Data Mining for Atmospheric Gravity Waves (Lee Waves)

Alfred Ultsch[1], Joachim Curtius[2] and Christof Maul[3]
ultsch@mathematik.uni-marburg.de
[1]*Data Bionics*
Philipps-University Marburg, Marburg, Germany
[2]*Institute of Atmospheric and Environmental Sciences*
Goethe-University Frankfurt, Frankfurt am Main, Germany
[3]*Institute of Physical and Theoretical Chemistry*
Technische Universität Braunschweig, Braunschweig, Germany

Abstract

Gravity waves can emerge as a result of the perturbation of atmospheric circulatory systems. They encompass periodic, yet geographically stationary, changes in temperature, pressure and vertical wind component. Occurrence of such waves is frequent if strong winds hit high mountains. Secondary effect of such waves may also be encountered as clear air turbulence (CAT) in commercial flights. Atmospheric gravity waves strongly influence weather phenomena and on a larger time scale climatic processes. They are responsible for the vertical transport and mixing of air from the stratosphere up to the mesosphere. First results from research flights in the Pyrenees during the spring 2015 measuring campaign are reported. Several flights with a sensor equipped unpowered glider in altitudes between 2000 and 7000m were undertaken. Data Mining and Knowledge Discovery methods were applied. The results point to interesting patterns (states) in the structure and formation of lee waves and lead to the understanding of such flights as Wave Track Flight scenarios.

Nomenclature

g	Acceleration of gravity, 9.81 m/s^2
h	Altitude
FL	Flight Level
GPS	Global Positioning System
k,i	indices of measurement points, with $k > i$
m	Average molecular weight of air
p^{pitot}	Total pressure
p^{stat}	Static pressure
R	Gas constant, 8.3144 J/(mol K)
t	time
T	Static temperature, absolute
u^{gross}	Gross vertical speed of a glider relative to ground
u^{net}	Net vertical speed of air-mass flow
u^{polar}	Vertical sink speed of a glider in still air
u^{stick}	Vertical speed of a glider induced by stick movement
v_{ne}	Never to exceed speed
v_i^{TAS}	True airspeed

Introduction

By far the longest and fastest soaring flights are nowadays reached in gliders flying in Lee Waves. For example, Klaus Ohlmanns 2010 world record flight over a distance of 1608 km and an average speed of 123 km/h was achieved in the lee waves of the Andes [1]. While much is known in principle about lee waves (see [2]) there are several open issues.

One of the most important points for the usability of lee waves for efficient long distance gliding is the leeward distance of the first upward lift from the wave generating obstacle. This certainly depends on the strength of the wind and other atmospheric parameters. Measured time series for parameters, such as temperature, dew point, density, wind, equipotential temperature, and moisture in the upward section of the wave vs. the rotor, respectively the air below the rotor, are still unavailable. As with the fine structure of thermals [3] there is a substantial lack of measuring time.

In this work we address the measuring of atmospheric parameters using low cost measuring equipment on low cost (zero cost) flights, i.e. the Open Glide Computer in non-profit, non-commercial (NC2) glider flights. For such measurements swarm data mining methods [3] are necessary to reduce errors from low-cost sensors and pilot as well as aircraft biases. The data presented in this work will show that

1. it is possible to distinguish between the main phases of flight, for example, wave distance gain, wave altitude gain, and rotor flight, by post data analysis, and
2. that the effective atmospheric vertical air speed in lee waves can be extracted from such data.

Fig. 1: Lee waves in the Pyrenee mountains

Classical Measurements

In early times when our modern ground based measurement devices were not available and aircraft less powerful than today, sailplane based data acquisition was largely used in field experiments for the exploration of lee wave phenomena. This is particularly true for the very first experiments performed in the 1930s in the Riesengebirge mountains [4, 5] and for the first Sierra Wave mission in the 1950s in the Owens Valley in East California [6].

The Sierra Wave experiment has been reviewed some 10 years ago by Grubišić who reports that "in the 1951–52 field project the main observational platform was a sailplane. The research fleet consisted of 2 two-seater Pratt-Read sailplanes, equipped with a clock, an altimeter, indicators for the rate of climb, airspeed and direction (compass), an accelerometer, an outside (fuselage) thermometer and a barograph. In order to produce a continuous record of the flight data, the instrument panel was photographed at 1 or 2 sec intervals on 16 mm film by two cameras in the rear of the cockpit. .....this system afforded the Sierra Wave Project researchers a continuous record of sailplane flights for the postanalysis. The total flight time of sailplanes was limited to 4.5 h by the oxygen supply, and the tracking operation was limited by the film length to 1.5 h." [6].

Later, the significance of sailplanes as measuring equipment carriers has decreased owing to the availability of more sophisticated equipment like Lidars, scintillometers, Doppler sodars, radar wind profilers, microbarographs, and radiosondes. The last systematic active participation of sailplane in such a measurement campaign took place during the French-Spanish PYREX mission in the early 1990s [7–9].

