SAFETY FACTORS FOR FULL-SCALE FATIGUE TESTS

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Summary

Accounting for the scatter in the fatigue-endurance of individual pieces may be made in different ways, according to the mode of certification and to other circumstances. For multi-piece test series we have developed a numerical confidence band calculation procedure. For single-piece tests a slightly modified form of Freudenthal's safety factor calculation can be recommended. A novel coordinate transformation formula is giving quite close monitoring of the fatigue-crack propagation process.

1. Introduction

Safety should be the first, economy a very strong second in sailplane fatigue design and development. We must avoid catastrophic in-flight structural failures at all costs but at the same time must strive to utilize the full amount of safe flying hours for every glider. The wide scatter of individual fatigue lives makes achieving this ideal a very hard task indeed. Not long ago a prominent aircraft factory advertised proudly, in an aeronautical journal of international fame, having achieved a best to worst fatigue life ratio of about 3.2. So much for this.
The scatter problem is covered formally by the introduction of the life safety factor m. Different views - and interests - regarding the seriousness of the possibility of an early service failure are manifesting themselves in the value of this factor.
Let us take a closer look at this problem. We can start at the assessment of a single load level fatigue test series.

2. Confidence band calculation for multi-piece test series

In view of the scatter problem, batch testing and statistical assessment are to be recommended in case of relatively cheap specimens. Raw test results take the form of a discrete series of load cycles to failure

$$N_i \;/\; i = 1 \div j \;/$$

and corresponding service hours to failure

$$t_i \;/\; i = 1 \div j \;/$$

chosen according to increasing fatigue lives.

Statistical assessment is based on the presumption of a suitable type of life distribution function P (N) and corresponding P (t) giving the probability of failure as a function of the fatigue life. From among the functions in general use we prefer the three-parameter Weibull-distribution. Most authors are using it in the form recommended e.g. by the ASTM Commitee on Fatigue (17):

$$P(N) \equiv W(N) = 1 - \exp[-(\frac{N - N_0}{\beta})^{\alpha}] \tag{1a}$$

rearranged for plotting as:

$$\frac{1}{1 - P(N)} \equiv \exp[(-\frac{N - N_0}{\beta^* - N_0})^{\alpha}] \tag{1b}$$

Instead of the scale parameter β we prefer to use the scatter parameter

$$\beta = \beta^* - N_0 \tag{2}$$

so our variant of the Weibull formula reads (5):

$$P(N) \equiv W(N) = 1 - \exp[-(\frac{N - N_0}{\beta^* - N_0})^{\alpha}]$$
$$= 1 - \exp[-(\frac{N}{\beta} - \varepsilon)^{\alpha}] \tag{3a}$$

with the safe life ratio

$$\varepsilon = \frac{N_0}{\beta} \tag{4}$$

For plotting the formula reads then:

$$\frac{1}{1 - P(N)} \equiv \exp[(\frac{N - N_0}{\beta})^{\alpha}]$$
$$= \exp[(\frac{N}{\beta} - \varepsilon)^{\alpha}] \tag{3b}$$

Eq. (3b) yields by double logarithmic transformation

$$\ln \ln \frac{1}{1 - P(N)} = \alpha \ln(N - N_0) - \alpha \ln \beta \tag{5}$$

i.e. a straight-line plot for the correct value of the minimum fatigue life N_0.
Test results do not follow perfectly the theoretical distribution function P (N), necessitating some form of smoothing and/or parameter identification. By such means we get a best fit representation for the continuous type life distribution function P (N) (see e.g.: Fig. 1 relating to Ref. Johnson (12) and Ref. (17)).

