FLUTTER ANALYSIS OF GLIDERS

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Summary

A method of flutter analysis of gliders is presented. The analysis is performed in two steps. The first one involves determination of the free vibration of the glider. The vibrations are computed or determined experimentally by ground vibration tests. The second involves determination of the critical flutter conditions (critical flutter speed, flutter frequency and flutter mode) and is based on the results obtained in the first step.
As an example of the analysis, the flutter calculations for a glassfibre glider are presented. Some relations between the results of calculation and experimental investigation are established.

1. Introduction

Flutter is a dynamic instability of an elastic aircraft in flight. The classic type of this aeroelastic instability is associated with non-stationary aerodynamic forces in a potential flow and depends on coupling between many elastomechanical degrees of freedom of the structure (1), (2). In designing a modern sailplane, the flutter analysis is very important and should start at early stage, as it affects the choice of the optimal glider configuration. In this step of the design process there exists only technical documentation on which to base the design of a computing model of the glider. The theoretical model can be represented physically by the flutter model which makes possible the verification of the aerodynamic theory being used in calculations. Flutter models are investigated for non-typical configurations only. When the glider prototypes are constructed then the static and dynamic tests can be applied to verify and/or to correct the theoretical model and the results of calculations. Flight tests yield the final verification and confirmation of the computed results. Correlation between calculations and tests is made as shown in Fig. 1.
The calculations on which the flutter analysis is based are performed in two steps (3). The first one involves determination of the natural vibrations of the structure (4). The calculation model in this case has about two hundred degrees of freedom. Natural vibration modes create the possibility of reducing the number of degrees of freedom to about twenty, and are used in the second step which is the determination of the critical flutter conditions. These conditions are defined as appearing at the lowest possible speed at which the damping ratio of any dynamic aeroelastic mode crosses zero. Critical flutter

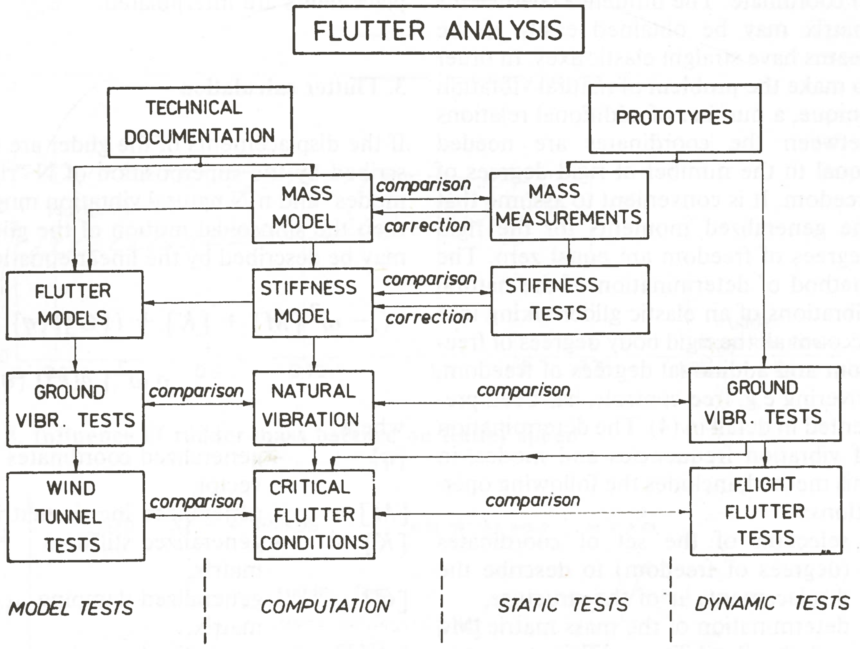

Fig. 1. Flutter analysis flow diagram.

conditions (critical flutter speed, flutter frequency and flutter mode) depend on many design parameters of the glider. The influence of some parameters on flutter speed is usually investigated by computing. In this way the required flutter properties of the glider can be determined.