Major setbacks in the employment of sailplanes are that their flight path cannot be chosen at will, but is highly dependent on (or rather dictated by) the meteorological conditions as well as on the skill of the pilot and that the availability of space and energy severely limit the type and number of measuring devices that can be supported. Several other major studies of lee wave phenomena like the ALPEX and MAP missions in Europe and the T-REX follow-up mission of the Sierra Wave project have therefore relied exclusively on the combination of ground based and powered aircraft data acquisition [10, 11].

Yet in quite another context, the sailplane has just recently experienced a renaissance as a means of data acquisition. The PERLAN project aims at reaching stratospheric altitudes in motor-less flight [12, 13]. Here, in-flight data have necessarily to be acquired (and ideally processed) by the sailplane equipment with the limited instrumentation available. Some recent work has been devoted to addressing the question how three-dimensional wind data can be extracted from the standard instruments available in the sailplane itself [14, 15].

Glider Based Measurements

In this paper we would like to advocate the use of sailplanes for a continuous monitoring of wave phenomena. While dedicated missions like the Sierra Wave or the T-REX missions have provided a wealth of information about mountain wave systems that cannot be compared to what would be possible by an isolated experimental approach, the major drawback of such large scale experimental missions are their huge organizational and financial efforts and that a continuous monitoring is not possible.

Fig. 2: Open Glide Computer

Sailplanes on the other hand are low-cost devices that are nearly ubiquitously distributed all over the world and fly the wave whenever it occurs. They may be equipped with state of the art miniaturized measurement equipment in accordance with the space and energy available such that the technological gap that has evolved in the last decades between powered aircraft based measurements and sailplane based measurements has become much smaller.

In addition, it should be remembered that sailplanes also do have genuine advantages over the much heavier powered aircraft. First, quoting Grubišić again, "sailplanes were perfectly suited for measurement of vertical velocities because, due to a much smaller wing loading, they were capable of responding to wind gusts within seconds or within horizontal distances in the order of 50 m. For powered aircraft, depending on the wing loading, that distance was closer to 500 m", [6]. Therefore, sailplanes are able to provide much finer spatial and temporal resolution of in-flight information than powered aircraft. While light motor-gliders or unmanned aerial vehicles might provide a similarly fine resolution, they suffer from other drawbacks as e.g. limited range or duration of operation.

Grubišić also states that "rotors, with their high degree of intermittence and small spatial scale, are very dangerous and difficult to sample using in situ aircraft measurements.", [6]. Indeed, rotors pose a severe danger to aviation and consequently any pilot not involved in a meteorological measurement campaign aims at avoiding any rotor contact. For sailplane pilots flying in the wave, however, using rotors as wave entry points is a standard procedure, so they can easily provide fine scale data of those parts of lee wave phenomena which are hardest to study.

Algorithmic Identification of Waves vs. Thermals

To the best of our knowledge, an algorithmic discrimination has been only published so far by Ohrndorf and Ultsch [16, 17]. The approach was to train a so called Artificial Neural Network (ANN) on several flights which contain thermals, as well as flights in lee waves. The ANN was a supervised neural network of the multilayer perceptron type and was trained using the backpropagation algorithm. The flights were classified by an expert pilot into 5 classes, three of which were "ridge", "thermal" and "wave". A Bayes Classifier, comparable with a Kalman filter, had a classification performance around 80% [16]. In [18] finally a trained ANN had a classification accuracy of more than 98%. This work proves the principal feasibility of an automatic classification approach, given enough and accurate data on lee waves are available.

Low Cost Measuring: the Open Glide Computer

With the advent of small yet very powerful all-in-one computers such as the Raspberry Pi or the Arduino nowadays the prices for such computers are $<<$ 100 EUR. A tremendous amount of low cost, yet powerful and reasonably accurate, sensors could be observed on the market. Driven by unmanned autonomous airborne observation bases, i.e. do it yourself drones (DIY Drones), the prices of such equipment dropped tremendously over the last years. The Open Glide Computer is an Open Source project to build a data logger for gathering meteorological and flight data for mountain wave research [19]. The hardware is built around the Raspberry Pi, a credit card sized computer running Linux. Included are pressure sensors for static and dynamic air pressures, a high resolution GPS, a real time clock, an outside air temperature probe, 3-axis gyroscope (L3G4200D), 3-axis accelerometer (ADXL345), 3-axis digital magnetic compass as well as air temperature and humidity sensors. The source code, including the board layouts, is available online for public use [19]. User Interfaces are a rotary encoder with push button for input and as output a 320x240 pixel 2.2" TFT. The IO part is designed such that it fits into a 57mm instrument hole for a typical glider instrument panel. The actual on-line measurements are combined into a pseudo IFR-flight-display, including an artificial horizon (see Fig. 2, top picture right side). Pure hardware costs are presently below 500 EUR and still dropping. Parallel to the Open Glide Computer nowadays gliders are equipped with a GPS recording system (logger) which produces so called IGC files containing the on-board GPS systems fixes in a special format encrypted to ensure data integrity [14]. However, standard recording intervals are only in the order of one to several seconds, and with the exception of a static pressure sensor most

