The Energy Loss in Pitching Manoeuvres

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Success in soaring depends on the efficient extraction of energy from the atmosphere and on its efficient utilisation. The first part of this process involves seeking regions of ascending air and avoiding regions where it is descending; the second part requires the pilot to follow some sort of optimised flight path, such as that indicated by the MacCready construction.

Now the MacCready analysis, even in its more sophisticated Calculus-of-Variations form, implicitly assumes that the load factor on the sailplane (i.e. lift/weight) is substantially unity (Refs. 1 and 2). In the course of the analysis, it also emerges that vertical flight paths with zero load factor are admissible. If there are vertical motions in the air traversed by the sailplane, then the pilot will have to adjust his speed accordingly, but the underlying assumption is that the drag at any instant is the same as the steady-state value at the instantaneous speed and hence it is possible to derive the usual relation between optimum speed and variometer readings by a calculation based on the steady-flight performance curve. In practice, if the speed adjustments are neither too sudden nor to great, this assumption is very reasonable and, in any case, the effects of the changes of load factor will mostly be self-cancelling. However, a pilot wishing to pursue low-loss flying will want to know how to deal with large adjustments of speed, as when getting out of or into a thermal. Since even the more sophisticated analysis only recognises load factors of unity and zero, it offers only rather impracticable advice: to indulge in vertical dives or climbs.

Trying to introduce the load factor as another variable under the control of the pilot is not very rewarding and it is clear that no analytical solution will emerge. It is also likely that the optimum manoeuvre in any particular circumstances would require even greater-than-usual powers of prophecy by the pilot and would, in any case, be too difficult to apply in real life. Attempts (Refs. 3 and 4) have been made to analyse dolphin-flying by computer calculations but, whilst they have been successful, it is rather difficult to disentangle the effects due to the manoeuvres of the sailplane from those due to the atmospheric motions.

It therefore seemed sensible to analyse in detail a single pull-up/push-over manoeuvre in an attempt to establish some easily-defined technique for minimising the energy loss in such a manoeuvre. To simplify the calculations, the pull-up was assumed to take place at a constant load factor, starting from level flight at 100 knots. When the sailplane had slowed down to a certain speed, a pushover was initiated – again at a constant load factor – until the sailplane regained level flight at about 40 knots. (See Fig. 1). The machine was assumed to have typical Standard Class performance: a maximum lift/drag ratio of 35 at 50 knots.

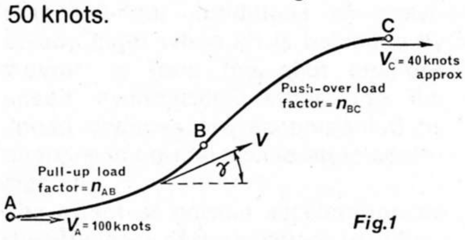


Fig.1

Diagram of the pull-up/push-over manoeuvre showing the notation used in subsequent graphs.

The equations of motion in these circumstances are such that there is no simple analytical solution relating, say, speed and flight path slope for a given load factor. However, they can be reduced to a first-order non-linear differential equation which can be solved numerically by a step-by-step process.

It is clear that when the speed has fallen to the chosen value at the end of the pull-up (the «intermediate speed»), there is only one possible push-over load factor which will take the machine from that particular combination of speed and flight path slope to the desired final conditions. It is therefore necessary to find, by a trial-and-error process, the load factor appropriate to each such push-over. Fortunately, a suitable value can be obtained from quite approximate calculations, since great accuracy in the final speed is not necessary.

For a given initial load factor, several speeds can be chosen at which to terminate the pull-up, each leading to its individual push-over. For each complete manoeuvre, the load factor and speed are known at all points, and hence it is possible to calculate the rate of loss of energy height at each instant and thus to find the total loss of energy height. The energy height represents the sum of the potential and kinetic energies per unit weight of the sailplane and is defined by

$$h_e = h + V^2/2g.$$

In fact, the calculations did not involve time explicitly but used flight path slope as the independent variable, as explained in Appendix I.