2. Natural vibrations calculation

By means of physical considerations or mathematical simplifications, the real structure, with an infinite number of degrees of freedom, was replaced by an approximate one with J degrees of freedom. The position of this structure with respect to an inertial reference frame can be described by a J-dimensional vector |u|, coordinates of which are independent and express the displacements of selected points of the structure and the rotations of elements in their vicinity (see Fig. 2). Owing to the symmetry of mass and stiffness distribution, the symmetric and antisymmetric vibrations can be calculated separately. The slenderness of the elements allows the structure to be replaced by an approximate model consisting of beams as shown in Fig. 2. Inertia properties of the structure may be described by a mass matrix $[M_I]$ of J degree. Direct replacement of the structure by isolated

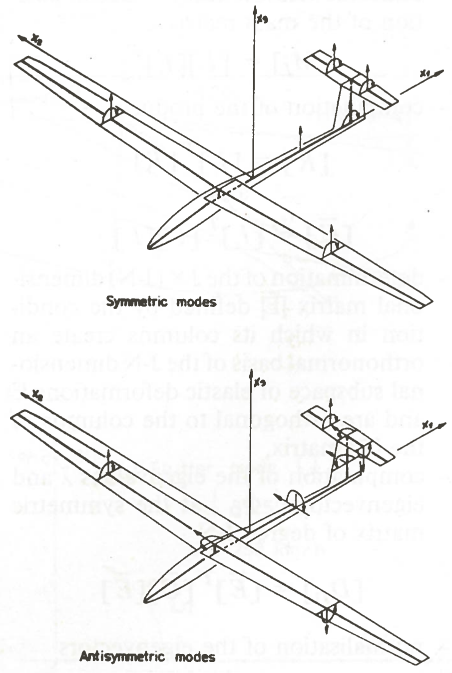

Fig. 2. Calculation model of a glider.

masses is not convenient. Better accuracy can be achieved by taking into account the continous mass and inertia distributions and using numerical integration methods to determine the equivalent isolated masses. The elastic properties of the structure may be described by a flexibility influence coefficients matrix [C] of J degree for the structure, in which rigid degrees of freedom have been eliminated by means of additional statically determinated constraints. The elements of the matrix [C] express the change of the i-th coordinate of the vector |u| produced by the generalized force appropriate to the j-th coordinate. The influence coefficients matrix may be obtained easily, if the beams have straight elastic axes. In order to make the problem of natural vibration unique, a number of additional relations between the coordinates are needed equal to the number of rigid degrees of freedom. It is convenient to assume that the generalized moments for the rigid degrees of freedom are equal zero. The method of determination of the natural vibrations of an elastic glider taking into account all the rigid body degrees of freedom and additional degrees of freedom, covering e.g. free-controls, has been presented in detail in (4). The determination of vibration frequencies and modes, in this method, includes the following operations:

- selection of the set of coordinates (degrees of freedom) to describe the displacements |u| of the structure,
- determination of the mass matric $[M_I]$ and the flexibility coefficients matrix [C],
- definition of the N possible rigid displacements of the structure and determination of the "rigid body degrees of freedom" matrix [R],
- Banachiewicz-Cholesky decomposition of the mass matrix

$$[M_I] = [L][L]^T$$

- computation of the products

$$[\bar{R}] = [L]^T [R]$$

and

$$[\bar{C}] = [L]^T [C][L]$$

- determination of the J×(J-N) dimensional matrix $[\bar{E}]$ defined by the condition in which its columns create an orthonormal basis of the J-N dimensional subspace of elastic deformations E and are orthogonal to the columns of the [R] matrix,
- computation of the eigenvalues λ and eigenvectors $| \Phi_D |$ at the symmetric matrix of degree J-N

$$[D_D] = [\bar{E}]^T [\bar{C}][\bar{E}]$$

- normalisation of the eigenvectors

$$\{\Phi_D\}^T \{\Phi_D\} = 1$$

- computation of the frequencies f and natural modes $| \Phi |$ from the relations

$$\omega = 2\pi f = \frac{1}{\sqrt{\lambda}}$$

$$\{\Phi\} = ([L]^{-1})[\bar{E}]\{\Phi_D\}$$

The data for the particular structure are involved in the first, second and third operations only. All the remaining operations can be performed by universal, stable numerical methods (5). As a result of the computations the frequencies and modes of natural vibrations defining displacements of the isolated masses are obtained. For future applications, the displacements are interpolated.

