Semi-Dynamic Thermalling

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Present theory and practice of thermalling (climbing in a thermal) is based on an assumption that the stationary circling is the only way to achieve the maximum rate of climb in an isolated thermal. This paper's main objective is to dispute the generality of this assumption. Therefore, a nonstationary computer analysis of thermalling flight has been carried out. A computer program capable of simulating the three-dimensional sailplane flight has been developed and used to assess the possibility of attaining a higher rate of climb, by performing "semi-dynamic" manoeuvers inside a thermal.

Dynamic effect

Dynamic effect in thermal soaring is still a little-known phenomenon among glider pilots. Thus, it was felt that some key aspects of this controversial issue should be explained here first.

Generally, the dynamic effect is the effect of an unsteady energy exchange between a sailplane and a non-uniform air current. The non-uniformity of the flow is essential in this case; hence, either a wind shear or a thermal lift gradient must be present.

There are two different approaches in the analysis of the dynamic effect in thermal soaring. The first considers the sailplane energy in a ground fixed coordinate system, as described by Gorisch (1). However, the sailplane energy exchange calculated in this way is incongruent with the energy exchange observed from the sailplane itself (by a total-energy variometer, for instance). This incongruity stems from the kinetic energy calculation, in which the sailplane ground speed is used. The second approach, which will be discussed here, is based on a non-inertial wind coordinate system that moves along with a sailplane. According to Gedeon (2), the apparent vertical acceleration in such a system is given by:

$$g_n = g + v\,w'\cos\theta \qquad (1)$$

where

$$w' = \frac{dw}{dl} \qquad (2)$$

is the thermal lift gradient along the sailplane flight path. From equation (1) it follows that if the w' is positive, i.e. if the thermal lift increases, the vertical acceleration g_n will be greater than g, and vice versa. Hence, for a certain height change the airspeed will change more for $w' > 0$ than for $w' = 0$, and least for $w' < 0$. For instance, a sailplane can be accelerated to a higher speed (for the same height loss) when approaching the thermal centre, which will result in a dynamic energy gain.

The energy exchange due to dynamic effect can be evaluated by comparing the change in sailplane kinetic energy for $w' \neq 0$ and $w' \neq 0$. According to the energy conservation law, the gain in kinetic energy is equal to the loss in potential energy, minus energy dissipation due to drag. So, for a non-inertial coordinate system (figure 1) we obtain:

$$dE_k = -m\,g_n\,dz^* - D\,ds \qquad (3)$$

and with g_n substituted from equation (1):

$$dE_k = -m\,(g + v\,w'\cos\theta)\,dz^* - D\,ds \qquad (4)$$

Note that dz^* is the height increment in relative wind coordinates, so it does not depend on the value of thermal lift w. For the constant vertical wind speed, the previous equation reads:

$$dE_k = -m\,g\,dz^* - D\,ds \qquad (5)$$

Comparing equations (4) and (5), the energy exchange differential due to dynamic effect can be defined as:

$$dE_d = -m\,v\,w'\cos\theta\,dz^* \qquad (6)$$

According to figure 1
dz can be substituted by:

$$dz^* = ds\sin\theta = v\,dt\sin\theta \qquad (7)$$

Equation (6) now becomes:

$$dE_d = -\frac{m\,v^2}{2}\,w'\sin(2\,\theta)\,dt \qquad (8)$$

This relation can also be expressed in terms of the equivalent rate of climb due to dynamic effect:

$$w_d = -\frac{v^2}{2\,g}\,w'\sin(2\,\theta) \qquad (9)$$

because

$$m\,g\,w_d = \frac{dE_d}{dt} \qquad (10)$$

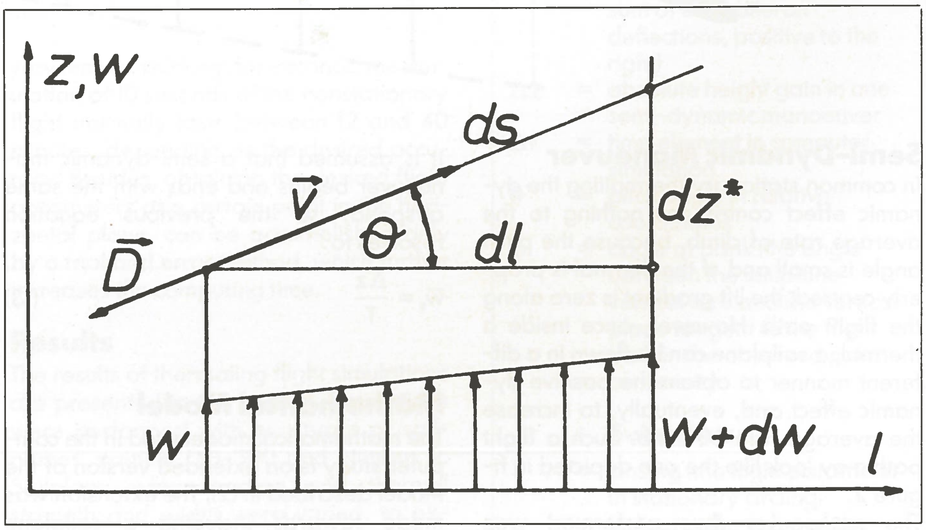

Figure 1. Flight in a non-inertial coordinate system

Equations (8) and (9) show that a positive dynamic effect, i.e. dynamic gain in energy, is obtained when w' and θ have different signs, otherwise a loss of energy occurs. The absolute value of w_d reaches its maximum for $\theta = \pm 45°$, and falls to zero for either $\theta = 0°$ or $\theta = \pm 90°$. For example, for $v = 50\,m/s$ $w' = 0.04\ s^{-1}$ and $\theta = -45°$, equation (9) gives $w_d = 5.1\ m/s$. This shows that the dynamic effect may have a substantial impact on the overall energy exchange between a sailplane and a thermal.
Finally, the total-energy exchange equation calculated in terms of climb rates, reads as follows:

$$w_{TE} = w + w_d - w_e \tag{11}$$

It should be noted that the TE variometer reading (which is equal to w_{TE}) is also affected by the dynamic effect. Possible magnitude of this influence is clearly shown by the foregoing example.

named "semi-dynamic", because a considerable portion of the average rate of climb achieved by it, is the result of the dynamic effect. Semi-dynamic thermalling consists of a series of these manoeuvers (which are in fact coordinated chandelles), performed around the centre of thermal.
The average rate of climb in this case can be calculated as:

$$\overline{w}_s = \frac{1}{T}\int_{t=0}^{t=T} w_{TE}\,dt \tag{12}$$

movement in all three dimensions. He[nce] the model is capable of describing the steady three-dimensional sailplane fl[ight] in variable thermal lift.
The main features of this model are as [fol]lows:
1. A sailplane is represented by a r[igid] body possessing definite moments of [in]ertia about longitudinal (rolling) and ve[rti]cal (yawing) axes. The moment of ine[rtia] about transverse (pitching) axis is [as]sumed to be zero.