It will be inferred that there was no gradation of load factor at the ends of the manoeuvre, nor at the point of inflexion. Clearly, going instantaneously from a load factor, of say, 2.0 to a value of 0.2 is unrealistic, but inserting a smooth gradation has a negligible effect on the overall energy situation.

One would not expect much variation of total energy loss as the initial load factors and intermediate speeds of the manoeuvres are changed because there are two swings-and-roundabouts situations prevailing:

1. To some extent, the increase in induced drag during the pull-up will be cancelled by the decrease in the push-over.
2. A large initial load factor will produce an appropriately large increase in the induced drag but, for a given intermediate speed, the larger the load factor, the shorter the time for which it is applied.

Figure 2(a) shows that, for a given initial load factor, there is an intermediate speed which minimises the total energy loss for the whole manoeuvre. For example, with an initial load factor of 2.0, the optimum intermediate speed is about 70 knots. As it happens, this is just about the mean of the initial and

(a) Loss of energy height, (b) push-over load factor and (c) flight path slope at point B, all plotted as functions of the speed at point B for various pull-up load factors.

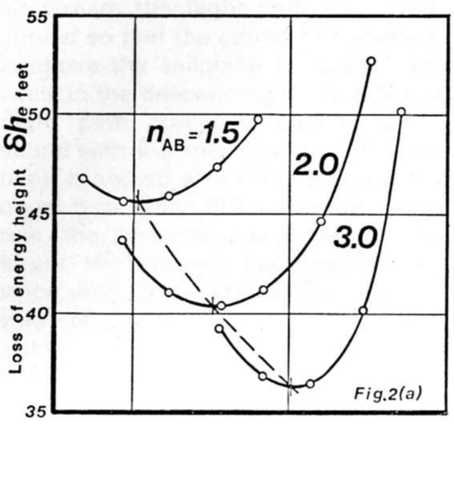
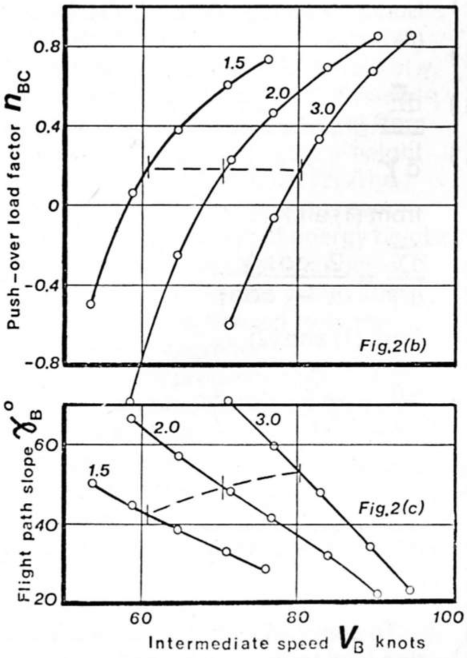


Fig.2(a)

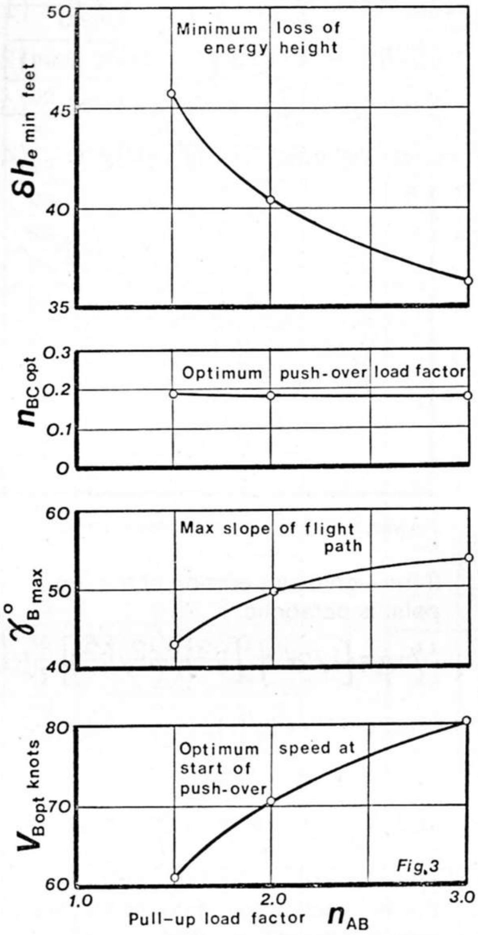


final speeds but it is clear from the other curves that this is not generally true: the higher the initial load factor, the higher should be the intermediate speed.

