Energy Exchange Between a Sailplane and Moving Air Masses under Nonstationary Flight Conditions with Respect to Dolphin Flight and Dynamic Soaring

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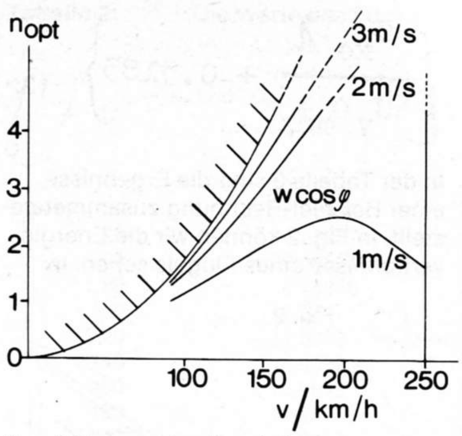


Figure 2. The curves show the optimal g-number vs. the velocity v for three values of thermal strength w cosφ. (φ is the angle between lift and air velocity vectors.)

Introduction

Previous calculations of energy gain during the penetration of thermals use a simple mechanical model: i. e. that the mechanical power gained during the lift equals the product of weight and resulting rate of height gain. This relation is valid where stationary flight conditions are fulfilled, i. e. where the resultant aerodynamic force vector equals the attraction force of the earth.

Recently the dolphin style has predominated during contest and performance flights. This style is characterized by a sequence of nonstationary flight figures and seems to lead to a surprising increase in cross country speed compared to that which the MacCready method would predict. This fact leads us to assume that the effect of dolphin style flight cannot be explained by the MacCready method.

We have nowadays a remarkably high standard in sailplane design and an extensive knowledge of aerodynamics, but we know very little about the energy transfer between sailplane and moving air masses. A better understanding of these interconnections will lead us to provide rules that contest pilots should follow and that, perhaps will affect sailplane design in the future.

Theoretical analysis:

A rectangular xyz co-ordinate system is used, with its origin being a fixed point on the earth's surface. All vectors, such as position, velocity and acceleration, hence forces and energies, are defined in this system. This inertial system allows for calculating mechanical problems like that here under study.

A basic relation of mechanics says that mechanical work equals the product of force and the distance over which the force is applied. Its derivative, the power, is given by the product of the force affecting a body and its velocity. In our case, the total energy derivative is prescribed by the product between two vectors: aerodynamic force and observed velocity. (See figure 1.) Since the aerodynamic force can be represented by an appropriate g-number, the rate of energy transfer must strongly depend on the g-loading, which is mainly under the control of the pilot.

On the other hand it is well known that higher load will increase drag. It is assumed that this trade-off will have an optimal g-number, depending on the velocity, the updraft velocity and the glider's performance.

In order to evaluate this factor, lift and drag will be replaced in terms of their coefficients as in (4):

Assuming:

$$(1) \qquad \dot{E} = (\vec{L}+\vec{D})(\vec{v}+\vec{w})$$

i. e.

$$(2) \qquad \dot{E} = \vec{L}\vec{v} + \vec{L}\vec{w} + \vec{D}\vec{v} + \vec{D}\vec{w}$$

where $\vec{L}\vec{v} = o$, because the vector $\vec{L}$ is perpendicular with respect to $\vec{v}$.

$\vec{D}\vec{w}$ will be neglected as both factors are small.

Thus:

$$(3) \qquad \dot{E} = \vec{L}\vec{w} + \vec{D}\vec{v} = Lw\cos\varphi - Dv$$

The angle φ is the angle between lift and air velocity vectors.

$$(4) \qquad \dot{E} = \frac{\varrho}{2} A v^2 (c_L w \cos\varphi - c_D v)$$

with air density ϱ and wing area A.

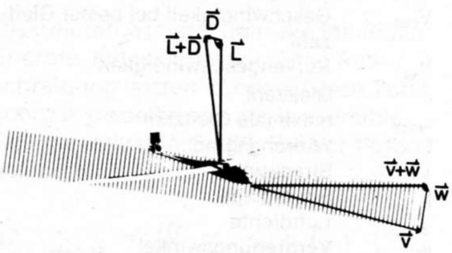


Figure 1. Vectors of forces and velocities affecting a glider and which are relevant for the calculation of energy transfer. The Lift $\vec{L}$ and the drag $\vec{D}$ are the parts of the resultant force vector $\vec{L}+\vec{D}$, whereas the velocity $\vec{v}$ and the wind velocity $\vec{w}$ combine resulting in the observed velocity $\vec{s}=\vec{v}+\vec{w}$.

