DYNAMIC ANALYSIS OF DOLPHIN-STYLE THERMAL CROSS-COUNTRY FLIGHT

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ABSTRACT

A dynamic computer study of sustained thermal cross-country flight using the so-called dolphin techniques has been started. The thermal model allowed for the sinking zone surrounding the core.

Preliminary results are showing the necessity of allowing for three different flight modes in the computer program. The best point for beginning the pull-up was found to be at some distance in advance of the thermal core.

Optimizing for the best normal load sequence in doing the pull-up and the push-down calls for more computer work. Relevant studies are going on and results will be compared with circling cross-country flight performance calculations.

NOTATION

c	thermal lift	[m sec^{-1}]
c_o	maximal thermal lift	[m sec^{-1}]
c_y	lift coefficient	
$c' = dc/dx$	thermal lift gradient	[sec^{-1}]
d	differential sign	
g	gravity acceleration	[m sec^{-2}]
n	normal load factor	
r	flight path radius (in wind co-ordinates)	[m]
t	time	[sec]
$\Delta t = t_2 - t_1$	time taken for a path element	[sec]
v	sailplane speed (in wind co-ordinates)	[m sec^{-1}]

w	sailplane rate of climb	$[m\ sec^{-1}]$
$\dot{w}_a$	vertical acceleration of wind co-ordinate system	$[m\ sec^{-2}]$
x	distance from centre of thermal	[m]
Δx	distance covered in a path element	[m]
y	height gain in wind co-ordinate system in a path element	[m]
G	sailplane weight	[kp]
H	height gain	[m]
$H_D=H+(v^2-v_s^2)/2g$	energy height gain	[m]
R	(nominal) thermal radius	[m]
R_∞	theoretical minimal radius of turn	[m]
S	spacing of thermals	[m]
X	drag	[kp]
Y	lift	[kp]
ϵ	glide ratio	
$\bar{\epsilon}$	glide ratio at v_{min}	
φ	thermal number	
$\varkappa$	thermal field number	
ξ	co-ordinate tangent to flight path	[m]
η	co-ordinate normal to flight path	[m]
θ	glide angle (in wind co-ordinates)	[rad, ^{o}]

SUBSCRIPTS

e	climbing	C	at point C
max	maximal		
min	minimal	1	at the beginning of the path element
s	gliding		
A	at point A	2	at the end of the path element
B	at point B		

INTRODUCTION

Since the advent of really high-performance sailplanes and modern cross-country flight methods the so-called dolphin-style crossing of some of the thermals (i.e. pulling up while entering the thermal and reaccelerating after flying through it in straight flight) should be in the repertoire of every keen sailplane pilot. Nevertheless, at the Euromech Colloquium at Oberwolfach last year most participants did not know exactly the best way to do it, but instead used feel or rule of thumb methods.

Since then two attempts to fill in this gap at least partially have come to the knowledge of the author. Hans Bohli (1) presented a first order approximation for a sailplane crossing an isolated thermal assuming unlimited manoeuvrability and no manoeuvring drag. Results showed the possibility of 9 seconds time saving per thermal by doing the crossing throughout with the appropriate momentary MacCready speed.

Mgr. inz. Andrzej Tomczuk (2) investigated dolphin cross-country speeds in thermal streets of constant lift with constant sink between them; likewise without accounting for dynamic effects.

Both studies are serving their intended purpose well, but rule of thumb calculations and practical experience showed time-speed restrictions and the influence of manoeuvring drag not to be negligible. So the necessity of a dynamic calculation was indicated. The present paper is an attempt in this direction.

At first, a two-parameter investigation was considered, but this number turned out to be too optimistic. Our present programs involve three parameters - thermal strength and diameter, and gliding and climbing speeds.

The resulting increase in computing work made it impossible to complete the whole investigation in time, so at present only the computing methods and preliminary results can be given.

1. THERMAL MODEL

For circling cross-country flight performance calculations most authors prefer to use a parabolic thermal velocity/radius relationship. Needless to say, this practical simple approximation does not work beyond the nominal thermal radius, that is to say in the downdraft region surrounding the thermal core. In MacCready calculations dealing with high climbs followed by straight descents of several kilometres, those few hundred metres could be safely neglected indeed, but here this is not the case. Therefore, a more appropriate thermal velocity distribution had to be sought.

The relationship has been written in the form:

$$c = c_o e^{-\left(\frac{x}{R}\right)^2}\left[1-\left(\frac{x}{R}\right)^2\right] \qquad (1)$$

with maximum updraft c_o and nominal thermal radius R as free parameters.

The thermal cross-section is seen in Fig. 1 in non-dimensional coordinates. No claim for theoretical exactness of this model is made because in developing it, the fulfillment of the continuity equation has been kept in mind only, other considerations being simplicity in handling and sound proportions.