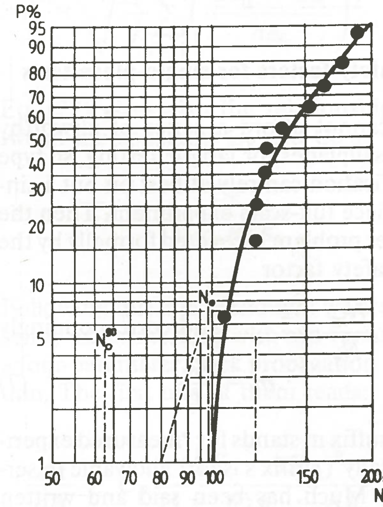

Fig. 1. Fatigue test evaluation using the Weibull distribution function (N_0^{90} indicates the 90% confidence band for N_0)

In the case of distribution functions giving positive safe fatigue lives N_0 in the process of evaluation a correct choice of an allowable service life seems to have been solved, at least in principle. But, sad to say, there is no guarantee for the best fit value of N_0 as calculated from a couple of specimen lives to be identical with the physical safe life of the whole production lot of J >> j pieces. We have to complement therefore the assessment with an appropriate confidence band calculation procedure.
There are some basic mathematical difficulties relating to the correct analytical calculation of confidence intervals for three-parameter Weibull-distributions. Several authors try to circumvent these by reducing the problem to an appropriate two-parameter case. So Johnson (12) and others work with zero minimum fatigue life. Amstadter (1) accepts the value of N_0 as calculated for the best fit representation to be right. Marialigeti (14) uses the best fit value of α for confidence calculations.
We accept none of these because none of the three Weibull parameters as given by the parameter identification calculation can be taken to the exactly right. Moreover, analytical confidence band calculations can be only reliable if the life probability distributions as determined from j specimens P_j (N) may be regarded as members of a convergence process, i.e.:

$$\lim_{j \to \infty} P_j(N) = W(N) \tag{6}$$

Sad to say, there are strong indications against the correctness of this postulation. A statistical evaluation of test series extension simulations done on 38 test series has given essentially negative results for the logarithmic-normal, for the arcus-tangens and for the Weibull type distribution functions also. In view of this, we use a numerical error-margin calculation for our Weibull assessments, details of which are to be found in Ref. (5).

3. Safety factors for single-piece tests

We cannot afford a series of airworthy new sailplanes for fatigue testing, so type certification can rely at best on but a single-piece full-scale experiment. Then the scatter problem is covered formally by the life safety factor

$$m = \frac{N_m}{N_s} \quad \text{and correspondingly} \quad m = \frac{t_m}{t_s} \quad . \tag{7}$$

The suffix m stands for "measured experimentally", suffix s is for "allowable in service". Much has been said and written about the correct choice of the value of m but - to say the truth - the usual values for m from 3 to about 5 originate more from tradition or personal preference than from a strict calculation procedure. A few years ago Freudenthal (4) strongly criticized the tendency for m to be decreased. He based his demonstration on the Weibull-distribution using the notation of Eq. (1b), the safe life ratio being

$$\varepsilon^* = \frac{N_0}{\beta^*} \tag{4a}$$

A short summary of his results reads as follows. If we have calculated the allowable service life from the median life value of j test specimens for a fleet size of J pieces with a safety factor m then the reliability level of this calculation is:

$$\mathcal{R} = \left[\frac{1}{1 + \frac{J}{j}\left(\frac{l - m\varepsilon^*}{m(l - \varepsilon^*)}\right)^{\alpha}} \right]^j \tag{8a}$$

In our notation this transposes to:

$$\mathcal{R} = \left[\frac{1}{1 + \frac{J}{j}\left(\frac{l + \varepsilon}{m} - \varepsilon\right)^{\alpha}} \right]^j \tag{8b}$$

Solving this for m gives

$$m = \frac{1 + \varepsilon}{\left[\frac{i}{J}\left(\frac{1}{\mathcal{R}^{1/j}} - 1\right)\right]^{1/\alpha} + \varepsilon} \tag{9}$$

Values of m giving R = 0.99 reliability for a single-piece test and for a fleet size of J = 100 and 1000 are to be seen on Fig. 2 and 3.

