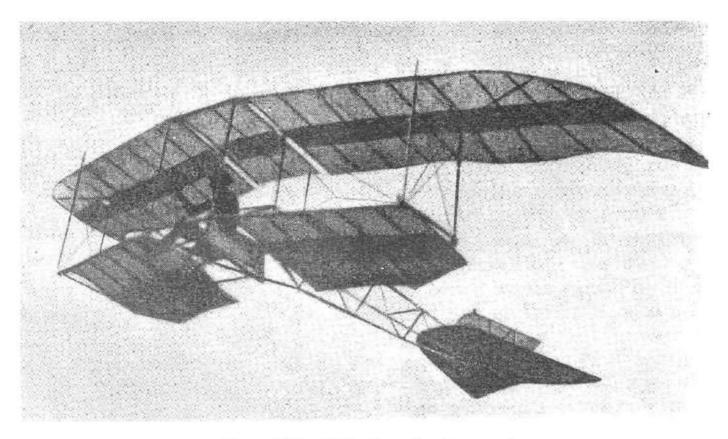


Fig. 8. Glider FSV-8 flown by Gutermuth



Fig. 9. Louis Mouillard



Fig. 10. Pierre Idrac



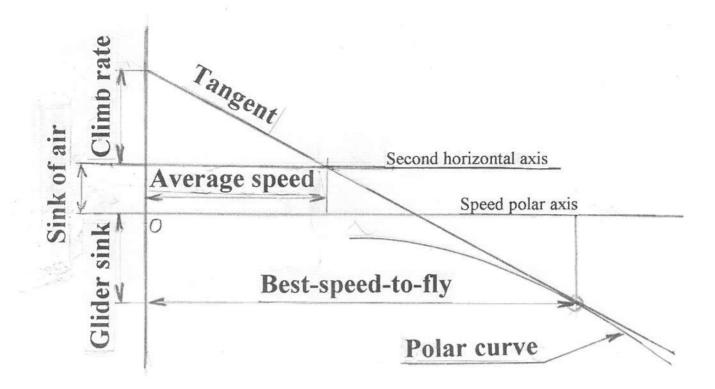
Fig. 11. Glider Thomas I and its variometer