The differential $d\dot{E}/dc_D$ is taken and then set equal to zero (i. e. to evaluate maximal $\dot{E}$:

$$(5) \qquad \frac{d\dot{E}}{dc_D} = \frac{\varrho}{2} A v^2 \left(\frac{dc_L}{dc_D} w\cos\varphi - v\right) = 0$$

This procedure results in an equation, which gives the optimal lift coefficient corresponding to the maximal $\dot{E}$:

$$(6) \qquad \frac{dc_L}{dc_D} = \frac{v}{w\cos\varphi}$$

The left side of the equation is simply the slope of the c_L/c_D polar – the right side is a number, which can be calculated from the aircraft velocity relative to the surrounding air, the wind velocity and the angle φ.

Equation (6) permits calculation of the optimal lift coefficient which gives the optimal g-number n_{opt} to fly with, where n_{opt} is given by: $c_L(opt)/c_L(v,n=1)$.

A standard sailplane, ASTIR CS with water ballast serves as an example. As figure 2 shows, the optimal g-number as a function of the velocity has been evaluated for three values of thermal strength: 1, 2 and 3 meters per second. We see that except for low thermal activity (1 m/s) the optimal g-number n_{opt} is very close to the maximal g-number which can be achieved at a given speed. It seems likely, that except for low air velocities, the real g-loading is restricted to a rather low value not far beyond 1 given by the real circumstances such as the spatial extent of thermals to be penetrated. It seems that the development of special instruments which indicate the "best load to fly" is not worthwhile, the reason being that in most cases it is impossible to follow the command.

Since the rate of energy exchange $\dot{E}$ is dependent on the load, $\dot{E}$ has been evaluated for several g-numbers. The same sailplane as mentioned above is used. The results are given in figure 3. It is concluded, that a weak updraft of about 1 m/s offers only small amounts of energy, even if the optimal g-number is realized. From 2 m/s upwards, the air velocity strongly enhances the possible energy gain by use of an increased g-number.

Although these calculations have been drawn from the polar curve of a common standard sailplane, they should be representative for all modern sailplanes. Having got this far, we will now continue to describe mathematically the mechanical problem of a motorless sailplane flying in turbulent air with respect to its energy exchange. We will obtain an expression which can be integrated to give the amount of energy gained or lost during integration time.

We will start with equation (2):

$$(2) \qquad \dot{E} = \vec{L}\vec{v} + \vec{L}\vec{w} + \vec{D}\vec{v} + \vec{D}\vec{w}$$

We recall that $\vec{L}\vec{v}$ vanishes as the two vectors are perpendicular to each other.

$$(7) \qquad \dot{E} = \vec{L}\vec{w} + \vec{D}(\vec{v}+\vec{w})$$

Three relations are now taken into account:

$$(8.1) \qquad M\vec{g} = M\vec{g}_V + M\vec{g}_L$$

represents the upward directed reactive force against the earth's attraction force. One of its two components is defined by the direction of the vector $\vec{v}$ and represents the inertia. The second component is now fixed in its direction and in its value. Since this component does not contribute to the acceleration (or deceleration) of the sailplane, it must be a part of the lift vector $\vec{L}$:

$$(8.2) \qquad \vec{L} = M\vec{g}_L + M\vec{f}$$

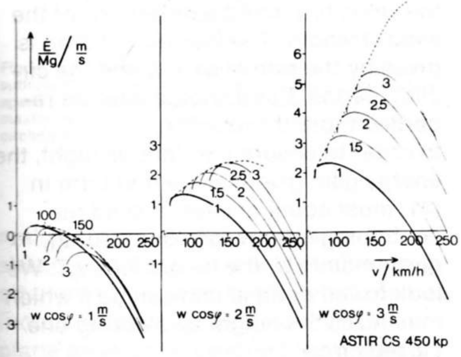


Figure 3. Rate of energy gain in terms of total energy compensated quasi-climb, for several values of the g-number and three values of thermal strength. The n = 1 curve represents the well known polar curve (shifted by the wind velocity). The dotted line is a kind of "envelope" and gives the highest possible rate of energy transfer corresponding to the optimal n_{opt} (see fig. 2).