Figure 2(b) shows the push-over load factor corresponding to various intermediate speeds for each pull-up load factor and Figure 2(c) shows the corresponding flight path slopes. Figure 3 summarizes the conditions corresponding to the minima of Figure 2(a).

It is clear from Figure 3 that the *minimum* loss of energy height decreases as the initial load factor increases – at any rate, up to any value likely to be employed in real life. Evidently, in situation (ii) above, the brevity of the pull-up wins. More generally, the optimum manoeuvre involves applying a large load factor for a short time when the speed is high and the induced drag is a small proportion of the total drag. Much of the manoeuvre occurs at a low load factor, thus keeping the induced drag small even at low speeds. One can infer that the optimum speed-increasing manoeuvre would consist of a push-over at a low load factor until quite a high speed had been attained, followed by a sharp, short pull-out.

A surprising feature of the results is that the optimum push-over load factor is almost constant, at about 0.18, for all pull-ups. There seems to be no analytical reason why this should be so: it simply emerges from the computations. In these examples, only one set of end-conditions has been considered so that this figure, and the various other features of Figures 2 and 3 are obviously



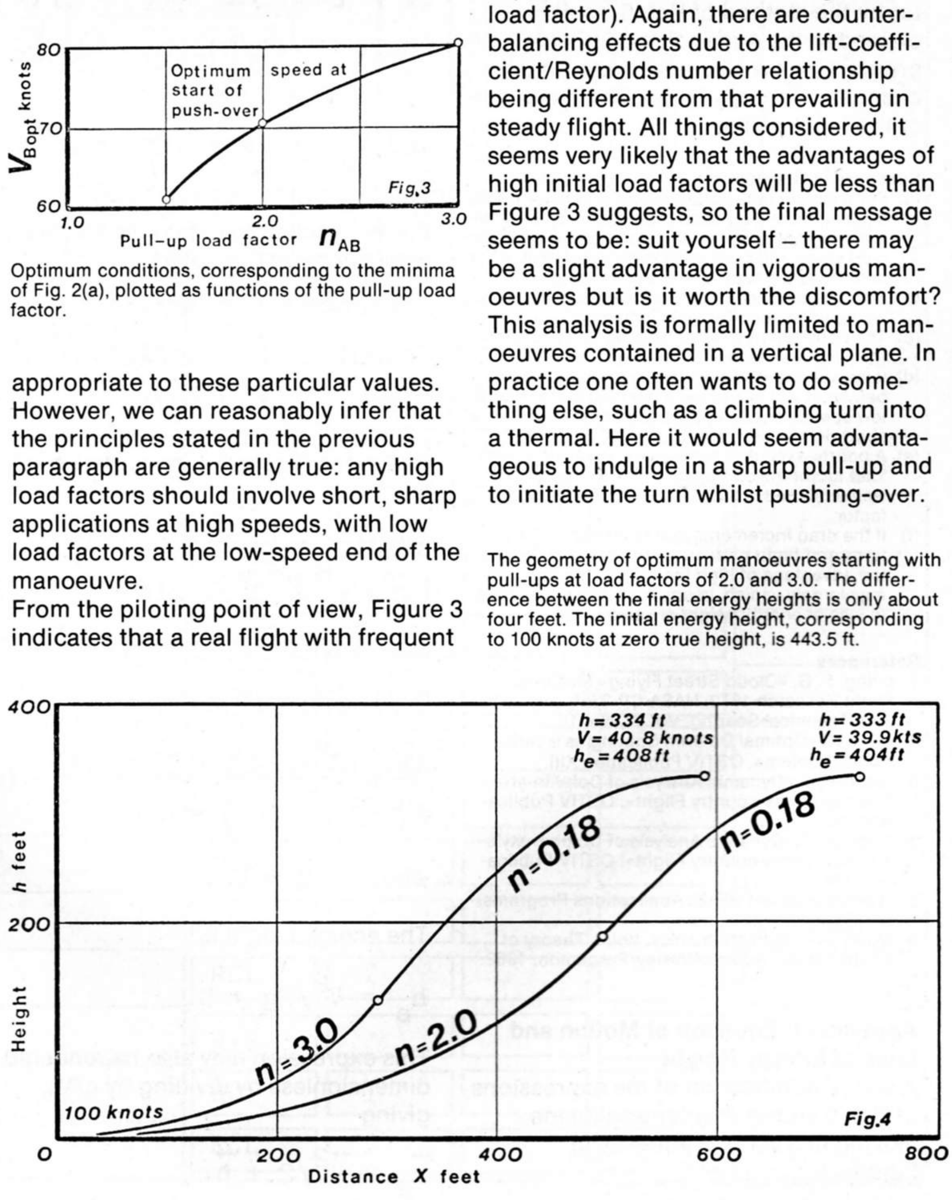
Optimum conditions, corresponding to the minima of Fig. 2(a), plotted as functions of the pull-up load factor.