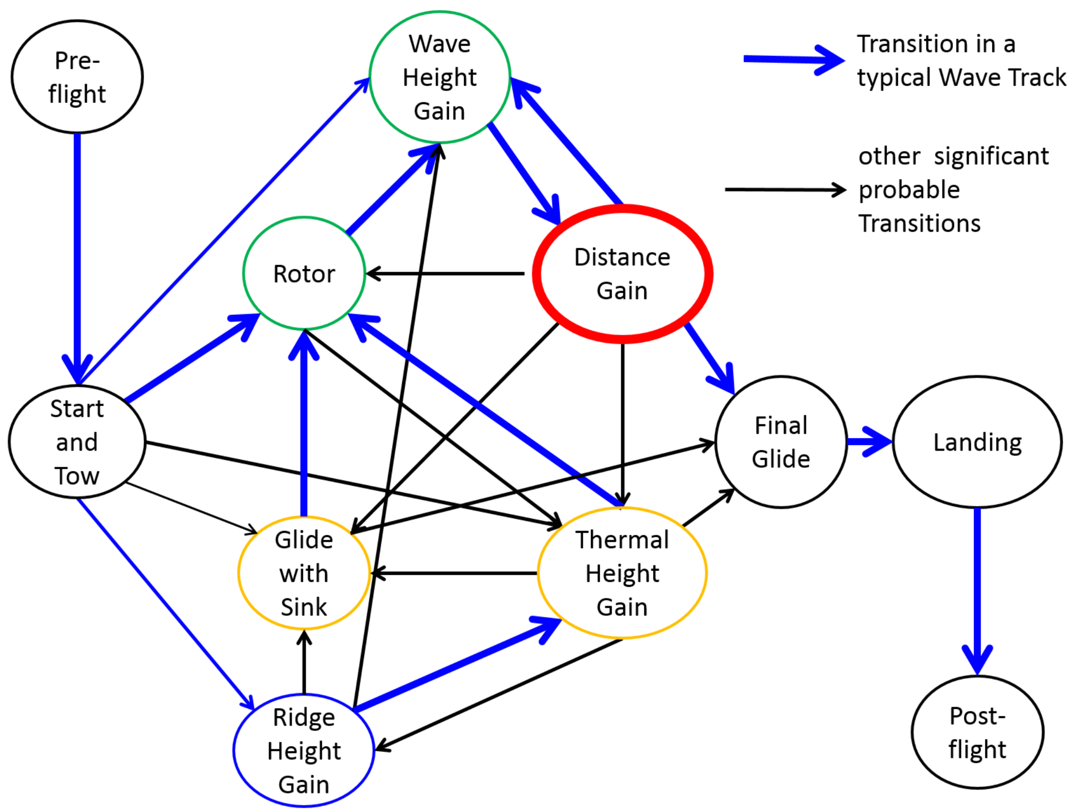

Fig. 3: States in a Wave Track Flight as Hidden Markov Model (HMM)

of such devices are not equipped with any of the above listed sensors.

Data Mining Methods

All the data considered above was not collected with the aim to measure atmospheric parameters in Lee waves. If several gliders fly the same area with the same goal in mind - to go as efficiently as possible on a large cross country flight, this is a typical example of so called ”Swarm Data”. A swarm of pilots in high performance sailplanes were sent out to find the best thermal or rotor lift, center as efficiently as possible and try to find the best possible climb in Lee Waves. The aim of each member of this swarm is to fly the longest distance or the fastest average speed at that day. This can only be done by making the best (i.e. most efficient) use of every meteorological situation. So we do not possess planned measures. However, we can assume that the pilots fly as best as they can.

The data gathering is not done for measuring, however, the ”need for speed” will urge each pilot to efficiently find thermal or rotor lift, then wave lift and then go along the wave track flight as efficiently as possible. In order to do so he or she must use the flight, respectively meteorological states (see next section), as efficiently as possible. Individual biases are, for example pilot performance, different search and usage strategies of the flight states, different gliders with different performances and, of course, different measurement calibrations. All this must be compensated by swarm data mining methods. See [3] for a successful example of the application of such methods for the fine structure of thermals using data from gliding competitions.

The data was analyzed using the R software (version 3.2.1 for Linux; http://CRAN.R-project.org/), in particular the CRAN packages ABC analysis, and Adapt Gauss, and Matlab (MathWorks, Natick, MS, USA) software packages in particular the databionics tool box (dbt) [20].