3. Flutter calculation

If the displacements of the glider are described by the superposition of N "rigid modes" and n-N natural vibration modes then the sinusoidal motion of the glider may be described by the linear equation:

$$(-\omega^2 [M] + [K] + i[G])\{q\} = \rho\, \omega^2 [A(k)]\{q\}$$

where:

$\{q\}$	- generalized coordinates vector,
$[M]$	- generalized inertia matrix,
$[K]$	- generalized stiffness matrix,
$[G]$	- generalized damping matrix,
$[A(k)]$	- generalized aerodynamic matrix, elements of which are functions of the reduced frequency,
ω	- circular frequency,
b	- characteristic length,
U	- free-stream velocity,
ρ	- free-stream density,
$i = \sqrt{-1}$	- imaginary unit.

The generalized mass matrix (of order n) has the form:

$$[M] = \begin{bmatrix} M_R & 0 \\ 0 & I \end{bmatrix}$$

where: $[M_R]$ - inertia matrix (of order M) of the rigid aircraft,
[I] - unit matrix.
The generalized stiffness matrix (of order n) has the form:

$$[K] = \begin{bmatrix} 0 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

where: [ω] - diagonal matrix (of order n-M) of circular frequencies corresponding to particular natural modes.
The generalized damping matrix defines the energy dissipation in the motion. It is usually assumed that [C] is diagonal

$$[G] = \begin{bmatrix} 0 & 0 \\ 0 & g \end{bmatrix}$$

where: [g_s] is the diagonal matrix (of order n-N) of structural damping corresponding to particular natural modes and can be determined experimentally.
If natural modes are determined by calculation then one global structural damping coefficient g for all modes is usually assumed. Elements of the generalized aerodynamic matrix [A(k)] of degree n are the complex numbers. In practice the aerodynamic matrix can only be determined for the pre-assumed values of the reduced frequency k which may be considered as a parameter in calculations. The calculation of the critical flutter conditions is equivalent to the determination of the pair of parameters ω and k for which the solution | q | of the flutter equation does exist. The solution which yields the lowest speed $U = \omega\, b/k$, defines the critical flutter speed $U_F = U$, the flutter frequency ω_F and the flutter mode | q_F |. Elements of the vector | q_F | are usually complex numbers which implies that a phase-shift between generalized coordinates (normal modes) exists. After transformation of the flutter equation to the form:

$$[D(k)]\{q\} = \lambda \{q\}$$

where:

$$[D(k)] = ([M] + \rho\, [A(k)])^{-1} ([K] + i\, [G]$$

It is possible to use standard numerical methods (5) to solve the eigenvalue problem of the complex matrix [D(k)].
If each eigenvalue is interpreted as

$$\lambda = \frac{\omega^2}{1 + ig}$$

then it is possible to calculate the parameters:

$$\omega = \frac{|\lambda|}{\sqrt{Re\,\lambda}}\ ;\ g = -\frac{Im\,\lambda}{Re\,\lambda}\ ;$$

$$U = \frac{\omega b}{k}$$

where g = artificial damping coefficient. From the plot of g as a function of U, the critical flutter speed can be defined as the lowest speed U_F, at which g is equal zero. The flutter mode is defined by the eigenvector | q_F | corresponding to the critical flutter eigenvalue. If normal modes used in calculation are normalized with respect to generalized masses equal to unity, then moduli of the eigenvector elements define the influence of particular modes on flutter. It creates the possibility of selecting the modes important in flutter calculations.
The calculation can be made in practice for simple linear-elastic models of airplane structure only. If calculation does incorporate the experimentally determined structural damping then it is possible to find the critical flutter conditions for small amplitudes of vibrations in flight with satisfactory accuracy.