appropriate to these particular values. However, we can reasonably infer that the principles stated in the previous paragraph are generally true: any high load factors should involve short, sharp applications at high speeds, with low load factors at the low-speed end of the manoeuvre.

From the piloting point of view, Figure 3 indicates that a real flight with frequent speed adjustments would be a vigorous – indeed possibly emetic – experience. It is also clear from Figure 2(a) that a poorly executed manoeuvre with a high initial load factor may be less efficient than a well-executed one at a lower initial load factor. The actual differences in minimum energy height loss are quite small: increasing the initial load factor from 1.5 to 3.0 saves about 9 ft in this case. In a more typical manoeuvre during a cross-country flight, the figure might well be 2 or 3 feet. If such manoeuvres occurred frequently in the course of a flight, the overall saving might become significant, perhaps equivalent to a turn or two in the last thermal. But these calculations take no account of the drag increments due to control deflections and to the curvature of the flight path (i.e. the fact that, relative to the aircraft, the free-stream streamlines are curved. This is quite a separate effect from the changes of load factor). Again, there are counterbalancing effects due to the lift-coefficient/Reynolds number relationship being different from that prevailing in steady flight. All things considered, it seems very likely that the advantages of high initial load factors will be less than Figure 3 suggests, so the final message seems to be: suit yourself – there may be a slight advantage in vigorous manoeuvres but is it worth the discomfort?

This analysis is formally limited to manoeuvres contained in a vertical plane. In practice one often wants to do something else, such as a climbing turn into a thermal. Here it would seem advantageous to indulge in a sharp pull-up and to initiate the turn whilst pushing-over.

The geometry of optimum manoeuvres starting with pull-ups at load factors of 2.0 and 3.0. The difference between the final energy heights is only about four feet. The initial energy height, corresponding to 100 knots at zero true height, is 443.5 ft.



It is, of course, more important to get quickly into the best part of the thermal than to fuss about the elegance of the entry manoeuvre. A further consideration is the structural strength: one needs to avoid superimposing a large manoeuvring load factor on a gust load. On the other hand, sailplanes are quite strong, maximum speeds in rough air are now quite high and at lower speeds it is quite difficult to cause damage. Figure 4 shows height/distance plots of typical manoeuvres. The loss of energy height is of the order of 10% of the initial value, taking the initial true height to be zero. It is worth noting that if the sailplane simply ascended vertically from an initial 100 knots to a final 40 knots, the loss of energy height would be only about 12 feet. All of the calculations relate to conditions near sea level. The solutions of the equation of motion were performed on a Hewlett-Packard HP-25 programmable calculator by the method of Ref. 5, as explained in the Appendices. Suitable programs were also devised to find the changes of energy height and the shape of the flight paths.