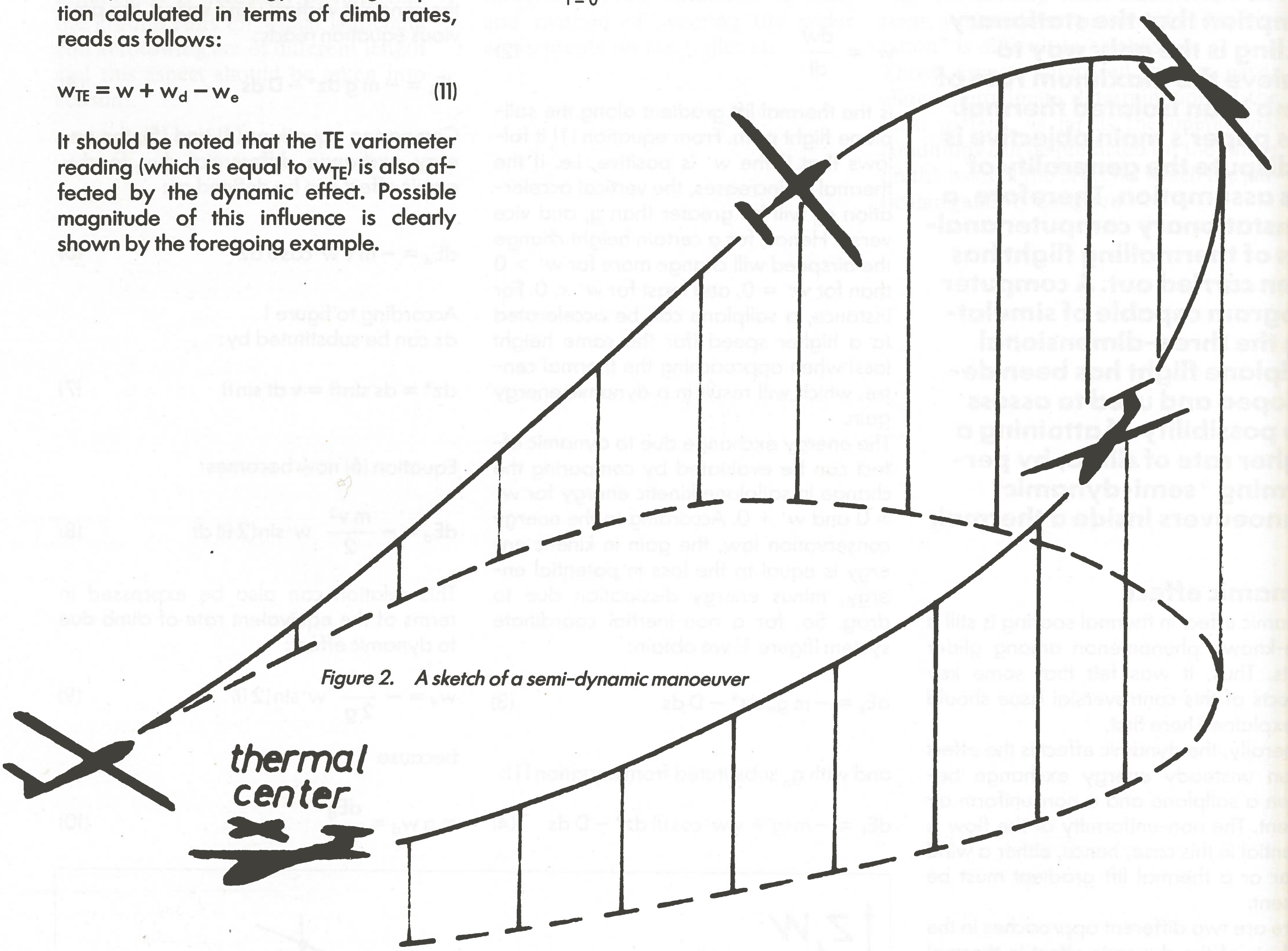

Figure 2. A sketch of a semi-dynamic manoeuver

Semi-Dynamic Maneuver

In common stationary thermalling the dynamic effect contributes nothing to the average rate of climb, because the pitch angle is small and, if the thermal is properly centred, the lift gradient is zero along the flight path. However, once inside a thermal, a sailplane can be flown in a different manner to obtain the positive dynamic effect and, eventually, to increase the average rate of climb. Such a flight path may look like the one depicted in figure 2.
The manoeuver thus performed was

It is assumed that a semi-dynamic manoeuver begins and ends with the same airspeed, so the previous equation resolves to:

$$\overline{w}_s = \frac{\Delta z}{T} \tag{13}$$

Mathematical Model

The mathematical model used in the computer study is an extended version of the model described in (2). The extension was made towards including a sailplane

2. Angular displacements and velocitie[s in] roll and yaw are the result of mome[nts] caused by control (aileron and rudd[er]) deflections, together with moments of [in]ertia and aerodynamic damping a[nd] coupling moments. Thermal lift gradien[t in] the lateral direction is also taken into [ac]count in the roll calculation. All the nec[es]sary coefficients and derivatives were [de]termined from the sailplane geometry, [us]ing the formulas given in (3) and (4).
3. Rudder deflection is always chosen [to] satisfy the zero slip condition.
4. Pitching manoeuvers are accomplish[ed]

by prescribing a value of the normal load factor, providing that the lift coefficient constraint is not violated. This is warranted by a necessity to keep a close control over the load factor during a computer run, and by simplicity.