The lift vector itself contains another, centripetal, force $M\vec{f}$ which is responsible for a curved flight path.

$$(8.3) \qquad M\ddot{\vec{s}} = -M\vec{g}_V + M\vec{f}$$

Lastly we introduce $\ddot{\vec{s}}$ which is the second derivative of the vector of position or, the first derivative of the flight path velocity $\dot{\vec{s}}$; $\dot{\vec{s}} = \vec{v} + \vec{w}$.

The actual rate of change in speed as seen from the earth is given by the accelerating (or decelerating) component of the earth's attraction pointing in the direction of the vector $\vec{v}$. The second term $M\vec{f}$ has already been mentioned above. It should be noted that $\vec{f}$ is under the control of the pilot and can be changed by manipulating the elevator; $\vec{f}$ also includes the effect of gusts which the plane meets during its flight. The centrifugal acceleration $\vec{f}$ is perpendicular to $\vec{v}$ and thus does not effect its value, but rather changes the flight path of the plane, and thus the direction of $\vec{v}$. The relations (8.1–3) are applied to equation (7) which leads to:

$$(9) \qquad \dot{E} = M(\ddot{\vec{s}}+\vec{g})\vec{w} + \vec{D}(\vec{v}+\vec{w})$$

Equation (9) represents the general solution of the mechanical problem dealt with. Since the flight conditions can be described by the $\ddot{\vec{s}}$ and $\vec{v}$ vectors and the air flow is known, it is then possible to evaluate the rate of energy exchange. The role of drag is shown in the second term; $\vec{D}\vec{w}$ should be neglected as both factors are small resulting in a negligible product value. We now get $-Dv$ as the rate of total energy loss due to the drag, which can be calculated depending on the known lift coefficient and on the c_L/c_D polar curve. The drag loss can be expressed as $-Mgv_s$ where v_s is the quasi sink speed of the sailplane.

In order to obtain the optimal flight path for any problem considered, with maximum total energy gain during the time interval Δt, equation (9) must be integrated:

$$(10) \qquad \Delta E = M\int_{\Delta t}(\ddot{\vec{s}}+\vec{g})\vec{w}\,dt - Mg\int_{\Delta t} v_S\,dt$$

It is assumed that numerical methods and variation theory must be used to solve such an equation, which it is not intended to do here. We will see later that under certain simplifying conditions the integral (10) can be evaluated more easily.

From (9):

$$(11) \qquad \dot{E} = M(\ddot{s}_x w_x + \ddot{s}_y w_y + (\ddot{s}_z+g)w_z) - Mg\,v_S$$

where all vectors of equ. (9) are written in component form.

We will now deal with the first example to be discussed:

Dolphin-style flight

Some special conditions are introduced:

$$(12) \qquad w_x = w_y = 0\,; \quad w_z = w$$

This means, that the air flow is restricted to vertical movements alone. We also assume, that the sailplane moves in a (vertical) x-z plane. Equation (11) then appears as:

$$(13) \qquad \dot{E} = M(\ddot{s}_z+g)w - Mg\,v_S$$

We replace now the vertical component $M(\ddot{s}_z+g)$ of the airplane's actual lift by the corresponding component of its actual g-number:

$$(14) \qquad M(\ddot{s}_z+g) = Mgn\cos\varphi$$

where φ represents the angle between the velocity vector v and the horizontal. As mentioned earlier, φ corresponds likewise to the angle between the vertically directed air velocity and the lift vector. Applying (14) to (13) we obtain:

$$(15) \qquad \dot{E} = Mgnw\cos\varphi - Mg\,v_S = Mg(nw\cos\varphi - v_S)$$

The rate of total energy exchange can now be evaluated. $\dot{E}$ normalized by Mg can be determined graphically:

The speed polar curve (v_s versus v) pertaining to the g-loading n will be shifted vertically by the "effective" air current $w \cdot \cos\varphi$ which itself is multiplied by the g-number n. As expected, this procedure leads to results which totally coincide with these obtained previously and which are shown in the drawings of figure 3.

Although these results are valuable, we have to answer the pilot's question for rules that ensure, for example, highest possible energy gain during the penetration of thermals.

In order to do this we must integrate equation (13).