For parametric studies non-dimensional parameters incorporating thermal parameters suitably related to appropriate sailplane ones might be useful. For climbing in an isolated thermal, the following sailplane parameters have been chosen: minimal sink w_{min} and theoretical minimal radius of turn (with $n = \infty$):

$$R_\infty = \frac{V_{min}}{g}\sqrt{1+\bar{\varepsilon}^2} \qquad (2)$$

Combining them with c_o and R yields the thermal number:

$$\varphi = \frac{c_o}{W_{min}}\frac{R}{R_\infty} \qquad (3)$$

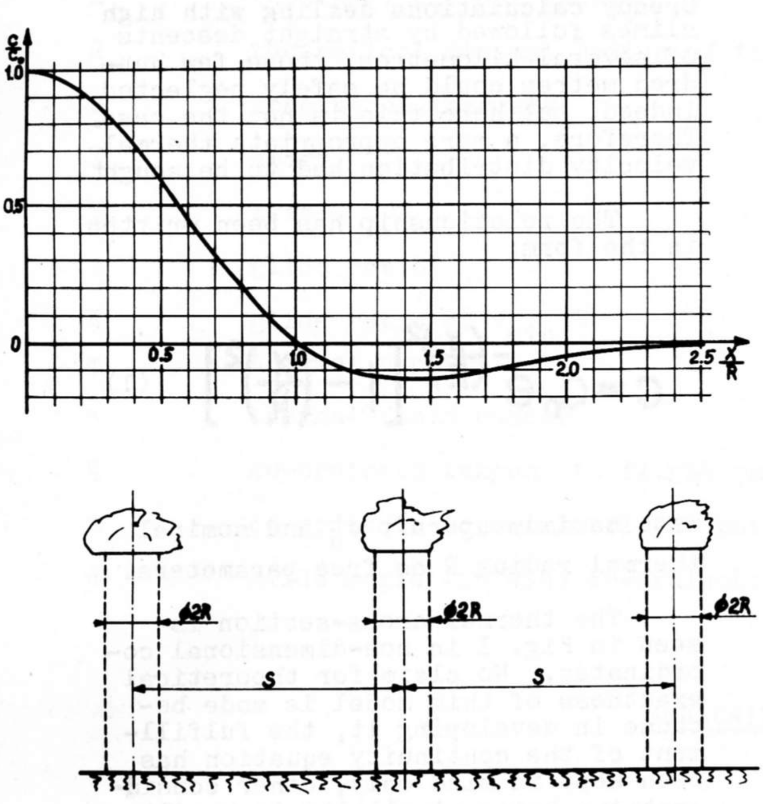

Figure 1. Thermal and Thermal Street Model

With increasing thermal numbers higher climbing speeds in circling and/or more height gain in crossing the thermal are to be expected.

The thermal field is supposed to consist of equally spaced rows of thermals, so one more parameter, the spacing of thermal rows, has also to be taken into consideration. Thus the three parameters defining the thermal field are: maximal updraft c_o, thermal radius R and thermal spacing S.

At first sight there are two "meteorological" parameters lending themselves to be combined with the thermal spacing: the greatest possible climbing height H_{max} - giving the necessary glide ratio between thermals H_{max}/S - and the thermal diameter 2R - forming the updraft distance to total distance ratio 2R/S. The first one would be the natural choice for circling flight, while the latter seems to be more closely associated with the needs of dolphin flight. The formula for the thermal street number used in the present paper reads:

$$\varkappa = \varphi\frac{2R}{S} = \frac{2c_oR^2}{W_{min}R_\infty S} \qquad (4)$$

The main consideration in its choice was that for a given thermal field it has the same value for circling and dolphinining, whereas the other is different for the two cases, making comparisons difficult if not impossible. 2R/S values over 0.4 are excluded from present investigations.

2. EQUATIONS OF MOTION

In terms of flight mechanics, dolphin-style thermal flight might be classified as the longitudinal motion of the sailplane under the influence of both varying vertical wind gradient and pilot-induced change of the lift coefficient. This complicated problem obviously calls for some kind of simplification not affecting the influence of the main dynamic parameters in the mathematical treatment.

Three possible grades of simplified computation have been considered.

a. Some kind of quasi-static calculation as in (1) and (2). While being necessary for estimating the possible gain in performance and thus establishing a reason for more elaborate work it is likely to have several serious limitations, e. g. the inability to account for losses due to abrupt dynamic manoeuvres.

b. Dynamic calculations with two degrees of freedom. The sailplane is modelled as a mass point (having, of course, lift and drag), neglecting the short period longitudinal oscillation and simplifying the phugoid mode. This kind of treatment seems to be the most promising one for the moment, giving already a useful amount of information with reasonable computer times and work.

c. Fully dynamic calculations involving both longitudinal modes, drag due to elevator deflection, etc., may add still something to accuracy but they require much more capacity both in preparatory work and in computer time than is feasible.