5. The sailplane drag coefficient c_D is calculated as a function of the lift coefficient c_L, aileron and rudder deflections and angular velocities in roll and yaw. No allowance is made for c_D variation due to angular velocity in pitch, sailplane elasticity, or Reynolds number changes. Thus, the calculation of c_D represents a compromise between simplicity and accuracy, which was found to give good results in a reasonable computing time.

Computer Program

The above mathematical model was used in writing a computer program capable of simulating the three-dimensional sailplane flight through a given thermal model. The thermal model used in this study is the same as in (5), and it is shown in figure 3 in relative coordinates. R_t and w_{max} are the free parameters which allow the formation of various combinations of thermal strengths and radii. Other thermal models can be easily implanted into the existing program, so the influence of the thermal shape could be investigated.

The program consists of three main parts. The first part performs the calculation of stationary circling. For a given sailplane and a given thermal, it determines the optimal flight parameters that lead to the maximum circling rate of climb. The calculation is based on the equations given in (6), with the sailplane drag coefficient being determined in the way described in the previous section. Hence, the calculated circling rate of climb is more realistic, though the difference between taking into account the drag increment due to aileron and rudder deflections, and neglecting it, is in this case very small.

The second part of the program does the nonstationary flight simulation. It solves the three-dimensional equations of motion, using a step-by-step procedure with a constant time element Δt. (The length of Δt was 0.6 seconds in preliminary runs and 0.2 seconds in final runs.) Before the calculation of each step, the normal load factor and the aileron deflection angle must be prescribed. Using these two values and the values of other flight parameters at the beginning of the step, the program calculates the coordinates and the flight parameters at the end of the step. If the given load factor calls for too high a lift coefficient, it will be automatically reduced to satisfy the $c_L \leqq c_{Lmax}$ constraint.

The third part of the program is in fact a subroutine which calculates rolling and yawing moments and the sailplane drag coefficient. It is addressed from both the first and the second part of the program.

In its current stage, the program is written in Basic for a ZX Spectrum home computer. It calls for an interactive work which is very time consuming; for instance, the simulation of 10 seconds of the nonstationary flight normally lasts between 12 and 40 minutes, depending on the desired accuracy. Besides, obtaining the desired flight parameters at a certain point in the horizontal plane, can be accomplished only by a trial and error method, which further increases the computing time.