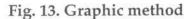


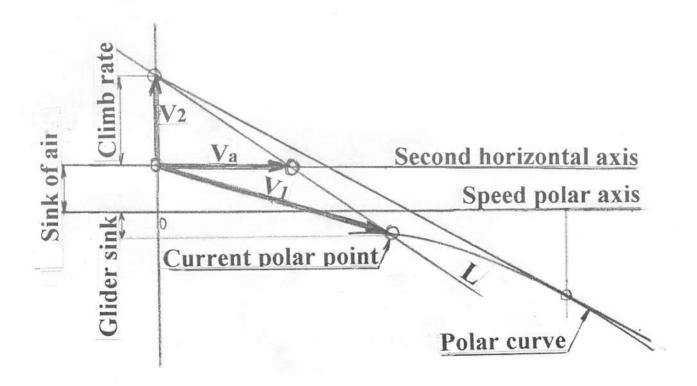
Fig. 12. Jean Hemerdinger



Fig. 12a. Robert Kronfeld







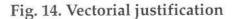




Fig. 15. Romuald Szukiewicz



Fig. 16 Pilot Szukiewicz

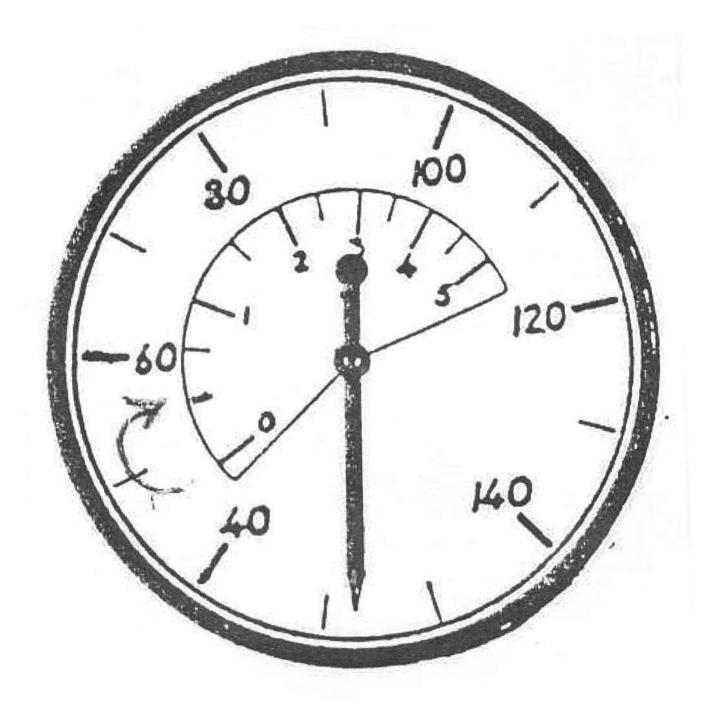


Fig. 17. Double scale on an ASI

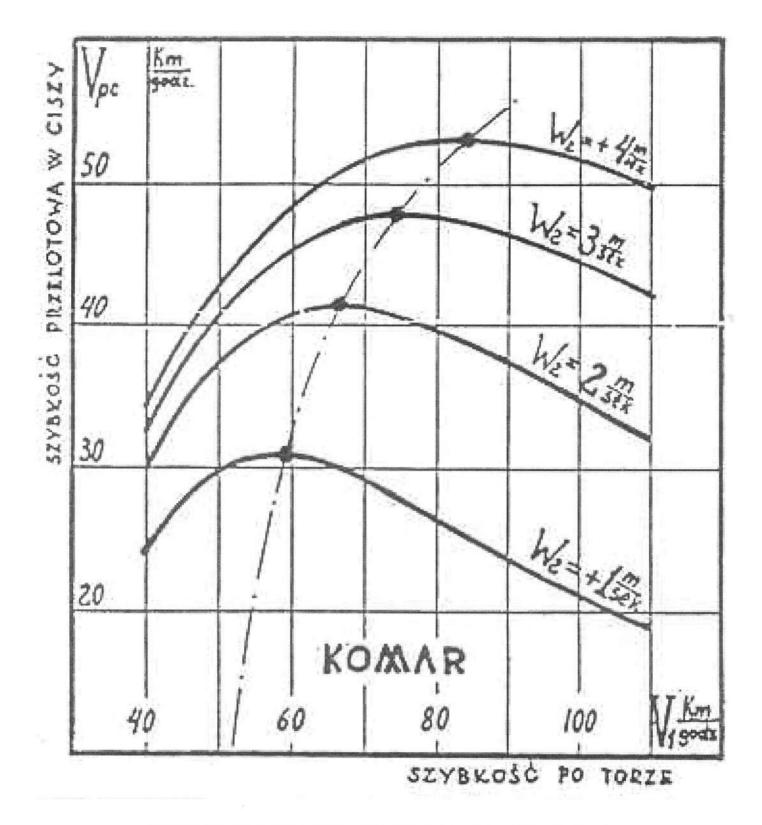


Fig. 18. Komar av. speed fn. of cruising speed for diff. climb rates



Fig. 19. Wolfgang Späte

Der Korpsfuhrer des AS-Aliegerkorps

Berlin W 15, Meierottofir, 8/9



Rhön - Segelflug - Wettbewerb 1938 (19. Rhön) Jielftrecken - Segelflug - Wettbewerb 1938 des Nationalfozialiftischen Fliegerkorps

Flugberichte

Fig. 20. Späte flight report

Diele Berichte werden als Sonderausaabe durch den Korpsführer des NS-Fliegerkorps herausgegeben undlind nicht kauflich zu erwerben.

Veröffentlichungen — buch auszugsweile — lind nur mit besonderer Genehmigung des herausgebers gestaltet.