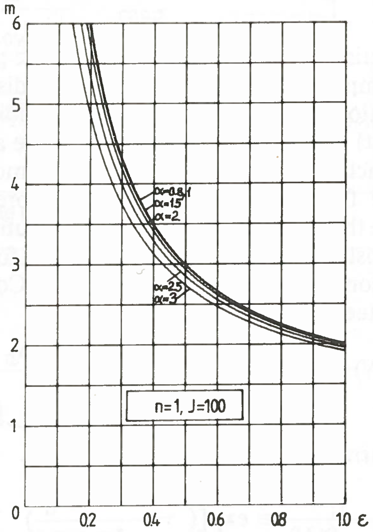

Fig. 2. Safety factors by the Freudenthal method for single-piece tests

(R = 0.99, J = 100)

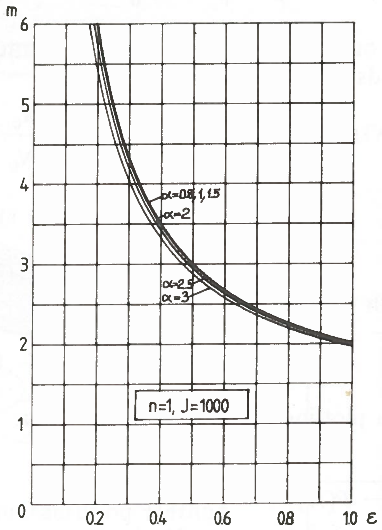

Fig. 3. Safety factors by the Freudenthal method for single-piece tests

(R = 0.99, J = 1000)

According to both graphs the dominant parameter determining the acceptable safety factor values is the safe life ratio ε of the respective Weibull-distribution. In the case of limited information we can stay on the safe side by taking

$$m \leq 1 + \frac{1}{\varepsilon} \tag{10}$$

Another practical conclusion from our calculations may be that safety factors below say 4 are rarely if ever justified. Covering against unexpected catastrophic service failures means giving up 65 to 80 per cent of actually safe service life of the fleet.

The ultimate goal of every fatigue design and development should be therefore the realization of a so-called safe-by-inspection mode certification without an absolute service life limitation for the type. This school of thought, under the name of fail-safe structures, has a long but not particularly victorious history in aeronautical engineering. For want of a proper crack-propagation theory based on solid laws of physics and lacking the necessary non-destructive inspection technique it was restricted practically to multiple load-path structures.

4. Statistical assessment of fatigue-crack propagation

Safety and costs of the safety-by-inspection method depend in addition to reliable crack detection on the knowledge of the crack propagation rate da/dN. According to fracture mechanics pioneered by Corten (2), Griffith (7), Irwing (9) and (10) and others, crack stability in metals depends on the stress-intensity factor

$$K = \sigma\sqrt{\pi a}\,\varphi \tag{11}$$

According to the same philosophy, fatigue-crack growth rate should be a function of the stress-intensity factor range

$$\Delta K = K_{max} - K_{\min} \tag{12}$$

In a corrosion-free environment the usual appearance of the crack propagation graph

$$\frac{da}{dN} = f(\Delta K) \tag{13}$$

is as shown on Fig. 4.

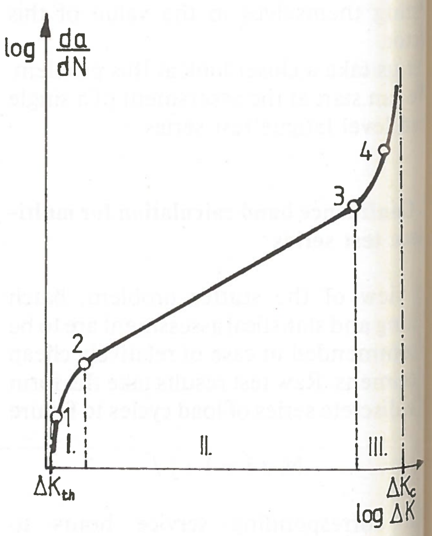

Fig. 4. Standard fatigue-crack diagram on a logarithmic scale

The middle part of this curve, section II, is a straight line on a log-log plot indication a power-law type relation. According to the Paris-Erdogan equation (15) this reads:

$$\frac{da}{dN} = C_P \Delta K^n \qquad (14)$$

Fracture occurs at the attainment of the critical stress-intensity factor K_c, i.e. in the fracture cycle:

$$K_{max} = K_c \qquad (15)$$

Our notation is based on the stress-intensity range Δ K. Introduction of the stress ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}} \qquad (16)$$

and combination of Eqs. (12) and (15) leads to

$$\Delta K_c = (1 - R) K_c \qquad (17)$$

For description of sections II and III of the crack propagation curve Forman (3) proposed the formula

$$\frac{da}{dN} = C_F \frac{\Delta K^n}{(1 - R) K_c - \Delta K} \qquad (18)$$

For description of section I of the diagram we have to know the treshold value ΔK_{th} and the character of this regime. Treshold calculation is always by extrapolation. Several authors have proposed four-parameter curve fitting formulae as listed e.g. by Kocanda (13, Section 4.3) but none of them has been universally accepted as on international standard as yet.