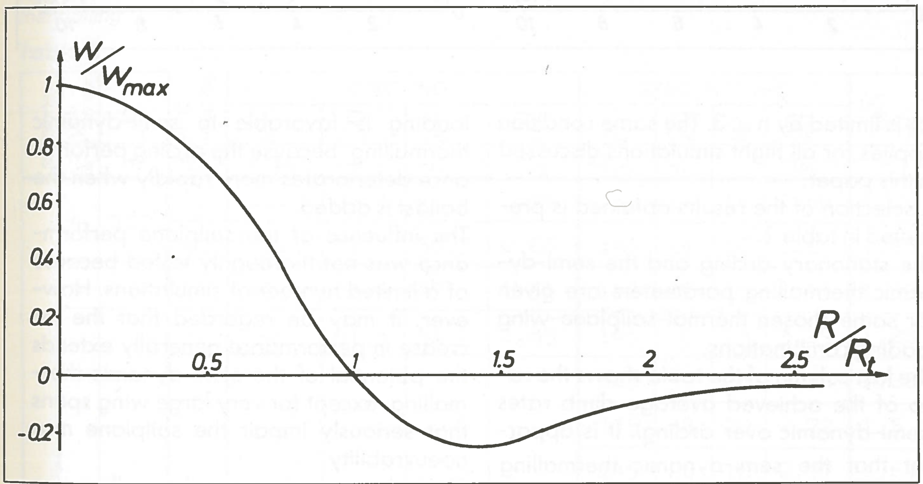

Figure 3. Thermal cross-section in relative coordinates

Results

The results of thermaling flight simulations are presented in this section. Simulations were performed with two types of sailplanes, namely DG-300 and Nimbus 2. Sailplane wing loading and thermal strength and width were varied, to examine their influence on the optimal thermalling trajectory. The air density was assumed constant (σ = 1.05 kg/m²) throughout the research.

List of Symbols

- c_L = lift coefficient
- c_D = drag coefficient
- d = differential sign
- dl = element of the flight path horizontal projection
- ds = flight path element
- dz^* = height increment in relative coordinates
- g = gravity acceleration
- g_n = equivalent acceleration of the non-inertial coordinate system
- m = sailplane mass
- n = normal load factor
- t = elapsed time from the beginning of the manoeuver
- v = airspeed
- v_i = indicated airspeed
- w = thermal lift (vertical wind speed)
- w' = thermal lift gradient along the flight path
- w_d = equivalent rate of climb due to dynamic effect
- w_e = sailplane sink rate
- w_{max} = maximum updraft velocity in the thermal centre
- w_{TE} = total-energy-height exchange rate, also the momentary reading of total-energy variometer
- $\overline{w}_c$ = maximum achieved circling rate of climb
- $\overline{w}_s$ = average semi-dynamic rate of climb
- z = absolute height
- D = sailplane drag
- E_d = dynamic energy exchange
- E_k = kinetic energy
- R = distance from the centre of thermal
- R_t = thermal radius
- T = period of semi-dynamic thermalling (duration of a single manoeuver)
- δ_a = aileron deflection angle (the sum of both aileron deflections, positive to the right)
- Δz = absolute height gain in one semi-dynamic manoeuver
- Δt = time element in computer simulation
- θ = pitch angle in relative coordinates
- φ = angle of bank (the angle between the sailplane vertical axis and the vertical plane tangent to the flight path in the observed point)
- ϱ = air density

Subscripts:

- ϑ = circling (the flight parameter in stationary circling)
- o = indicates the value of the flight parameter at the initial (the lowest) point of a semi-dynamic manoeuver