Wave Track Flying States and Scenarios

A primary aim of the analysis is to see differences in the atmospheric parameters, for example, in the uplift part of a Lee wave vs. the downswing part or the rotor vs. thermals. Here we define for the first time systematically different states of a wave track flight. This definition serves as the basis of a state based flight analysis (time series analysis) using Hidden Markov Models (HMM) [21], also called Kalman Filters [22]. A typical wave track flight will follow this scenario (see the blue arrows in Fig. 3).

From Preflight and Start the pilot will normally enter a rotor for further height gain. Alternatively he might aim for the windward side of a mountain in order to gain height in ridge soaring mode. Along this way some sink may occur. If ridge soaring has reached its top altitude the pilot will enter the Rotor eventually. On the rising side of the Rotor the pilot will (hopefully) gain altitude so he or she can fly upwind into the upward swing of the Lee Wave - Wave Height Gain (WHG). In the WHG state the flight will usually gain a first top altitude as a preparation/prerequisite for the next flight state: Wave Distance Gain (WDG). Ideally, once the glider has obtained the desired cruising altitude, further flight is done in straight flight such that the altitude is maintained. This can be achieved by adjusting the horizontal speed of the sailplane in such a way that wave lift will just compensate for the sink rate of the sailplane. The track of the sailplane is then aligned with the line of maximum wave lift, and the gross vertical speed is zero while the horizontal speed is variable. The variation of wave lift in the laminar flow of the wave is smooth and occurs on timescales of tens of seconds.

In strong wave conditions it can occur that the sink rate of the sailplane cannot be made large enough owing to having reached the maximum speed of the sailplane, so that instead of further increasing the speed the flown track needs to be detuned from the line of maximum wave lift. Now all speeds have fixed values: the horizontal speed of the glider is its maximum speed v_{ne}, wave lift has the same magnitude but opposite sign as the sink rate of the glider traveling at v_{ne}, and the gross vertical speed is 0. These two only slightly different flight modes occur most likely when airspace regulations forbid further height gain, a situation which is not uncommon under good wave conditions.

While Wave Distance Gain (WDG) is the most desirable state from the point of view of a pilot all the flights are limited by regulations which usually forbid glider flights at night time. So the final states of a typically wave track flight are Final Glide, which can be well over 100 km distance, Landing and the Post flight phase with the glider on the ground. A flight is only termed a wave track flight (WTF) if WDG is achieved for a substantial proportion of the total flying time. Wave height gain mode represents flying towards the next turn point, but in contrast to the distance gain mode discussed before, now the speed of the glider will be fixed to the speed of minimum sink, and the gross vertical speed of the glider will be the meteorological wave lift diminished by the relatively small minimum sink rate of the glider. Of course the height gain mode is also needed when altitude needs to be gained before attempting to cross to another wave system or, most often, after having crossed to another wave system.

All speeds of the glider and the air will undergo large changes on rather small time-scales of a few seconds only: horizontal and gross vertical speeds of the glider, vertical speed of the air and sink rate of the glider.

Data

Data was gathered in the Cerdanya Research Camp (CRC) measuring campaign in 2015 [23]. The CRC is organized and supported by the academic flying group of Frankfurt Main, Germany (Akaflieg) [24]. While several IGC files from different flights and days were available only the measurement sets from the Open Glide Computer of one glider was available in 2015. A swarm data analysis, however, allowed to geo-referentially locate the positions of the wave lifts in the Spanish Pyrenees (yellow circles in Fig. 4). The presumed locations of the wave triggering obstacles were assumed as the highest windward watersheds of the mountain ridges (red circles in Fig. 4). These positions were extracted on n=6 IGC files of different glider flights. The wind was estimated from prediction and calculated from the Open Glider Computer (OGC) log. OGC logged data were n= 31300 data points (cases, fixes) in d = 25 different time series (variables, dimensions).

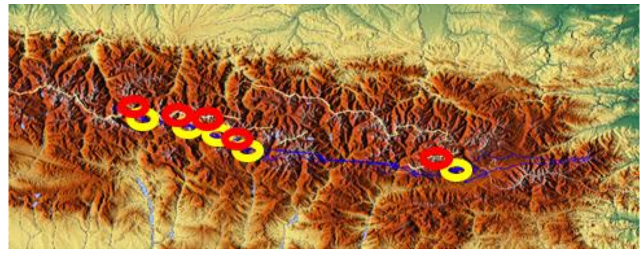

Fig. 4: Locations of wave lift and presumed wave triggering obstacles i. e. the highest upwind watersheds (luv)