4. Example

An example of practical application of the method described is the flutter analysis for glassfibre-epoxy KROKUS sailplane. Typical T-tail glider flutter mode which must be eliminated is a coupling between rudder deflection and a tail bending-and-torsion. The critical flutter speed in this case depends generally on the rudder mass balance and control system stiffness. The required rudder mass balance for free control was investigated in designing the glider. Three "rigid modes" and nineteen computing natural vibration modes were used in the flutter analysis. Results of the tested KROKUS glider prototype are presented at Fig. 3.
For illustration, U-g plot is shown at Fig. 4.
The line g = 0.02 represents the structural damping assumed in the analysis. For examined value of rudder mass balance 0.5 kg a typical "rudder flutter" mode (marked S) exists in a limited flight speed range and another flutter mode (marked Q) exists at higher speeds. Flutter modes S and Q are presented on complex planes at Fig. 5.
The vectors indicate the participation of particular natural modes in the flutter mode. The relative location of the vectors on the complex plane defines the phase-shift of the natural modes.
Natural modes dominating in the flutter mode S are shown at Fig. 6, 7, 8.
Fig. 9 and 10 show the deformations of sailplane (flutter mode S) at times (wt of O and II/2) in critical flutter conditions. For the rudder mass balance installed on the prototype (1.5 kg) the flutter does not exist up to the maximum diving speed of glider. It is calculated that flutter (marked K), exists only above the required speed range, the mode of which is shown at Fig. 11.
Ground vibration tests on the prototype KROKUS glider were performed. Agreement between the calculated natural modes and the measured vibration modes was quite good. On the basis of the modes determined by vibration tests critical flutter conditions were computed. Agreement between these results and the theoretical ones is good (flutter mode K) and is presented at Fig. 3. In agreement with the numerical analysis, no flutter was found in flight tests up to the maximum dive speed of the glider.

5. Concluding remarks

The method of flutter analysis presented is applied practically in the design process of gliders and aeroplanes. The computing system based on it permits carrying out preliminary and certification flutter calculations. Calculated or measured vibration modes may be used for flutter computing. The modes measured in ground vibration tests are usually non-orthogonal. The