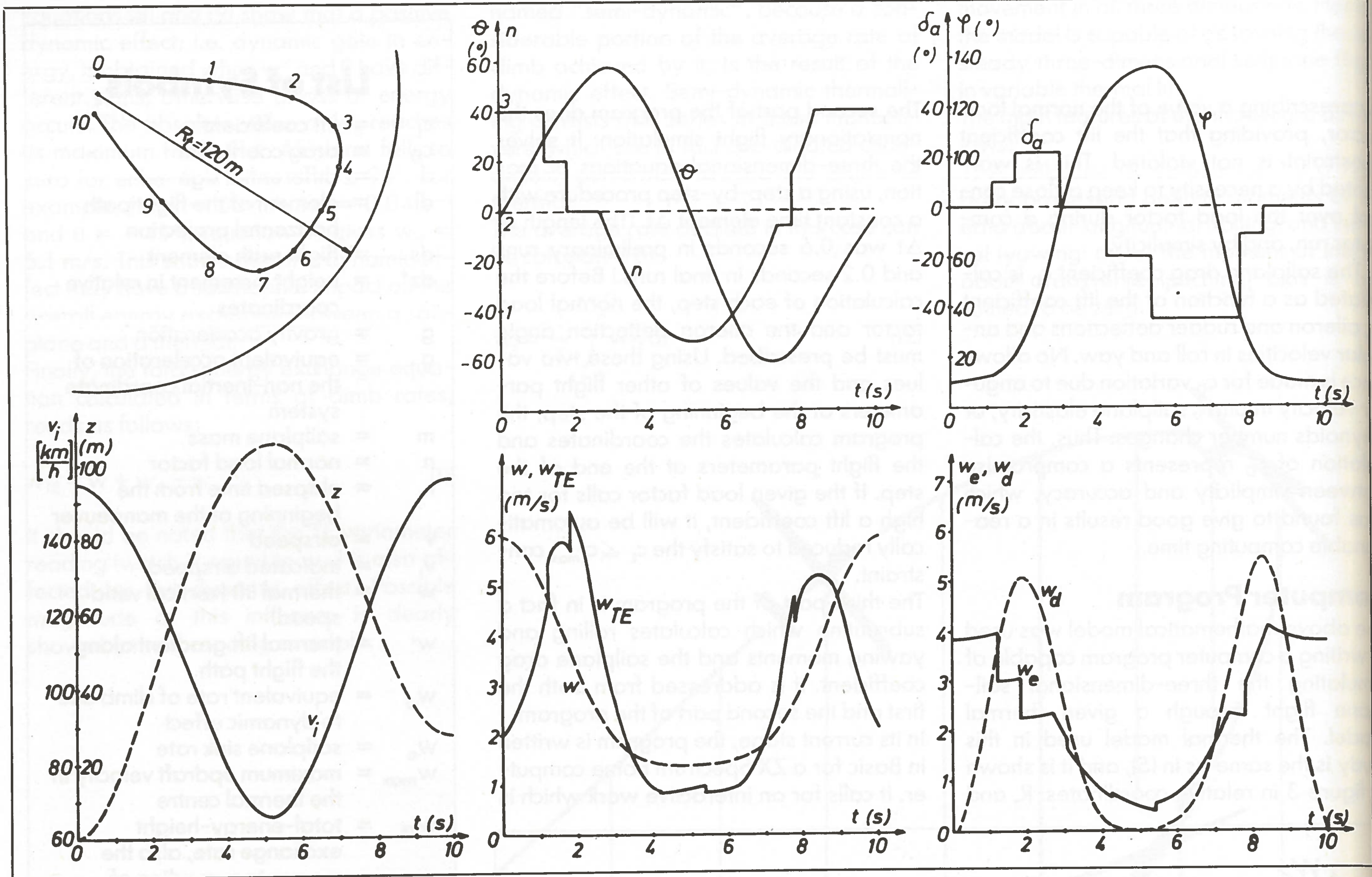

Figure 4. Example of a semi-dynamic manoeuver

An example of a simulated semi-dynamic manoeuver with a DG-300 sailplane is given in figure 4. The wing loading is 40 kg/m² and the thermal parameters are w_{max} = 6 m/s and R_t = 120 m. The drawing in the top left corner shows the flight path viewed from above, with the numbered dots indicating the elapsed time in seconds. The rest of diagrams show the momentary values of flight parameters versus the elapsed time, so they can be associated with the trajectory plot. It is evident that the highest airspeed and the lowest altitude are reached at the initial and the final points of the manoeuver, which are also the closest to the thermal centre. The lowest v_i and the greatest z are attained around point 5 near the thermal edge. All this is done to produce a favorable dynamic effect, shown in the last diagram through its equivalent rate of climb w_d. It is obvious from the same diagram, however, that this manoeuver also entails a drag penalty which increases the sailplane sink rate w_e. Still, the overall total-energy exchange rate w_{TE} is appreciably high.

Note that w_{TE}, which also indicates the total-energy variometer reading, does not, even remotely, represent the profile of the net thermal lift along the flight path. This is partly due to the manoeuvring drag increase, but, the main reason is the dynamic effect.

Finally, looking at diagrams which show the values of pitch and bank angles, this particular manoeuver can be regarded as aerobatic, although the normal load factor is limited by $n \leqq 3$. The same condition applies for all flight simulations discussed in this paper.

A selection of the results obtained is presented in table 1.

The stationary circling and the semi-dynamic thermalling parameters are given for some chosen thermal-sailplane-wing loading combinations.