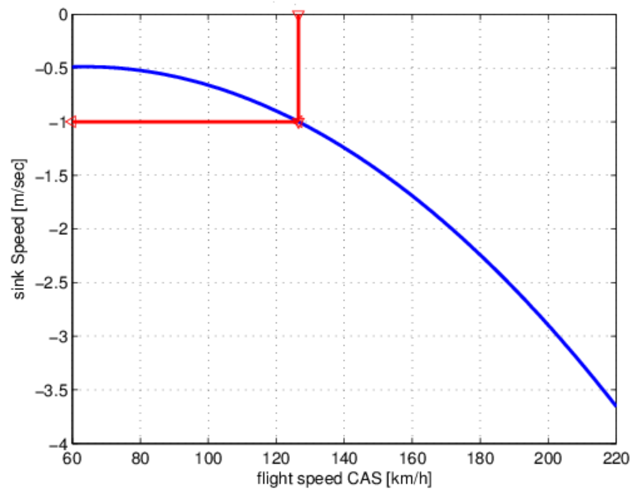

Fig. 5: Sink speed vs. calibrated airspeed (CAS) taken from the glider's airspeed polar (schematic representation)

The speed dependent vertical sink was calculated from the calibrated air speed using the gliders airspeed polar, in this case a Duo Discus, see Fig. 5. Formulas for calculation are given in the appendix.

First Results

A barogram, i.e. GPS altitude vs. flight time, is shown in Fig. 6. The flight was conducted by Christof Maul as pilot and co-pilot Phillip Illerhaus on March 27th 2015 [25]. Start and landing airfield was the La Cerdanya airport in the Spanish Pyrenees (LECD).

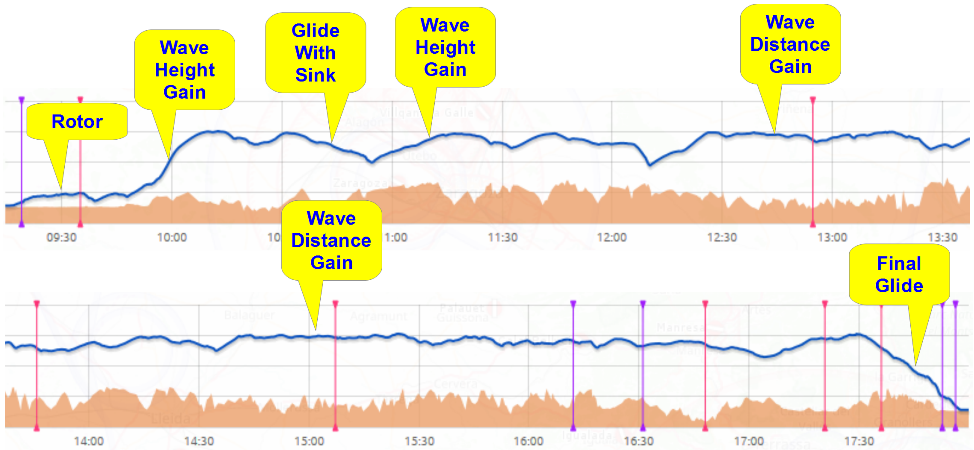

Fig. 6: Barogram of wave track flight

Predicted wind field showed a North-Westerly direction, Fig. 7. This is consistent with Fig. 4. Wind speeds were predicted above 50 km/h and a positive, i.e. increasing, wind shear was predicted and encountered.

Using the formulas for calculation of vertical air mass movements (see appendix) we calculated from the sensor data:

- u^{net}, the net vertical speed of the air-mass flow in [m/sec],
- v_i^{TAS}, the true airspeed of the glider in [km/h],
- u^{polar}, the vertical sink speed of the glider in still air according to it's airspeed polar in [m/sec],
- u^{stick}, the vertical speed induced by stick movement from the pilot that leads to a dynamic reaction of the glider (e.g. height gain due to deceleration) in [m/sec].

This allowed also to calculate the wave "strength" that day with a mean at 5 m/sec (standard deviation ± 1.8 m/sec), see green time series in Fig. 8. Using the stated velocities as defined above, the Rotor, Thermal Wave Climb and Distance Gain (DG) states could be identified. The state Rotor is quite dramatic. As can be seen in Fig. 9, which is on a seconds time scale, the up-down component of the Rotor shows a variation from more than +15 m/sec to -10 m/sec within seconds time. Such turbulent air is a heavy burden for pilots, planes and passengers.

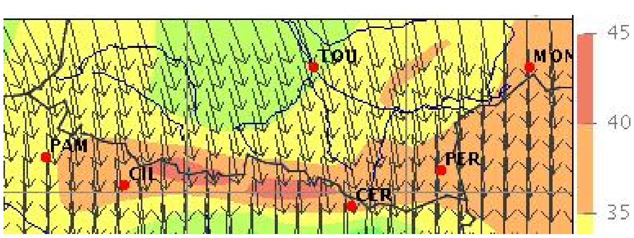

Fig. 7: Predicted wind field at FL 85, colored scale in knots

Discussion

Many GPS tracks of lee waves have been published. However, there are extremely few published measurement data sets on the atmospherics in these conditions. In order to elucidate such atmospheric dynamics capturing wave data from many flights in different weather conditions and landscapes would be necessary. However, nowadays the policy to obtain our own flight data back from the OLC database (www.onlinecontest.org) for scientific purposes is unfortunately too restrictive to allow such research. A submission of our flight data to an open source database such as, for example, https://skylines.aero/flights will help to overcome this problem.