After weighing all the odds, a two degree of freedom dynamic analysis was attempted. In the ξ, η co-ordinate system with axes tangential and normal to the flight path, and without thermal action, the equations of motion read as follows, Fig. 2:

$$\frac{G}{g}\frac{dv}{dt} = -G\sin\theta - \varepsilon n \theta \qquad (5)$$

$$\frac{G}{g}\frac{v^2}{r} = nG - G\cos\theta \qquad (6)$$

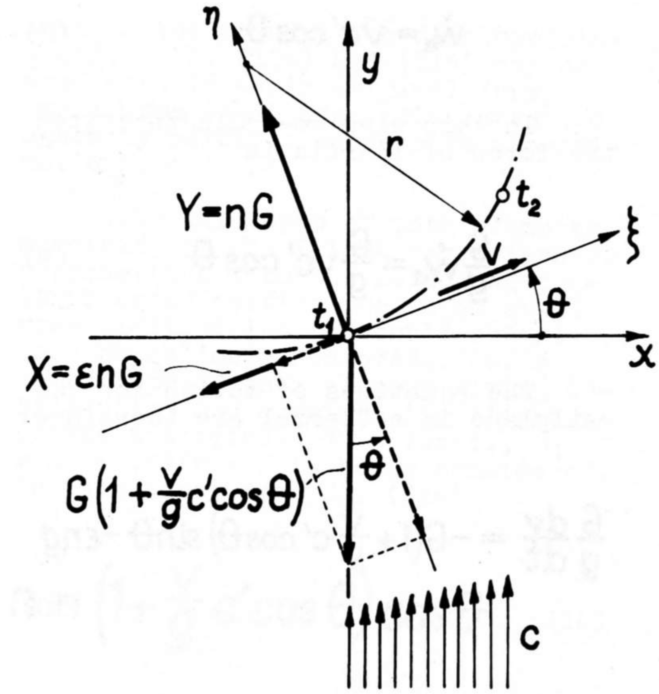

Figure 2. Forces Acting on the Sailplane

In accounting for thermal lift, a somewhat unusual method has been followed. As is known, steady thermal lift does not affect the balance of forces acting on the sailplane and dynamic effects originate exclusively from the gradient of the thermal velocity. This gives the possibility to bypass the aerodynamic calculation of varying lift coefficients by working in an accelerating wind co-ordinate system. The apparent vertical acceleration of this system moving with the sailplane may be determined as follows.

The thermal velocity at x+dx is

$$c(x+dx) = c(x) + \frac{dc}{dx}dx = c(x) + c'dx \qquad (7)$$

giving for a plane moving with the horizontal velocity component $v.\cos\theta$ the apparent vertical acceleration

$$\dot{w}_a = vc'\cos\theta \tag{8}$$

In this wind co-ordinate system the force of inertia is

$$\frac{G}{g}\dot{w}_a = \frac{G}{g}vc'\cos\theta \tag{9}$$

The equations of motion for the sailplane in a thermal are therefore:

$$\frac{G}{g}\frac{dv}{dt} = -G\left(1+\frac{v}{g}c'\cos\theta\right)\sin\theta - \varepsilon ng \tag{10}$$

$$\frac{G}{g}\frac{v^2}{r} = ng - G\left(1+\frac{v}{g}c'\cos\theta\right)\cos\theta \tag{11}$$

or

$$\frac{dv}{dt} = -g\left[\varepsilon n + \left(1+\frac{v}{g}c'\cos\theta\right)\sin\theta\right] \tag{10a}$$

$$\frac{1}{r} = \frac{g}{v^2}\left[n - \left(1+\frac{v}{g}c'\cos\theta\right)\cos\theta\right] \tag{11a}$$

In view of the nonlinear relationship $\epsilon = \epsilon(c_y) = \epsilon(v,n)$ it was felt impractical to attempt an analytical solution of this differential equation system. A kind of finite element method has been tried instead.

3. COMPUTER PROGRAM

The path of the sailplane through the thermal may be divided into parts according to the character of flying, Fig. 3. Before entering the thermal, a constant gliding speed v_s is supposed to be held. In the second part the pilot is pulling up to slow down to the climbing speed v_e (section A-B). In the third part a constant speed (v_e) climb follows (section B-C). The fourth part is a re-acceleration to the gliding speed v_s (C-D) followed by the fifth part, a constant speed (v_s) glide to the next thermal.

Hence, from the computing point of view, there are constant speed and variable speed sections to be dealt with.