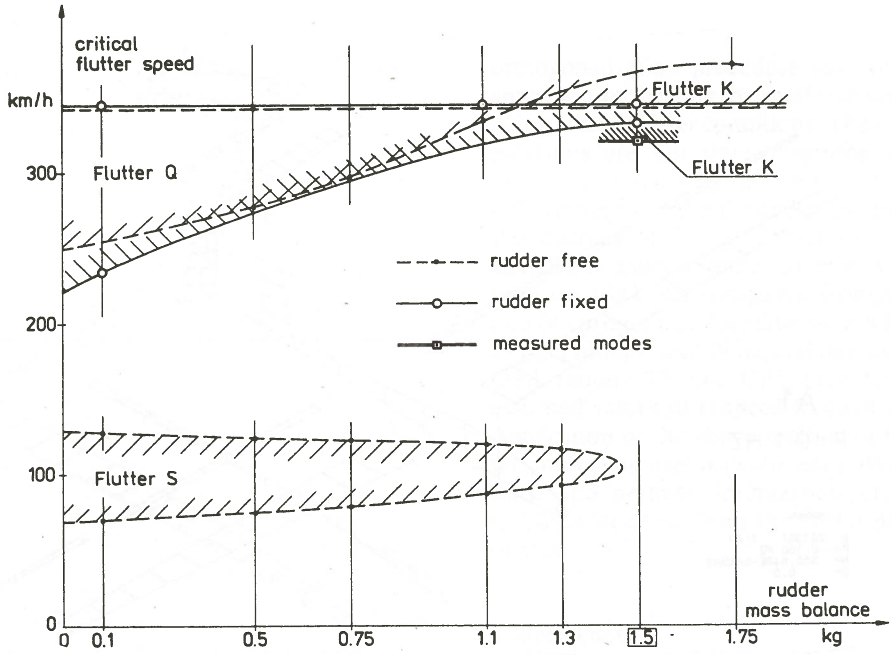

Fig. 3. Influence of rudder mass balance on flutter speed

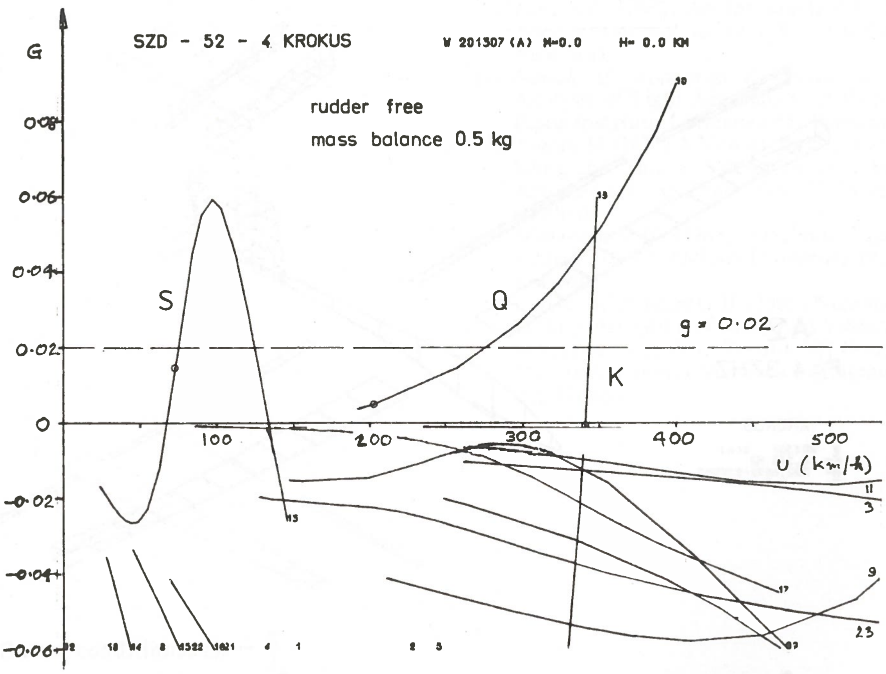

Fig. 4. U-g plot.

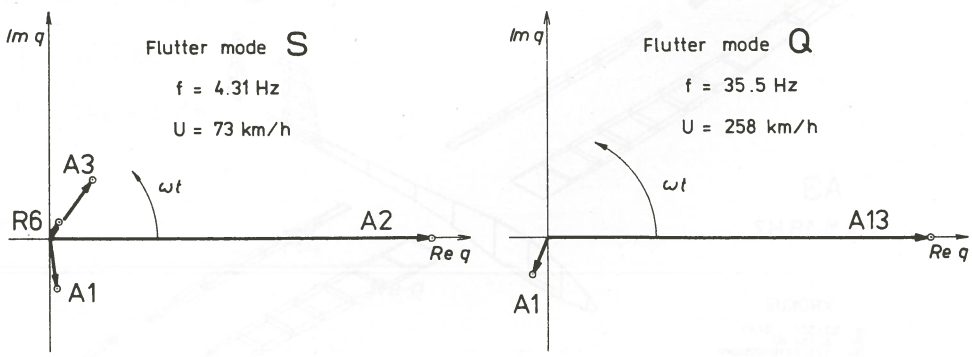

Fig. 5. Flutter modes S and Q.

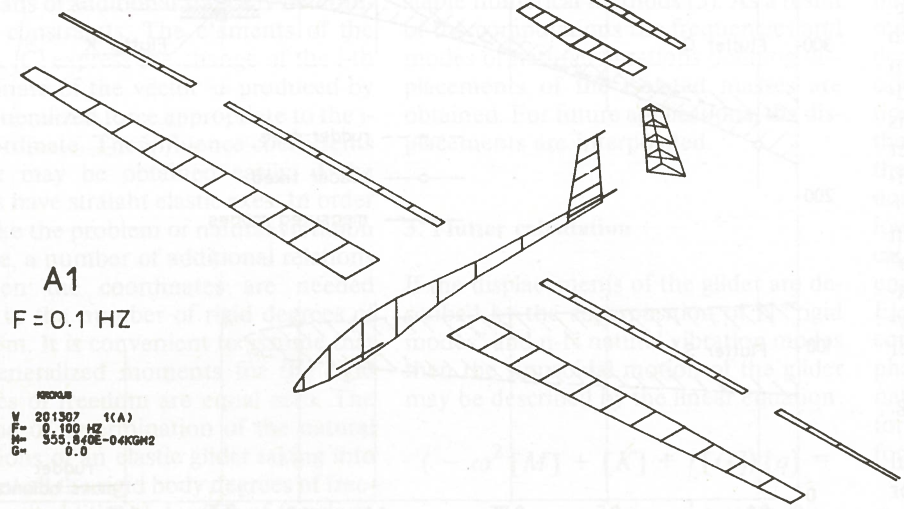

Fig. 6. Natural vibration mode A1.