Publication only with autorisation

Fig. 21. Secret report

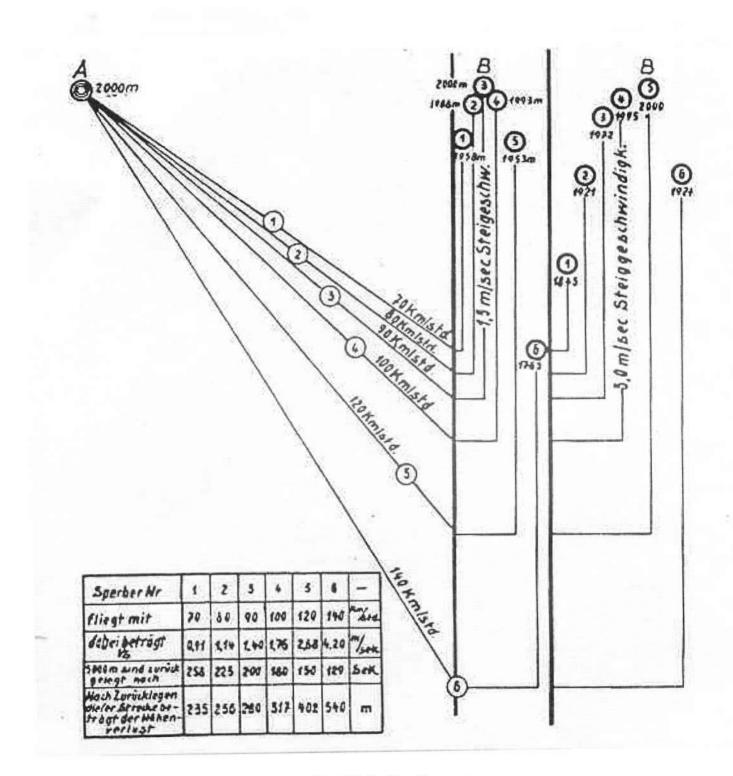


Fig. 22. Späte diagram

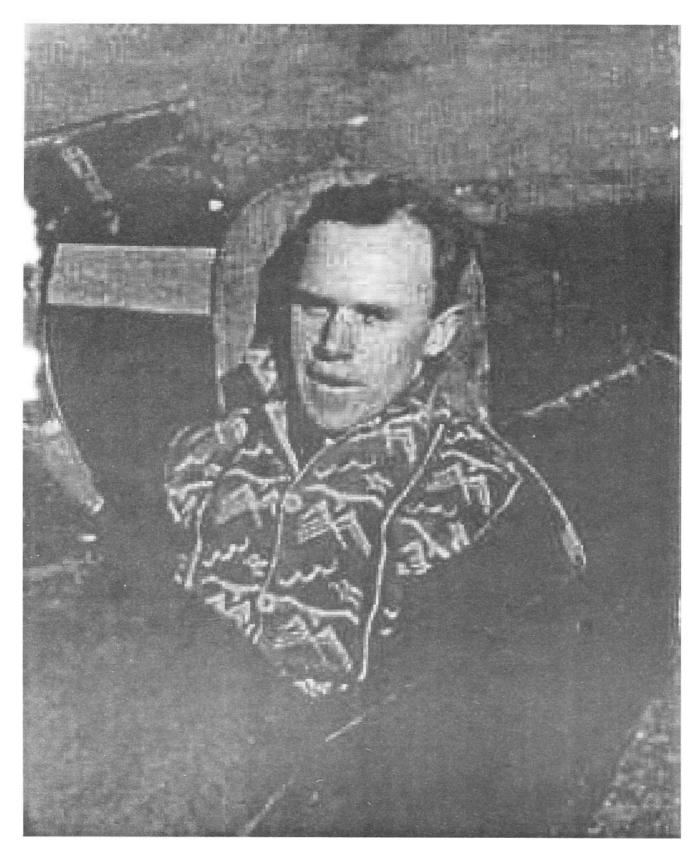


Fig. 23. John Eliot Sylvanus Fox



Fig. 24. Fox and Murray in a Falcon III



Fig. 25. Witold Kasprzyk

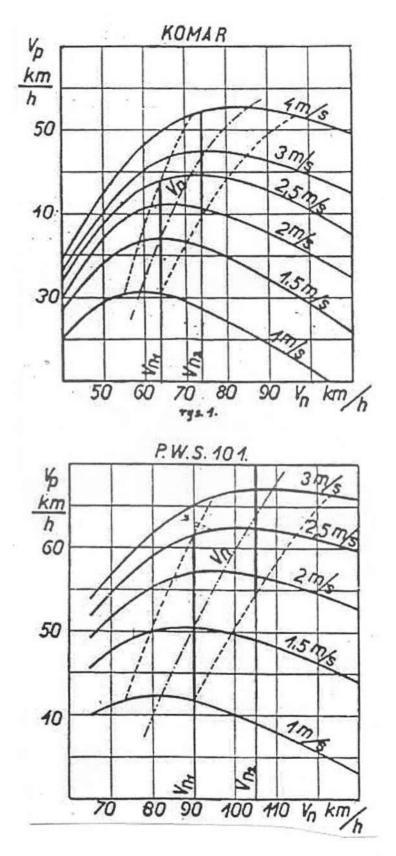


Fig. 26. Av. speed fn. of cruising speed for diff. climb rates

Oct. 26, 1947 Jop Secret ainspeed calculation Oct. 26, 1947 NE Nav= av. rue AT in themal NS 6 Iplane to go mit distance Fo requires A=alt. lat A N ÷ No + NS NS -Nor + Nos + ---e(v -+ NS