A first measuring campaign on n = 34 flights, however with only n=1 Open Glide Computer, allowed the automatic identification of the flight states (see section "Data Mining Methods"). A first report was given in [26].

In particular the wave triggering watersheds and the average wave lift could be calculated from the data.

Conclusion

We demonstrate here that with low-cost and easily available instrumentation it is possible to analyze the extreme conditions prevailing in turbulent and laminar regimes of lee wave systems. Above all, such data can be acquired almost casually without active participation of the pilot or any need to compromise on his or her flying related goals. Thus, a continuous monitoring of lee waves becomes feasible, particularly in the hard-to-access rotor regimes and in ascending laminar flow which is of significant importance in view of the temporal and spatial variations of lee wave systems reported in previously performed large scale field missions. With swarm data mining methods [3] there is, however, a necessity to reduce errors from low-cost sensors and pilots, as well as aircraft biases. First results presented here demonstrate the feasibility and usefulness of this approach.

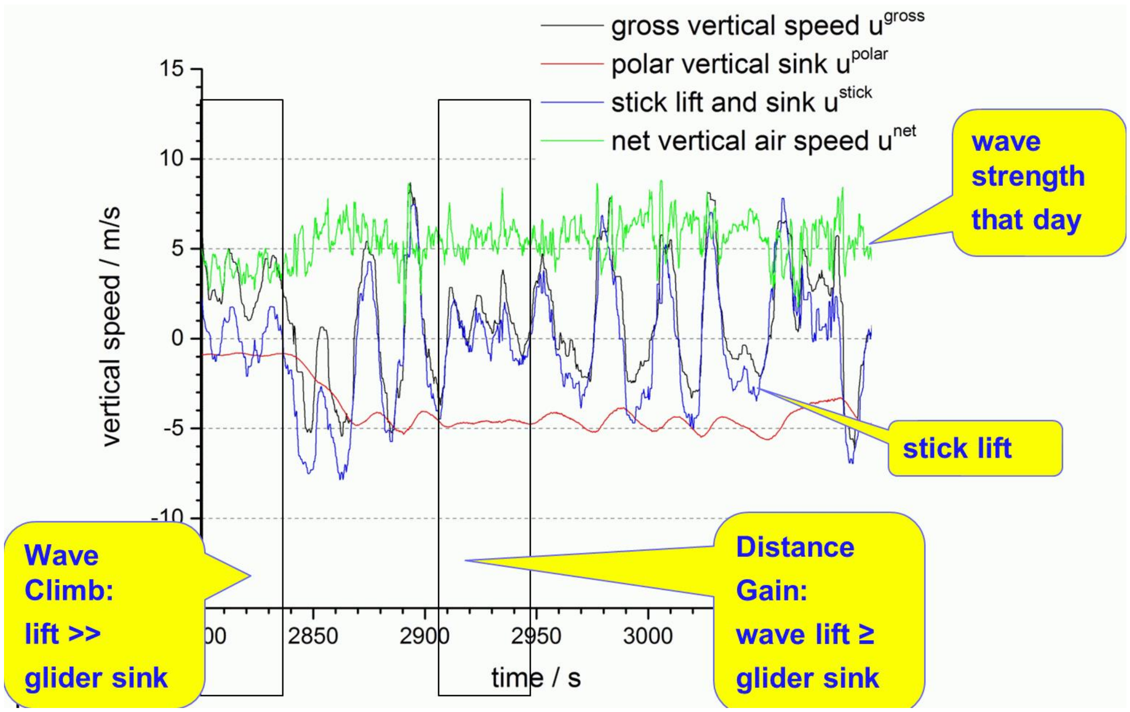

Fig. 8: Identified Wave Track Flight (WTF) states, shown are Wave Climb (WC) and Distance Gain (DG)