For inspection schedule planning we have first to convert the function

$$\frac{da}{dN}(\Delta K) \;\; into \;\; \frac{da}{dN}(a)$$

using Eq. (11). Let us suppose that we can safely detect any incipient crack of (half-) length a_l. Then the nominal number of load cycles from crack detection until failure would be for side cracks:

$$\Delta N_{lc} = \int_{a_l}^{a_c} \left[\frac{da}{dN}(a)\right]^{-1} da \qquad (19)$$

The same formula holds for central cracks but we have to remember that for those a denotes half the crack length. In the case of corner cracks and other non-standard fractures mechanical modelling problems reference should be made to the appropriate literature.

This nominal interval between inspections being based on statistical averages incorporation of a safety factor m becomes mandatory here too. This gives for the planned inspection interval

$$\Delta U = \frac{1}{m} \Delta N_{lc} \qquad (20)$$

Crack propagation life safety factors are not subject to international or legal standards but it is customary to work with about m ≅ 2. Let us see if it is sufficient and if it can be improved upon.

For proper and efficient averaging an exact determination of the parameters K_c, C_F, n, ΔK_{th} and a realistic four-parameter crack propagation formula is necessary. We can start with the Forman equation. There is a well-proven standard for the determination of the critical stress-intensity factor in tension K_{Ic} on compact specimens. But thin-gauge metal sheets do not fail in this mode so the effective K_c has to be calculated by extrapolation from fatigue test data. Forman's original concept for proving his formula is not particularly effective in this case. The author has proposed therefore the following coordinate transformation (6).

Let us expand the right side of Eq. (18) by ΔK_c and rearrange it giving

$$\frac{da}{dN} = \frac{C_F}{\Delta K_c} \Delta K^n \frac{\Delta K_c}{\Delta K_c - \Delta K}$$

We can write this also as

$$\frac{da}{dN} = \frac{C_F}{\Delta K_c}\left[\Delta K\left(\frac{\Delta K_c}{\Delta K_c - \Delta K}\right)^{1/n}\right]^n$$

Introducing the coordinate transformation

$$x = \Delta K\left(\frac{\Delta K_c}{\Delta K_c - \Delta K}\right)^{1/n} \qquad (21)$$

gives then a power-law type equation:

$$\frac{da}{dN} = \frac{C_F}{\Delta K_c} x^n \qquad (22)$$

On a logarithmic coordinate scale this yields a straight-line fit because

$$\log\frac{da}{dN} = n \log x + \log\frac{C_F}{\Delta K_c} \qquad (23)$$

as shown on Fig. 5.

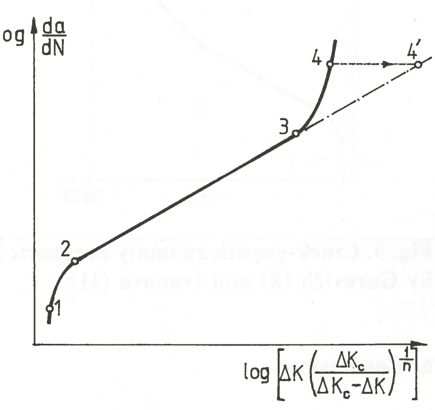

Fig. 5. Forman transformation proposed in Ref. (6)

Error-margin calculations for formula (22) and for all other smoothing functions can be based on the value of the relative error standard deviation Δ. Let us have for the crack propagation curve j measured points ΔK_i – da_i remembering that i = 1 ∸ j and let us denote the corresponding crack propagation values as given by say Eq. (22) as da_{fi}. The formula giving the relative error standard deviation would be then:

$$\Delta = \sqrt{\frac{1}{j}\sum_{i=1}^{j}\left(\frac{da_{fi} - da_i}{da_{fi}}\right)^2} \qquad (24)$$