Summary of Conclusions

(a) For a simple pull-up/push-over manoeuvre with a given initial load factor, there is a value of the intermediate speed (with a corresponding flight path slope and push-over load factor) which minimises the total loss of energy height.
(b) The minimum loss of energy height diminishes as the initial load factor is increased.
(c) The optimum push-over load factor is substantially independent of the pull-up load factor.
(d) It may be inferred that, in any pitching manoeuvre, it will pay to keep the load factor low at low speeds and to apply a high load factor for a short time at high speeds.
(e) A poorly-executed manoeuvre involving a high load factor may dissipate more energy than a well-executed manoeuvre with a lower load factor.
(f) If the drag increments due to control deflections and flight path curvature are introduced, the advantage of high load factor manoeuvres may largely vanish. In any case, the differences in loss of energy height are small.

References

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Appendix I: Equation of Motion and Loss of Energy Height

A slight modification of the expressions of Ref. 6 shows that for a sailplane moving in a vertical plane as in Figures 5:

$$dx/dt - V\cos\gamma = 0 \qquad (1)$$

$$dh/dt - V\sin\gamma = 0 \qquad (2)$$

$$D + m\,(g\sin\gamma + dV/dt) = 0 \qquad (3)$$

$$L - m\,(g\cos\gamma + V d\gamma/dt) = 0 \qquad (4)$$

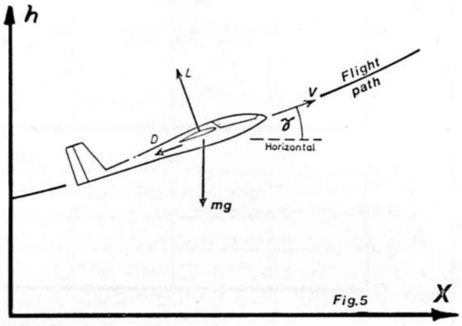

Diagram to illustrate the equations of motion.

If the significant portion of the drag polar is parabolic,

$$D/mg = \left[1/2E^*\right]\left[\bar{V}^2 + (n^2/\bar{V}^2)\right], \qquad (5)$$

where $E^* = (L/D)_{max}$, $n = L/mg$,

$$\bar{V} = V/V_R,$$

and V_R = speed for max (L/D)

(All speeds are „true").

It is convenient to define the following dimensionless quantities:

Distance $\bar{X} = Xg/V_R^2$;

Height $\bar{h} = hg/V_R^2$;

Time $\bar{t} = tg/V_R$.

Equations (3) and (4) may then be written:

$$d\bar{V}/d\bar{t} = -\left[1/2E^*\right]\left[\bar{V}^2 + (n^2/\bar{V}^2)\right] - \sin\gamma, \qquad (6)$$

$$\text{and } \bar{V}d\gamma/d\bar{t} = n - \cos\gamma. \qquad (7)$$

Dividing (6) by (7) leads to

$$\frac{dZ}{d\gamma} = \frac{(Z^2+n^2)/E^* + 2Z\sin\gamma}{\cos\gamma - n} \qquad (8)$$

where $Z = \bar{V}^2$.

The energy height is

$$h_e = V^2/2g + h.$$

This expression may also be rendered dimensionless by dividing by g/V_R^2, giving

$$\bar{h}_e = \bar{V}^2/2 + \bar{h}. \qquad (9)$$

Hence, from equations (9), (2), (7) and (8)

$$\frac{d\bar{h}_e}{d\gamma} = \frac{Z^2 + n^2}{2E^*\,(\cos\gamma - n)}\,; \qquad (10)$$

from (1) and (7)

$$\frac{d\bar{X}}{d\gamma} = \frac{Z\cos\gamma}{n - \cos\gamma}; \qquad (11)$$

from (1) and (2)

$$\frac{d\bar{h}}{d\gamma} = \frac{d\bar{X}}{d\gamma}\tan\gamma\,; \qquad (12)$$

from (7)

$$\frac{d\bar{t}}{d\gamma} = \frac{\bar{V}}{n - \cos\gamma}. \qquad (13)$$

To summarize, the equations of motion lead to (8), which relates V and γ. Equation (10) gives the changes of energy height, (11) and (12) describe the geometry of the manoeuvre and (13) enables time to be introduced. All of these equations have been rendered dimensionless.