Fig. 27. MacCready's notes



Fig. 28. Flt. Lt. Neubroch

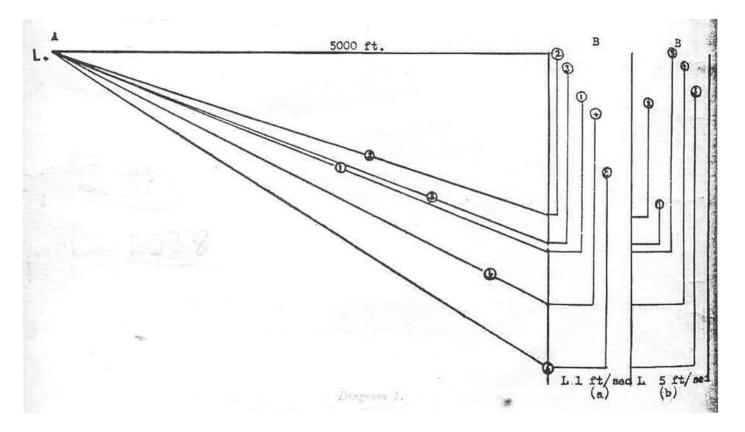
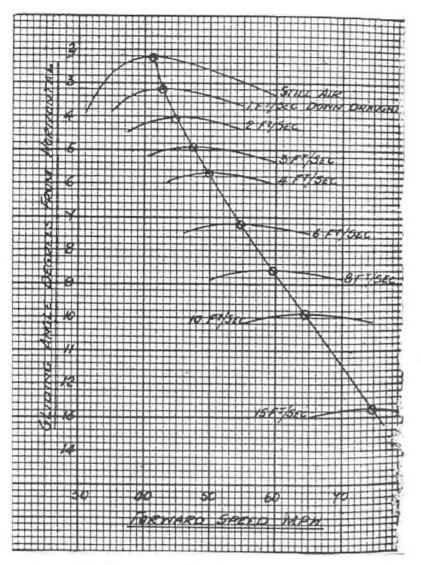
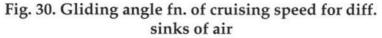
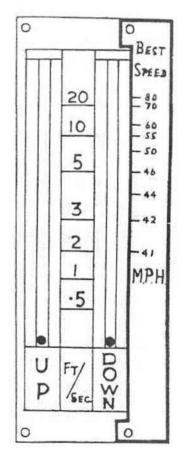
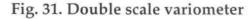


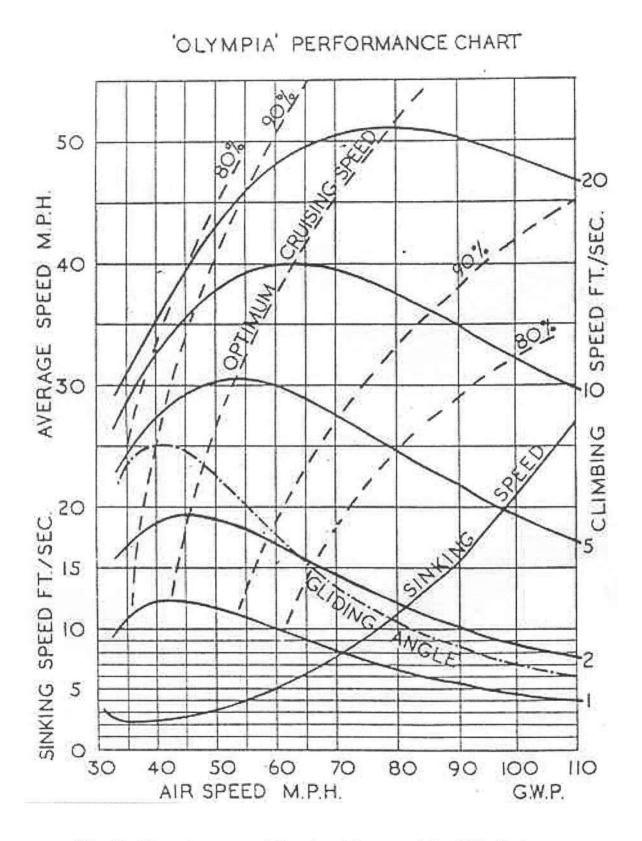
Fig. 29. Neubroch diagram similar to Späte diagram