Assuming that the air current remains steady during the time interval Δt, equation (13) can now be integrated easily. The result is:

$$(16) \qquad \Delta E = M\Delta\dot{s}_Z\,w + Mg(w - v_S)\Delta t$$

We conclude that the amount of energy transfer is dependent on the vertical speed difference $\Delta\dot{s}_z$ between entry and exit, as illustrated in figure 4, which can be considered as a nonstationary contribution to the energy exchange and, as a rule, must be maximized. The second term represents the well known relation, where the energy difference is proportional to the time interval during which the sailplane remains inside the air current; the rate of sink v_s must also be taken from the appropriate polar curve depending on the g-number n.

It is noted at this point, that the time independent "nonstationary" energy term $M\Delta\dot{s}_z w$ from equation (16) can have positive or negative sign which means gain or loss of energy respectively. The sign is positive when $\Delta\dot{s}_z$ and w are both positive or both negative, whereas it is negative when $\Delta\dot{s}_z$ and w are opposite in sign. In other words: energy is lost in those cases, where the vectors of the air current and the change of the vertical velocity point in opposite directions.

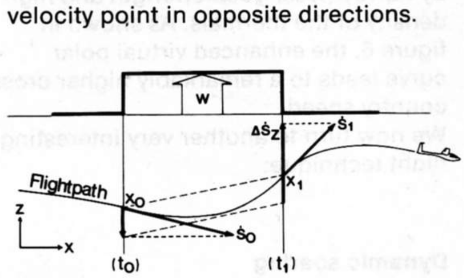
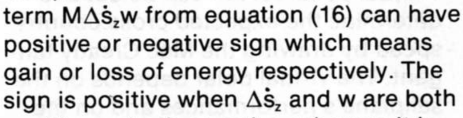


Figure 4. A flight path which offers advantages in energy gain. The vertical component $\Delta\dot{s}_z$ of the velocity must be increased during the time interval between entry and exit points x_0 and x_1 of the thermal. This flight movement also satisfies the previous demand for an increased average g-loading to be maintained inside the thermal. Since only the difference in vertical velocity components is considered, there is no further need for reference to the angle φ.



	a,	b,	c,	d,
ΔE_{instat}:	pos.	neg.	neg.	pos.
ΔE_{stat}:	pos.	pos.	neg.	neg.

Figure 5 shows four possible cases, where the separate energy terms are different in sign. ΔE_{instat} is the nonstationary energy term which is connected to $\Delta\dot{s}_z$. It can by positive or negative, depending on the two directions (signs) of w and $\Delta\dot{s}_z$. ΔE_{stat} depends on the plane's period of stay inside the moving air mass of strength ±w. Its sign is given by the sign of w (upward = pos.). A third energy term, given by the drag, is always negative.

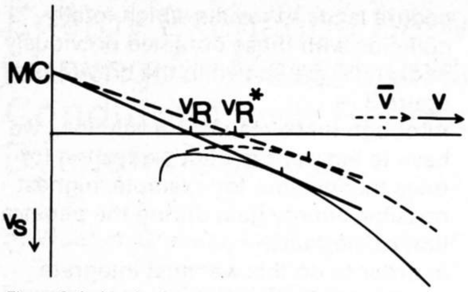


Figure 6. A virtual polar can be used to determine the average "best speed to fly" in order to maximize the cross country speed by drawing the tangent through the climb speed found in thermals, the MacCready value MC. The virtual polar depends mainly on the flight technique, but, additionally depends on the thermal's feature e. g. strength, narrowness and thermal spacing ie. density.

Figure 5 shows the four possible cases a) to d) and the signs pertaining to the energy terms (positive: gains, negative: losses). With respect to a positive energy balance the pilot should avoid the two situations b) and c) because the "nonstationary" contributions to the energy interchange are loss-producing. The main aim of this paper is to provide the reader with a better understanding of the energy balance of a sailplane flying through moving air masses, where the real flight conditions are taken into account. The results should make us think over the role of the "best speed to fly". Since higher average speeds increase the possible energy gain due to nonstationary flight techniques, the speed polar values can be "improved" markedly at these higher speeds. This "virtual polar curve" can be used to determine the expected cross country speed by drawing the Mac Cready tangent. The virtual polar depends on the sailplane's performances and on the flight technique, by which the pilot takes advantage of the air flow nonhomogeneities he meets during flight. The desirable "dolphin weather" is characterized by narrowness, good strength and high density of the thermals. As shown in figure 6, the enhanced virtual polar curve leads to a remarkably higher cross country speed.
We now turn to another very interesting flight technique:

Dynamic soaring

In cases where horizontal wind strength varies with height, i. e. in the zone of wind shear, there exist manoeuvres which ensure long flights for motorless planes. This kind of soaring, called dynamic soaring, is exploited by large sea birds e. g. albatrosses, in the wind shear region near the sea surface. It is a wave-like or circling flight containing steep rise and fall movements with turns very close to the water surface.
A mathematical analysis of this flight technique can be derived using the universal equation (11):

(11) $$\dot{E} = M(\ddot{s}_x w_x + \ddot{s}_y w_y + (\ddot{s}_z + g) w_z) - M g v_s$$

We remember that the total energy E and 1st derivative $\dot{E}$ are defined here using an inertial co-ordinate system fixed relative to the earth. Other authors use a system which is connected to the air masses surrounding the aircraft. In the writer's opinion, this non-inertial system is not well suited for the treatment of the energy exchange during dynamic soaring.
We assume, that w_y und w_z do not exist, w_x being dependent on the height z:

(17) $$w_y = w_z = 0; \quad w_x = w(z)$$

We write:

(18) $$\dot{E} = M \ddot{s}_x w(z) - M g v_s$$

The total energy exchange during the time interval Δt can be given by the integral:

(19) $$\Delta E = M \int_{\Delta t} \ddot{s}_x w(z)\, dt - M g \int_{\Delta t} v_s\, dt$$

Although equation (18) is the general description of the problem under discussion, it is not easily applied.
The integral can be performed easily under one simplifying condition: that the wind strength remains constant during the integration period Δt, i. e. the height is not changed during Δt, we obtain:

(20) $$\Delta E = M \Delta \dot{s}_x w(z) - M g \int_{\Delta t} v_s\, dt$$

The last term gives the energy loss due to the drag. The factor $\Delta \dot{s}_x$ is the difference between the horizontal x-components of the observed velocities at the two integration boundary points.

We can conclude that a closed circle flown at constant height leads to a vanishing term of dynamic energy exchange, as expected. In contrast, a certain amount of energy can be gained (or rather, retained) where the two semicircles are flown at heights with different wind strength. Similar thoughts were first published by the famous physicist Lord Rayleigh in NATURE (1883).
We will now discuss a simple model (see figure 7) where the wind velocity follows

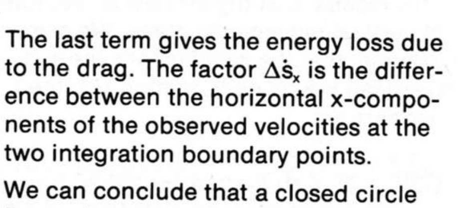


Figure 7. Model situation for dynamic soaring: a horizontal plane divides the space into two parts of different wind strengths w_1 and w_2. The flight path can be described as follows: the airplane curves from A to B remaining above the plane, at the point B it dives into the area of lower wind strength w_2. After completing the second semicircle the glider comes up again at the point A'. A' differs from A because the glider is being shifted by the average wind strength $(w_1 + w_2)/2$. The angle ε is considered mathematically to be zero.

a step-like dependence on height. Consider a complete circle flown in such a way that one semicircle is completed in the direction of a wind whose strength is w_1, and the other semicircle is finished against a wind of lower strength w_2. The energy balance over a closed circle can be derived from (20) as follows:

(21) $$\Delta E_{circle} = 2 M v (w_1 - w_2) - M g v_s T$$

The dynamic energy gain depends on the velocity v and the difference of the wind strength. The loss due to drag is given by the rate of sink v_s and the circling period T and should later be regarded more thoroughly.
In order to ensure continuous flight, the energy gain given by the first term in (21) must equal or even exceed the continuing energy loss due to drag, given mainly by the height loss $v_s T$. We look for an optimal bank angle β which minimizes the height loss during one closed circle.
The rate of sink depends on the angle-of-bank as follows:

(22) $$v_s(\text{curve}) = v_s(\text{straight}) \cos^{-3/2} \beta$$

where v_s(straight) is taken from the common polar curve at a (lower) speed, where the same lift coefficient is reached. The period T depends on:

(23) $$T = 2 \pi v / (g \tan \beta)$$

Minimal loss is achieved where the product $\tan\beta \cos^{3/2}\beta$ has its maximum and results in the angle $\beta = \arctan \sqrt{2} \approx 54.7$ degrees. It is remarkable that Idrac who investigated the flight of albatrosses published an average of 55 degrees, as taken from film recordings. It seems that the simple model, assuming a step-like distribution of wind velocity, is well suited to describe the problem.
Straight forward calculations lead to a necessary minimal wind shear $(w_1 - w_2)$ of about five times the rate of sink, which is taken from the common speed polar curve:

(24) $$\Delta w = w_1 - w_2 \gtrsim 5 v_s$$

The "performance" data of albatross ($v_s \approx 1$ m/s at a velocity of about 20 m/s) requires a total wind shear exceeding 5 m/s which can in practice be observed near the sea's surface.
Of course this source of energy exploited by birds cannot be utilized for man's flight. A suitable and perhaps sufficient wind shear may occur in jet streams which one day might be used for dynamic soaring. A possible flight path is shown in figure 8. For this more

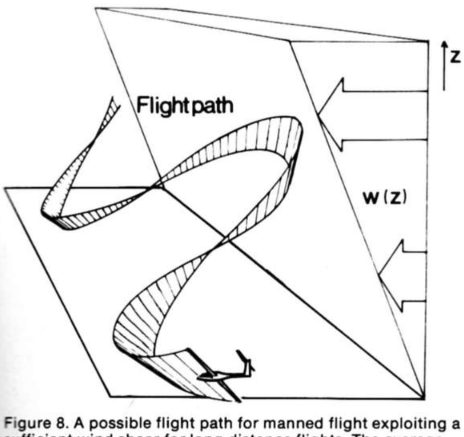


Figure 8. A possible flight path for manned flight exploiting a sufficient wind shear for long distance flights. The average speed to fly must be high enough to take advantage of maximum likely wind shear during the wave-like (or closed) circling flight.

realistic situation the available wind shear depends on the height difference attained during one circle, which can be increased with higher velocity. A sailplane especially designed for dynamic soaring will tend to a low rate of sink at high speeds. The g-loading the pilot must withstand is related to the bank of about 55 degrees and only amounts to about 1.7 times the acceleration due to gravity.

Summary

The system sailplane – moving air masses is treated by the method of classical mechanics using a co-ordinate system fixed to the earth. The rate of total energy exchange is completely analyzed for any flight condition and any flow of air masses. The g-loading proves to have an important meaning.

The results give reasons for, and describe, the dolphin flight technique and dynamic soaring by applying the derived equations to the special conditions found in thermals and wind shear respectively.

Zusammenfassung

Das System Segelflugzeug – bewegte Luftmasse wird dem Formalismus der klassischen Mechanik unterworfen, wobei ein erdgebundenes Koordinatensystem als Bezugssystem für alle Bewegungsgrössen dient. Es werden Formeln angegeben, die für jeden Flugzustand und jede Form der Luftbewegung die zeitliche Änderung der Totalenergie des Flugzeugs beschreibt. Das geflogene Lastvielfache hat dabei eine zentrale Bedeutung.

Die Ergebnisse begründen und beschreiben den Delfin-Flugstil und den dynamischen Flugstil, indem die speziellen Bedingungen, wie sie in Aufwinden bzw. in Windscherungen vorliegen, in die Gleichungen eingesetzt werden.

References:

Pirker, H., Über die Reisegeschwindigkeit von Segelflugzeugen, 2. Teil: Der Delphinflug, Flugsportzeitung 1 (1975).
Meyer, R., Segelflug im Delphinstil, Aero-Revue 12, 671–678 (1975).
Lord Rayleigh, The Soaring of Birds, Nature, 27, 534–535 (1883).
Idrac, P., Experimentelle Untersuchungen über den Segelflug, Verlag von R. Oldenbourg, München und Berlin (1932).
Trommsdorff, W., Flugmechanische und Technische Voraussetzungen für den Dynamischen Segelflug mit bemanntem Fluggerät, OSTIV Publication XIII (1974) Congress at Waikerie, Australia (1974).
Gorisch, W., Instationärer Segelflug, Aerokurier 4 (1975).