The last column of the table shows the ratio of the achieved average climb rates (semi-dynamic over circling). It is apparent that the semi-dynamic thermalling gives considerably higher rate of climb for strong and narrow thermals; in some extreme cases it can amount to more than double the maximum circling climb rate. For all the other parameters constant, decreasing the thermal radius makes the semi-dynamic thermalling more profitable with respect to circling. The same thing happens when the maximum thermal lift is increased. In both cases the dynamic effect becomes more significant because the thermal lift gradient gets sharper.

For a fixed maximum thermal lift and a given sailplane with a constant wing loading, exists a critical thermal radius at which the best semi-dynamic manoeuver can only match the circling rate of climb. This radius is highly dependent on the sailplane wing loading. For a DG-300 sailplane and w_{max} = 6 m/s the critical radius lies between 150 and 210 metres for the wing loading between 32 and 44 kg/m². The research has shown without an exception that the higher wing loading is favorable to semi-dynamic thermalling, because the circling performance deteriorates more rapidly when the ballast is added.

The influence of the sailplane performance was not thoroughly tested because of a limited number of simulations. However, it may be regarded that the increase in performance generally extends the potential of the semi-dynamic thermalling, except for very large wing spans that seriously impair the sailplane manoeuvrability.

Since the semi-dynamic thermalling is a complicated unsteady motion, it is of great importance to perform it properly. Several main parameters, such as the maximum airspeed v_o, the maximum load factor etc., influence the average rate of climb. Only the influence of the maximum airspeed was investigated, and the result is shown in figure 5. It applies to DG-300 sailplane with 40 kg/m² wing loading in a thermal with W_{max} = 6 m/s and R_t = 120 m. It is apparent that the semi-dynamic rate of climb is not very sensitive to variations of the maximum airspeed in the vicinity of its optimal value. Figure 5 also shows that the stationary circling can be regarded as a special case of semi-dynamic thermalling. The transition between them is continuous not only in speed, but, in other parameters as well. Looking back at table 1 it can be noticed that the minimal radius from the thermal center R_o and the minimal angle of bank φ_o increase as the maximum speed decreases towards the circling value. This is necessary in order to execute the manoeuver correctly, i.e. to complete it with the same flight parameters it was started with.

Practical utilization of the semi-dynamic thermalling is not a straightforward matter as it is in stationary circling. There are a few problems which may impair, or even negate the advantage in the average rate of climb.

The first obstacle is the improper instrumentation for the task. A common TE variometer is very hard to interpret when there is the dynamic effect involved together with high load factors. So, the chances are that the thermal will be lost after a short time.

A dynamically compensated netto variometer would be much more suitable for the semi-dynamic thermalling. It should not respond to changes in the load factor, nor to the dynamic effect. However, the latter is not easy to achieve, due to an implicit relation between the vertical wind speed and the dynamic energy exchange. Also, an instrument that could show the lateral thermal lift gradient, would be very useful in determining the location of the thermal centre.

Actually, any kind of thermal visualization would be of great help in the semi-dynamic thermalling. A soaring bird, a dust devil, or a puff of smoke left intentionally in the thermal centre, can make the difference between success and failure.

Finally, it should be pointed out that the semi-dynamic thermalling involves certain hazards inherent in its aerobatic nature. Also, the risk of mid-air collision is much higher than in circling flight. Therefore, this kind of thermalling should only be performed away from other sailplanes.

Conclusion

The semi-dynamic thermalling was in some cases found to produce a higher rate of climb than the stationary circling. It can be performed within the limits of sailplane structure and human capabilities, although a more sophisticated instrument than a total-energy variometer is needed. However, the risks associated with this kind of thermalling may in some cases preclude its practical use.