Fiig. 32. Olympia av. speed fn. of cruising speed for diff. climb rates

44



Fig. 33. Paul, 12 years old



Fig. 34. Glider pilot MacCready

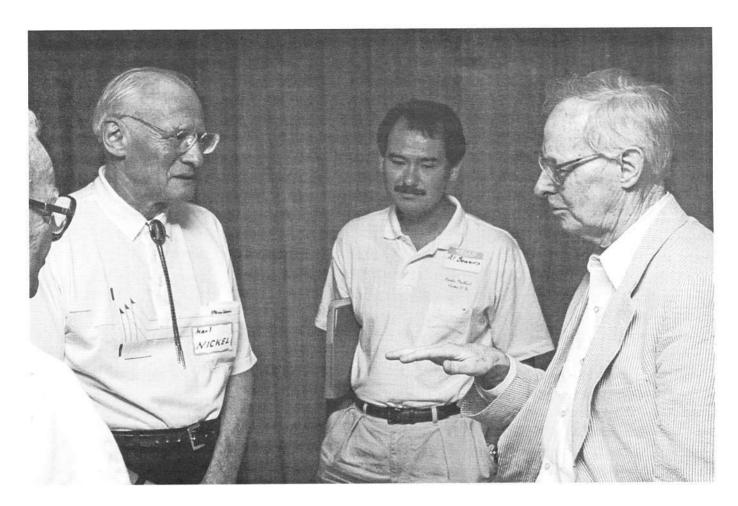


Fig. 35. MacCready (right) & Nickel (left)

(ray w = 0, 5, 10, 20, 24.) CARD FOR Wt=6F **Orlik** glider VARIOMETER (f.p.s.) mat UD V should = 71 m.p.h. 0 Dow when W= (variometer reading) = 4.3 f.p.s. 66 20 15 10 70 91 86 77 80

Fig. 36. Original MacCready ring

Note: I rotatable card ruffice if To lay line

Fig. 37. MacCready's note



Fig. 38. Kalle Temmes

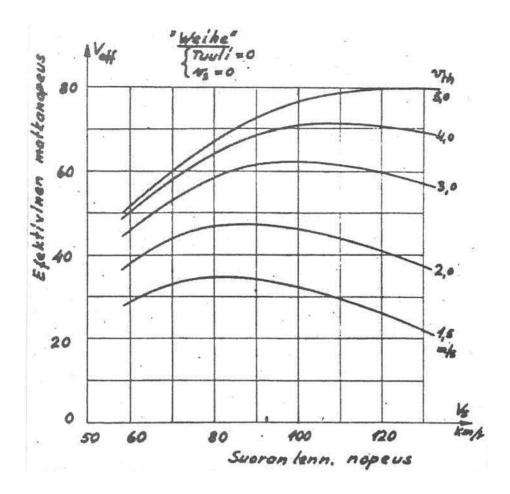


Fig. 39. Weihe av. speed fn. of cruising speed for diff. climb rates

Termikisso	. Suona kento		
Variometri m/s	Variometri m/s	Vopt. km/h	Veff km/h
+1,0	-1,0	~80	40
	-2,8	~ 90	27
+2,0	-1,3	~ 90	55
	-2,7	~ 100	42
+3,0	-1,8	- 100	63
	-3,1	~110	53
+4,0	- 2,2	~+10	.70
	-3,4	-115	62