Eq. (22) gives also the relationship between the constants C_P and C_F as

$$C_P = \frac{C_F}{(1 - R) K_c} \qquad (25)$$

Following this line of thought we are presently experimenting with two variants of a four-parameter crack propagation function. The first one of them reads:

$$\frac{da}{dN} = C_F \frac{(\Delta K - \Delta K_{th})^n}{(1 - R) K_c - \Delta K} \qquad (26)$$

Regression is being done after the transformation

$$x = (\Delta K - \Delta K_{th})\left(\frac{\Delta K_c}{\Delta K_c - \Delta K}\right)^{1/n} \qquad (27)$$

Our second formula is:

$$\frac{da}{dN} = C_F \frac{\Delta K - \Delta K_{th}}{(1 - R) K_c - \Delta K} \Delta K^r \quad where \;\; r = n - 1 \qquad (28)$$

and the corresponding regression transformation reads:

$$x = \Delta K\left(\frac{\Delta K - \Delta K_{th}}{\Delta K} \frac{\Delta K_c}{\Delta K_c - \Delta K}\right)^{1/n} \qquad (29)$$

Both variants have given promising results in the first trials. As reported in Ref. (6) relative error standard deviations Δ = 0.0925 ∸ 0.1138 have been obtained for multi-piece test series. After a prolonged testing period we shall select the more exact of the two variants. If the first indications prove to be correct then safety factors of say m ≅ 1.5 might be acceptable using this formula.

5. Anomalies in fatigue-crack propagation

Environmental conditions, especially corrosion, can have an adverse influence on crack propagation rate. Corrosion-fatigue crack diagrams may be made up of several seemingly different sections (see e.g.: Vosilovsky (18)). Most authors – if indeed they care for fitting such graphs – assess such test data by local Paris equations. This piecemeal treatment makes a unified evaluation of the process as a whole most difficult if not impossible (see also Speidel (16)).

We have tried to improve upon this situation by reference to a statistical correlation between the Paris constant C_P and the exponent n. Several authors (listed e.g. in Ref. (6)) found a close relationship as shown on Fig. 6 to be

$$\log C_P = \log p - n \log q \qquad (30)$$

p and q being constants. Eq. (30) can be put in the form:

$$C_P = p \frac{1}{q^n} \qquad (31)$$

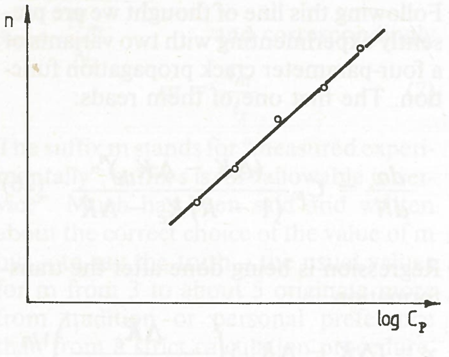

Fig. 6. Statistical correlation between the Paris constants

Eqs. (30) and (31) imply that for a given material Paris straights for different (non-corrosion) conditions pass through the pole point

$$\Delta K^* = q\,, \quad \frac{da^*}{dN} = p$$

(see Fig. 7).

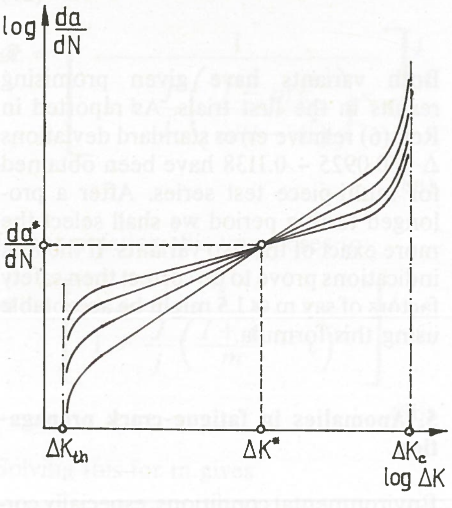

Fig. 7. Geometrical interpretation of the correlation

In corrosion fatigue anomalies in crack propagation rate are to be expected in section II, as shown of Fig. 8a–b, accompanied sometimes also by a decrease of ΔK_{th} (see e.g. Refs. (16) and (18)). The critical value ΔK_c and section III of the graph remain essentially unchanged. We can exploit this the following way.