For a given value of n, (8) is of the form $y' = f(x,y)$ and may be solved for given initial conditions by the method of Ref. 5 using a Hewlett-Packard HP-25 Programmable Calculator. At first sight, there seem to be insufficient available steps to insert $f(\gamma,Z)$, but there are several redundant steps elsewhere in the published program. The present program is given in Appendix II.

Suitable intervals of γ for the pull-ups are 0.02 or 0.04 radians. For each pull-up, a few convenient values of Z corresponding to various values of V_B were taken. It was then necessary, for each Z, to find the load factor n_{BC} which made Z_C about 0.64 when γ_C was zero. This was done by trial-and-error, using the same program, initially with quite coarse intervals of γ. Great accuracy is not necessary at this stage since the final energy height is not particularly sensitive to errors in V_C.

Appendix II: HP-25 Program Flight Paths for Sailplanes

Comments
Line 16: Flag in R_4
Line 18: γ or $\gamma + \delta\gamma$ in R_5
Line 44: Updated γ in R_1
Line 48: Updated Z in R_2
Line 49: Displays Z
Lines 18–36 inclusive represent the routine for $f(\gamma,Z)$.

Using the values of n_{BC} a more accurate calculation of the flight path was then carried out using smaller intervals of γ. Further programs were then devised to find the change of energy height from equation (10) and the distance-height relationship from (11) and (12). The accuracy of these calculations seems good: the total change of energy height calculated from the total changes of height and speed agrees within about one foot with that derived from the step-by-step integration.

The flight-path program given in Appendix II can obiously be applied to manoeuvres other than those described here, which is why it seemed useful to display it in detail. If many such calculations are to be done, the limitations of a small programmable calculator become rather obtrusive and it would pay to use a full-sized computer.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY
00		
01	34	CL x
02	23 04	STO 4
03	24 02	RCL 2
04	24 01	RCL 1
05	13 18	GTO 18
06	22	R↓
07	23 03	STO 3
08	24 00	RCL 0
09	61	X
10	24 02	RCL 2
11	51	+
12	24 01	RCL 1
13	24 00	RCL 0
14	51	+
15	01	1
16	23 04	STO 4
17	22	R↓
18	23 05	STO 5
19	14 04	f sin
20	02	2
21	61	X
22	21	x≷y
23	61	X
24	14 73	f last x
25	15 02	g x^2
26	24 07	RCL 7
27	15 02	g x^2
28	51	+
29	24 06	RCL ·6
30	71	÷
31	51	+
32	24 05	RCL 5
33	14 05	f cos
34	24 07	RCL 7
35	41	−
36	71	÷
37	24 04	RCL 4
38	15 71	g x=0
39	13 06	GTO 06
40	22	R↓
41	24 03	RCL 3
42	51	+
43	24 00	RCL 0
44	235101	STO+1
45	61	X
46	02	2
47	71	÷
48	235102	STO+2
49	24 02	RCL 2

REGISTERS	
R_0	$\delta\gamma$ (Radians)
R_1	γ
R_2	Z
R_3	$dZ/d\gamma$
R_4	FLAG
R_5	γ OR $\gamma+\delta\gamma$
R_6	E^*
R_7	n

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Key in program						
2	Store intervals of γ	$\delta\gamma$ rads	STO	0			
3	Store initial	γ_o rads	STO	1			
	conditions	Z_o	STO	2			
4	Store other data	$E*$	STO	6			
5	Insert chosen n	n	STO	7			
6	Set to radians		g	RAD			
7	Initialize		f	PRGM			
8	Solve first step		R/S				
			RCL	1			γ_1 rads
			RCL	2			Z_1
9	Repeat as often		R/S				
	as desired		RCl	1			γ_2, etc
			RCL	2			Z_2, etc