Fig. 40. Olympia chart

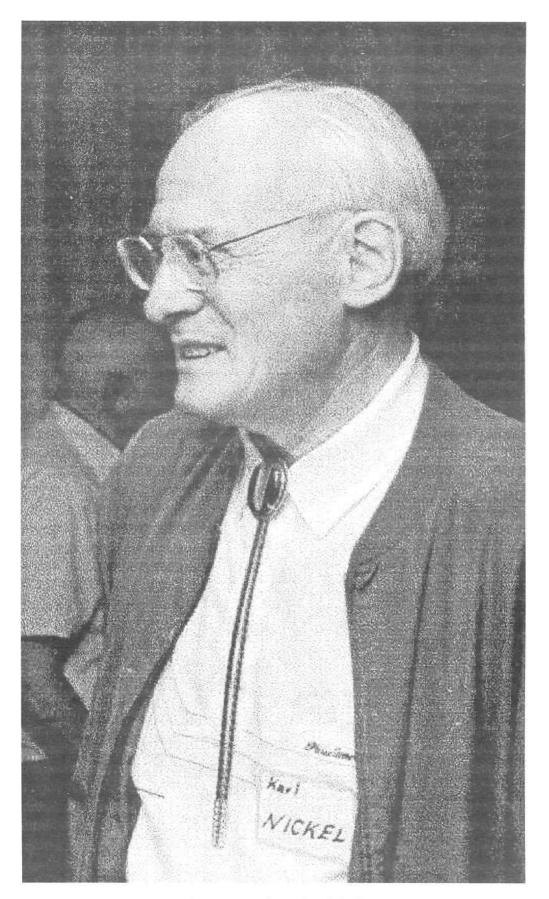


Fig. 41. Prof. Karl Nickel

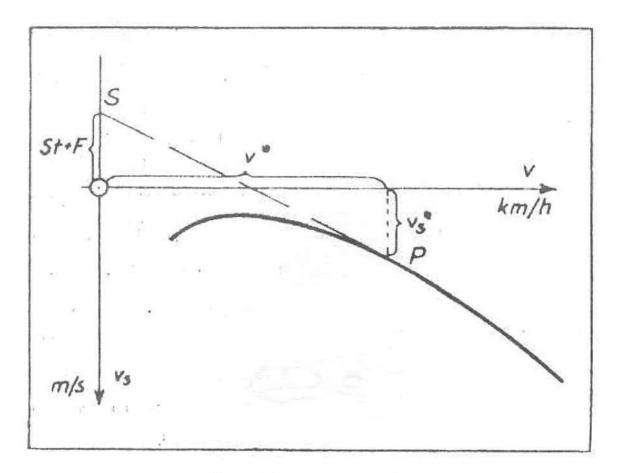
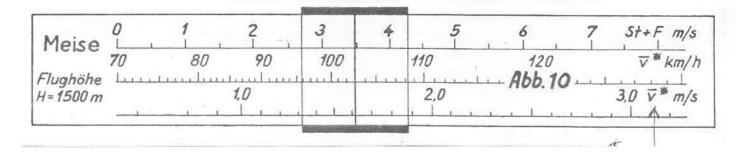


Fig. 42. Tangent construction



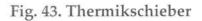




Fig. 44. Adam Zientek

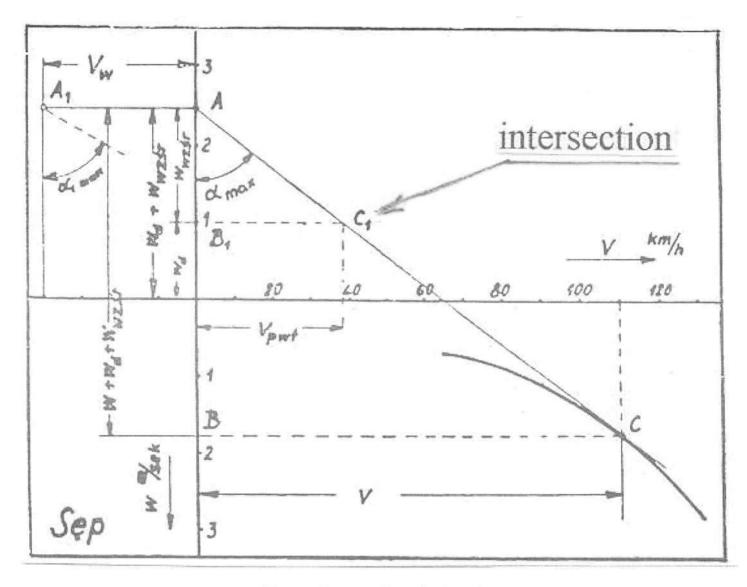


Fig. 45. Intersection of Zientek