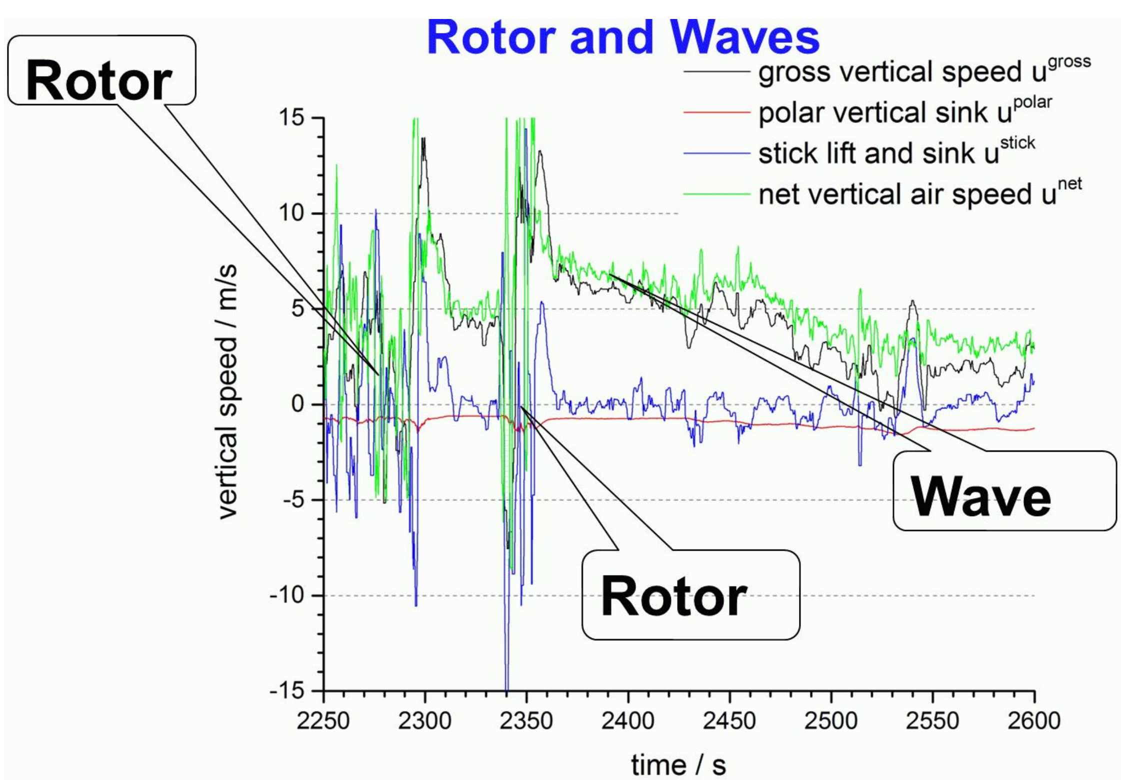

Fig. 9: Rotor vs. wave states

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References

[1] Ohlmann, K., "Neue Segelflug-Weltrekorde in den Leewellen-Systemen der Anden," *Beilage zur Berliner Wetterkarte*, Verein Berliner Wetterkarte e.V. c/o Institut für Meteorologie der Freien Universität Berlin, 2011, pp. 1–4.

[2] Etling, D., *Atmospheric Gravity Waves and Soaring Flight.*, Institute of Meteorology and Climatology Leibniz University Hannover, Hannover, Germany, 2014.

[3] Ultsch, A., "Swarm Data Mining for the Fine Structure of Thermals." *Technical Soaring*, Vol. 36, No. 4, 2013, pp. 37–44.

[4] Küttner, J., "Moazagotl und Föhnwelle." *Beiträge zur Physik der freien Atmosphäre*, Vol. 25, 1938, pp. 79–114.

[5] Küttner, J., "Zur Enstehung der Föhnwelle." *Beiträge zur Physik der freien Atmosphäre*, Vol. 25, 1939, pp. 251–.

[6] Grubišić, V. and Lewis, J. M., "Sierra Wave Project revisited: 50 Years Later." *Bulletin of the American Meteorological Society*, Vol. 85, No. 8, 2004, pp. 1127–1142.

[7] Bougeault, P., Jansa Clar, A., Benech, B., Carissimo, B., Pelon, J., and Richard, E., "Momentum Budget over the Pyrenees: The PYREX experiment." *Bulletin of the American Meteorological Society*, Vol. 71, No. 6, 1990, pp. 806–818.

[8] Bougeault, P., Benech, B., Bessemoulin, P., Carissimo, B., Jansa Clar, A., Pelon, J., Petitdier, M., and Richard, E., "PYREX: A summary of findings." *Bulletin of the American Meteorological Society*, Vol. 78, No. 4, 1997, pp. 637–650.

[9] Benech, B., Attie, J. L., Blanchard, A., Bougeault, P., Cazaudarre, P., Druilhet, A., Durand, P., Koffiand, E., Prudhomme, P., , and Tannhauser, D. S., "Observation of the waves above the Pyrenees (French-Spanish 'PYREX' experiment)." *Technical Soaring*, Vol. 18, No. 1, 1994, pp. 7–12.

[10] Drobinski, P. e., "Föhn in the Rhine Valley during MAP: A review of its multiscale dynamics in complex valley geometry." *Quarterly Journal of the Royal Meteorological Society*, Vol. 133, No. 625, 2007, pp. 897–916.

[11] Grubišić, V., "The Terrain-Induced Rotor Experiment." *Bulletin of the American Meteorological Society*, Vol. 89, No. 10, 2008, pp. 1513–1533.

[12] NASA, "Soaring to 100,000 ft on Stratospheric Mountain Waves." Tech Brief DRC-00-08.