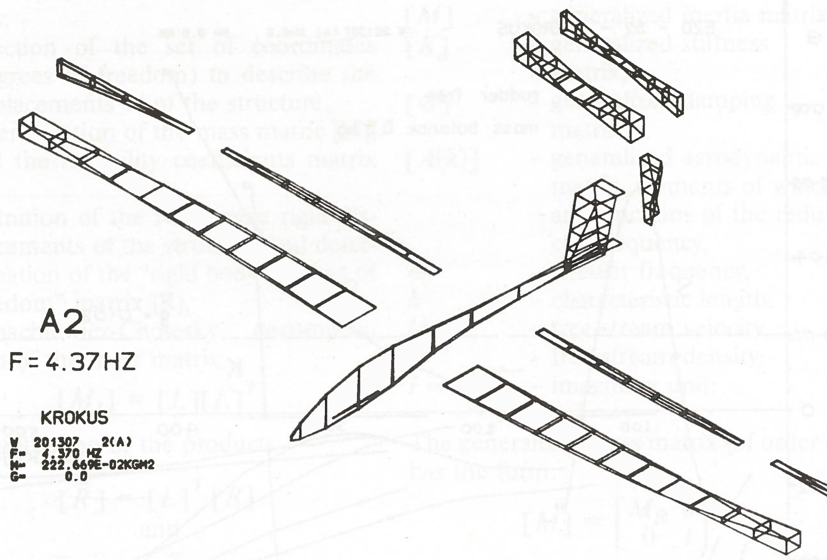

Fig. 7. Natural vibration mode A2.

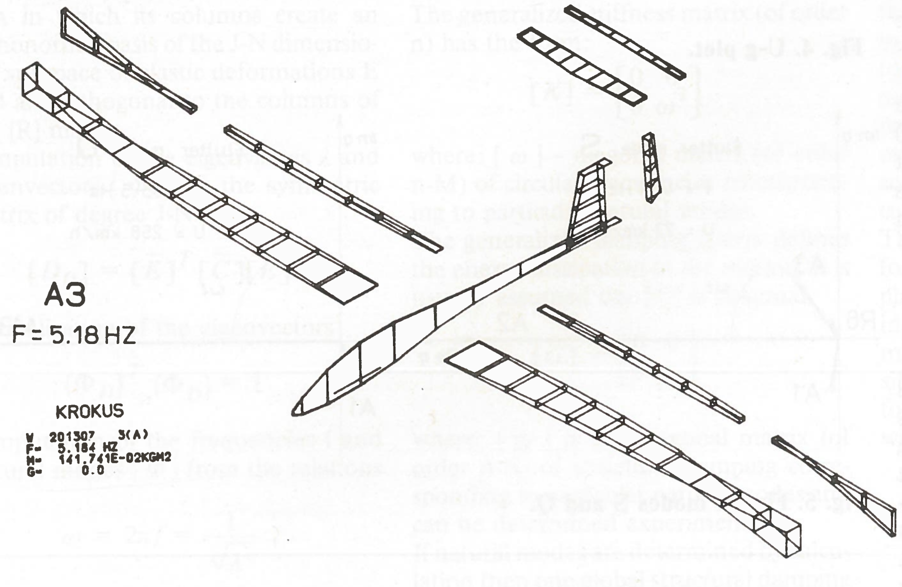

Fig. 8. Natural vibration mode A3.

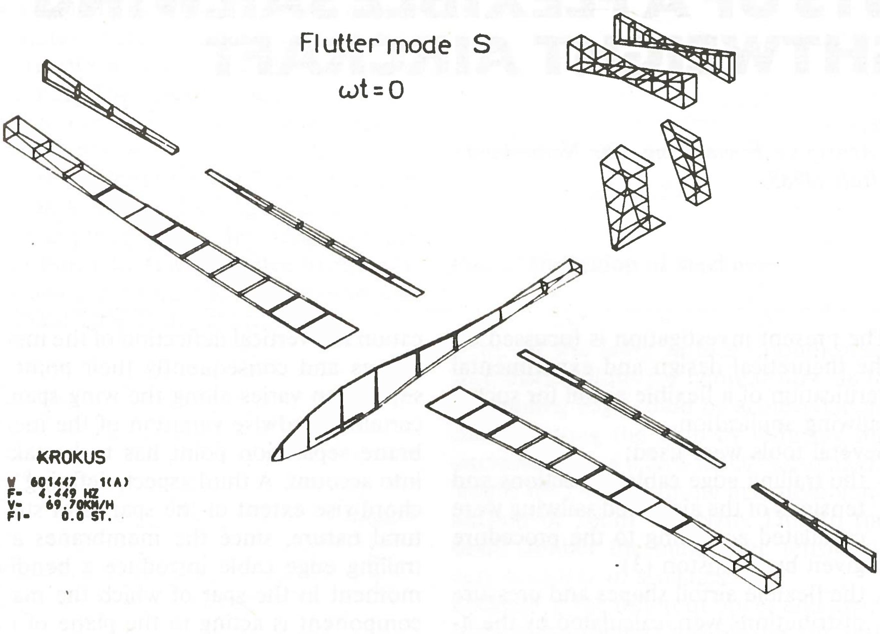

Fig. 9. Sailplane deformations at critical flutter conditions, $\omega t = 0$.