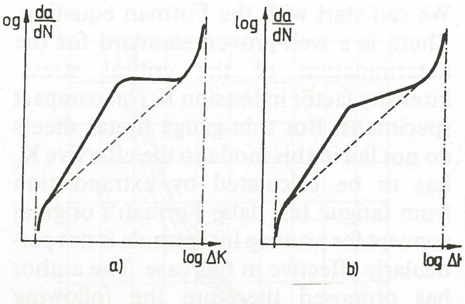

Fig. 8. Typical shapes of corrosion-fatigue graphs

After assessment of ΔK_c from section III of the graph using the Forman formula, regression of the remaining parts can be made with the same ΔK_c using the Forman formula by means of Eqs. (26) and (27) or Eqs. (28) and (29). Now anomalies can be defined in terms of a growth in da* and /or a lowering of ΔK^* resulting in a much more coherent picture of the process.

Allied with this we also refer to an observation reported by Gurevich and his coworkers (8) and by Ivanova and her coworkers (11). Some metals display even without a corrosion influence a breakpoint in section II as shown in Fig. 9. The increase in crack growth rate from ΔK_a is connected with and may be explained by a change in the character of the micrographs of the fracture. We also have observed this phenomenon. The possibility of the identy of ΔK with ΔK_a cannot be fully excluded at this stage although speaking of a definite probability would be premature.

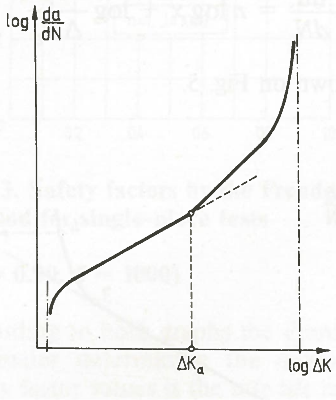

Fig. 9. Crack-growth anomaly as reported by Gurevich (8) and Ivanova (11)

6. Conclusions

Because of the scatter problem incorporation of an appropriate life safety factor m in every fatigue test evaluation is essential. For multi-piece test series this can be done in the form of an individual confidence band calculation. A realistic and safe life factor for a single-piece test can be calculated only from the scatter distribution, i.e. in the knowledge of the respective Weibull parameters. From among these the safe life ratio ε is the dominant one.

The most safe and economical way of covering fatigue problems is by safety-by-inspection mode certification and operation. Fracture mechanical methods of crack propagation rate evalution can give us realistic data for an inspection schedule calculation.

Corrosion fatigue results in a manifold increase of the crack propagation rate. Our four-parameter crack propagation formula and regression method is adaptable to these conditions, too.

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Appendix notation:

a	half crack length, mm
j	number of test pieces, number of data points
m	life safety factor
n	exponent
p	constant, mm/c
q	constant, $N/mm^{3/2}$
r	exponent
x	transformed ΔK coordinate, $N/mm^{3/2}$
t	service life, h
J	fleet size
K	stress-intensity factor, $N/mm^{3/2}$
ΔK	stress-intensity range, $N/mm^{3/2}$
N	number of load cycles, number of cycles to failure
ΔN_{Ic}	nominal inspection interval
$P(\,)$	failure probability
R	stress ratio reliability level
ΔU	inspection interval
$W(\,)$	Weibull function
α	Weibull shape parameter, Weibull exponent
β	Weibull scatter parameter
β^*	Weibull scale parameter
ε	safe life ratio
ε^*	Freudenthal safe life ratio
φ	stress concentration factor
σ	far-field normal stress, MPa
Δ	relative error standard deviation

Subscripts:

c	critical value
i	rank order of specimen
f	as given by the function f ( )
j	as determined from j specimens
m	measured experimentally
max	maximum
min	minimum
s	allowable in service
th	treshold value
F	Forman formula
P	Paris-Erdogan formula
o	for zero failure probability