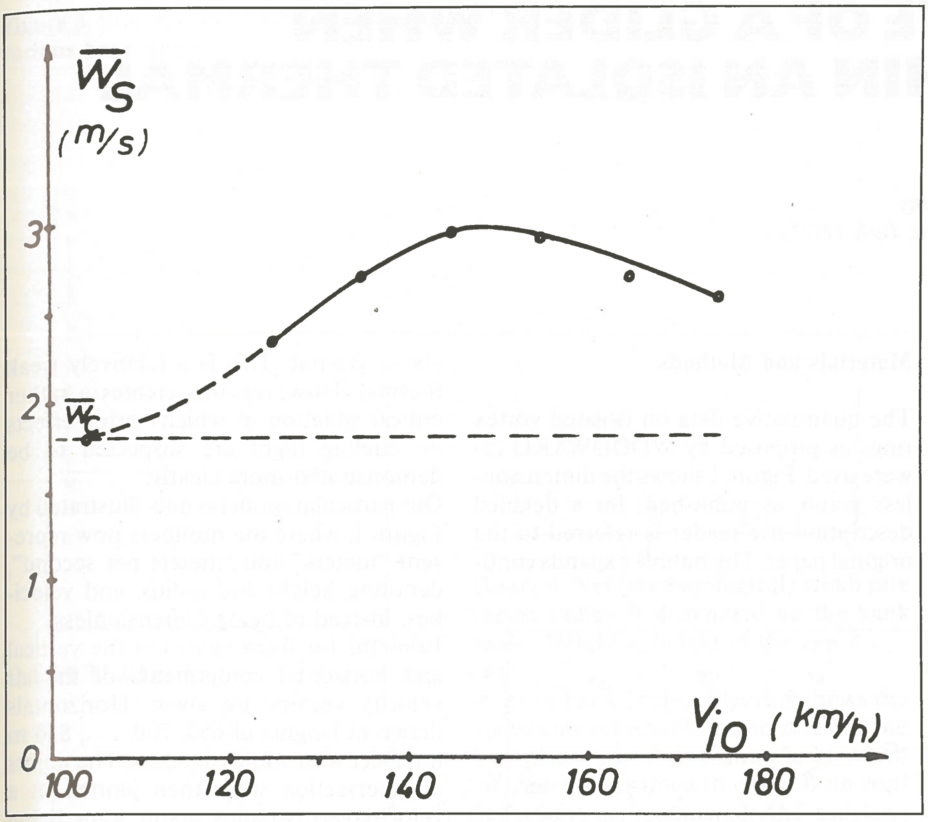

Figure 5. The influence of the maximum airspeed on the average rate of climb in semi-dynamic thermalling

Table 1

sailplane	W_{max}	R_t	wing loading	CIRCLING V_{ic}	R_c	φ_c	$\overline{W_c}$	SEMI DYNAMIC V_{io}	R_o	φ_o	$\overline{W_s}$	$\overline{W_s}/\overline{W_c}$
	m/s	m	kg/m²	km/h	m	°	m/s	km/h	m	°	m/s	–
DG-300	4	120	32	88.3	64.3	48	1.44	135	14	18	1.56	1.08
			40	100.9	77	50.5	0.65	145	24	16	1.2	1.85
	4	150	40	97.4	81.2	47	1.31	145	27	16	1.43	1.09
	6	120	32	91	61.5	51	2.85	145	0	13	3.29	1.28
			40	104.3	73.9	53.5	1.81	125	50	28	2.32	1.28
								135	38	20	2.72	1.5
								145	20	16	2.97	1.64
								155	7	10	2.92	1.61
								165	9	10	2.7	1.49
								175	6	10	2.61	1.44
			44	111.3	79.6	55	1.23	160	12	10	2.71	2.2
	6	150	32	87.8	64.8	47.5	3.49	145	5	13	3.49	1
			40	100.9	77	50.5	2.72	155	17	16	3.33	1.22
			44	106.8	83.3	51.5	2.3	160	12	10	3.01	1.31
	6	180	40	97.9	80.6	47.5	3.26	155	20	10	3.5	1.07
			44	103.6	87.1	48.5	2.94	160	17	10	3.27	1.11
	6	210	44	101.1	90.6	46	3.35	160	20	10	3.38	1.01
Nimbus 2	6	120	37	101	69.3	53.5	2.31	120	50	30	2.69	1.16
								130	35	25	3.04	1.32
								140	20	20	3.23	1.4
								148	10	15	3.46	1.5
								160	7	20	3.16	1.37
								180	0	10	2.99	1.29

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