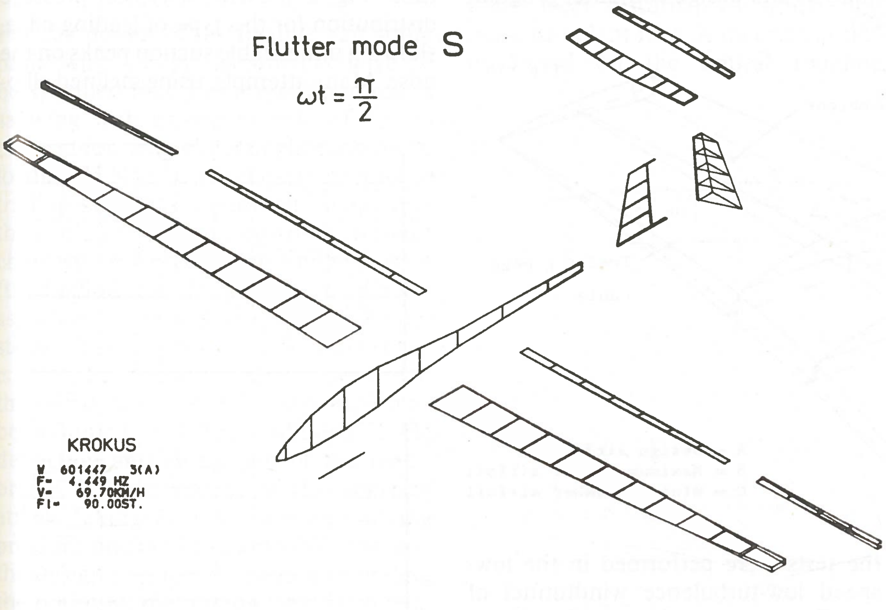

Fig. 10. Sailplane deformations at critical flutter conditions, $\omega t = \frac{\pi}{2}$

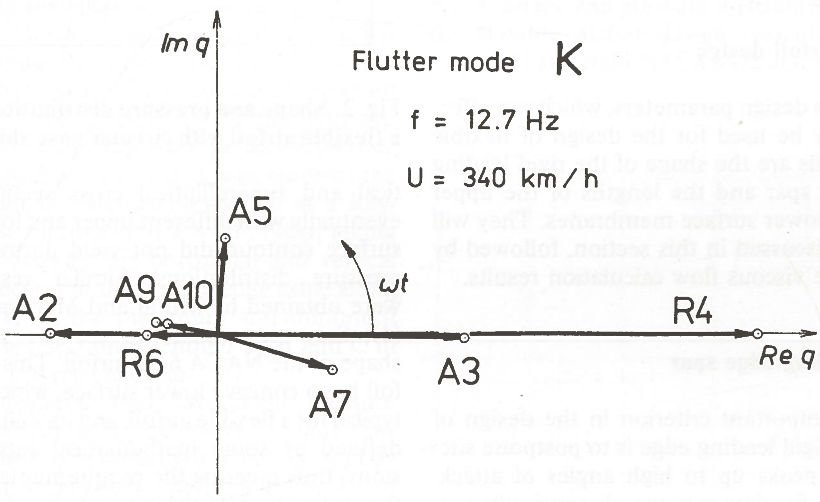

Fig. 11. Flutter mode K.

orthogonalization procedure (6) can be applied prior to using the modes in calculating critical flutter conditions. The computations are not timeconsuming. For example, computation of symmetric and antisymmetric natural modes of about 150 degrees of freedom model of the KROKUS glider require 20 min. CPU time on IBM 360 computer. Computation of critical flutter conditions, in which 3 "rigid modes" and 19 natural modes are used, require 25 min. CPU time for 30 assumed values of reduced frequency. Application of the above method of flutter analysis is useful in the early design stage and permits optimization of the sailplane structure from the flutter point of view.

6. References

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