[13] Brahic, C., "The Perlan Project: flying on the thinnest air." *New Scientist*, Vol. 213, No. 2846, 2012, pp. 34–37.

[14] FAI-IGC, *Technical Specification for GNNS Flight Recorders.*, 2^{nd} ed., 2011.

[15] Millane, R. P., Stirling, G. D., Brown, R. G., Zhang, N., Lo, V. L., Enevoldson, E., and Murray, J. E., "Estimating Wind Velocities in Mountain Waves Using Sailplane Flight Data." *J. Atmos. Ocean. Technol.*, Vol. 27, 2010, pp. 147–158.

[16] Ultsch, A. and Ohrndorf, P., "Die Erkennung von Leewellen in GPS-Dateien von Segelflügen in den Alpen." Mittelgebirgs-Leewellen-Forum, Leibniz University Hannover, 2009.

[17] Ultsch, A., "Identification of Waves in IGC Files." Workshop Pfaffstaetten, OSTIV Scientific Section and Meteorology Panel, Austria, Sep. 25th–27th, 2009.

[18] Ohrndorf, P., "Die Identifikation von Leewellen mit Hilfe von Flugwegaufzeichnungen am Beispiel ausgewählter Segelflüge im Alpenraum." Thesis, Department of Geography, Philipps-University Marburg, Germany, 2009.

[19] Hoeth, H., "Open Glide Computer." website, June 2014.

[20] Ultsch, A. and Moerchen, F., "ESOM-Maps: tools for clustering, visualization, and classification with Emergent SOM." Technical report no. 46, Department of Mathematics and Computer Science, Philipps-University Marburg, Germany, 2005.

[21] Rabiner, L. R., "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition." *Proceedings of the IEEE*, Vol. 77, No. 2, 2012, pp. 257–286.

[22] Grewal, M. S. and Andrews, A. P., *Kalman Filtering: Theory and Practice.*, Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1993.

[23] Ultsch, A., "Computer versus Realität." *Segelfliegen*, Vol. 8, No. 4, 2010, pp. 28–29.

[24] Gehrmann, S., "Lilienthals Erben: Die Akademische Fliegergruppe Akaflieg." *UniReport 4-09, Goethe-Universität, Frankfurt am Main*, 2009, pp. 19.

[25] Maul, C. and Illerhaus, P., "Wave Track Flight, March 27^{st}, La Cerdanya, Spain." https://skylines.aero/flights/43781, May 2017.

[26] Ultsch, A., Rogos, C., and Maul, C., "Data Mining in Atmospheric Gravity Waves," *Proc. European Conference on Data Analysis*, University of Essex, Colchester, UK, 2015, pp. 45–.

Appendix

In the following, equations and formulae used for the calculation of the vertical air-mass movement are presented. The basic idea is that the gross vertical speed (or climb/sink rate, respectively) of the glider relative to the ground, u^{gross}, is given by the sum of the net lift of the air-mass flow, u^{net}, which is the unknown of interest, the sink speed of the glider at it's actual flight velocity relative to the air-mass, u^{polar}, and the vertical speed of the glider in dynamic maneuver due to stick movement, u^{stick}:

$$u^{gross} = u^{net} + u^{polar} + u^{stick} \tag{1}$$

From the GPS measurements u^{gross} can be determined according to:

$$u^{gross}(GPS) = \frac{h_k - h_i}{t_k - t_i} \tag{2}$$

Alternatively, u^{gross} may be calculated from the rate of change of the measured static pressure in climb (or sink) with time using the aerostatic equation (sometimes also referred to as hydrostatic equation, Laplace 1805):

$$u^{gross}(baro) = \frac{(p_k^{stat} - p_i^{stat})}{p_i^{stat}(t_k - t_i)} \frac{RT_i}{mg} \tag{3}$$

Here, static temperature T_i is also part of the set of measured data. With the additionally available total pressure p_i^{pitot} (and, hence, dynamic pressure $(p_i^{pitot} - p_i^{stat})$) the actual true airspeed of the glider is derived from

$$v_i^{TAS} = \sqrt{\frac{2RT_i}{mg} \frac{(p_i^{pitot} - p_i^{stat})}{p_i^{stat}}} \tag{4}$$

True airspeed then is the basis for the determination of the glider's sink speed u^{polar} from its airspeed polar.

Calculation of the vertical speed of the glider in dynamic maneuvers due to stick movement is based on the assumption that the exchange between kinetic and potential energy occurs without losses (conservation of total energy). This assumption can be justified by the fact that especially sailplanes have a lift to drag ratio in the order of 40-50, so aerodynamic losses are in general very small for this type of airplane and may be neglected in first approximation:

$$u_i^{stick} = \frac{1}{2g} \frac{(v_k^{TAS})^2 - (v_i^{TAS})^2}{(t_k - t_i)} \tag{5}$$