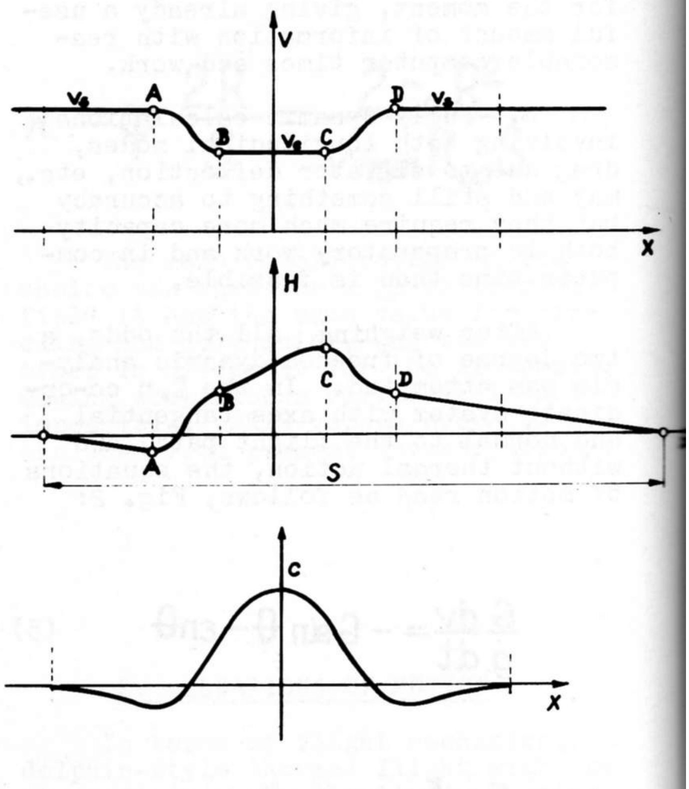

Figure 3. Sketch of Dolphin-Flight Path Through a Thermal

3.1 Constant Speed Sections

At v=const., dv/dt = 0 Eq. (10a) assumes the following shape:

$$0=\varepsilon n+\left(1+\frac{V}{g}c'\cos\theta\right)\sin\theta \tag{12}$$

From which:

$$n=-\frac{\sin\theta}{\varepsilon}\left(1+\frac{V}{g}c'\cos\theta\right) \tag{13}$$

Eq. (11a) remains unchanged.

The practical computation is made in the following manner:

The section is broken down into path elements corresponding to 0.1 sec. or even less flying time. In each element normal acceleration n, thermal lift gradient c', glide ratio ε and (air) flight path radius r are assumed to remain constant and equal to their respective mean values at $x=(x_1+x_2)/2$.

The functions sin θ and cos θ, respectively, are also computed from the mean value $\theta=(\theta_1+\theta_2)/2$. Dynamic equilibrium is attained by multiple successive approximations.

3.2 Pull-Up and Re-Acceleration Sections

Initially, for these parts a simple constant longitudinal acceleration or deceleration path element had been thought of. In several futile attempts it failed to give acceptable transition to the next constant speed section, so a path element with prescribed constant normal load factor n had to be developed instead. Eqs. (10a) and (11a) may be used here in their original form, equilibrium conditions being attained again by multiple successive approximations.

With this type of path elements, smoothing of the flight parameters to a transition into the following constant speed section presented still some difficulties and could not always be reliably achieved. So, a third type of path element giving decelerating and accelerating flight at constant attitude θ = const., r = ∞, 1/r = 0 is being considered. In this case, from Eq. (11a)

$$n=\left(1+\frac{V}{g}c'\cos\theta\right)\cos\theta \tag{14}$$

Combining Eqs. (14) and (10a) gives - after the usual successive approximations - the required flight parameters. A transition section of constant attitude elements will be formed after each of the pull-up and re-acceleration sections.

3.3 Program Structure

A flow-chart of the ALGOL computer program may be seen in Fig. 4. After reading the input data, the flight path, starting from point x = -2.5R, and other flight parameters are computed. Constant air speed is held on this section, so procedure SIKL (for GLIDE) has been built up around Eqs. (13) and (11a). After each consecutive flight path element, a control for reaching x_A is made.

After passing x_A, in the pull-up section, the change for procedure, DELF (for DOLPHIN) using Eqs. (10a) and (11a) is made. The correct manipulation of the normal load factor n

so as to provide for reliable push-over to the intended climbing speed v_e is not easy. The presently adopted solution is rather computer time consuming, therefore, the insertion of a short transition section is intended. Using this, after touching the correct (air) glide angle for constant speed climb θ_e - at whatever speed - it will be switched over to constant-attitude flight. The procedure KIFUT (for RUN DOWN) employs Eqs. (14) and (10a) and a regular check is made for attaining the intended climbing speed v_e.

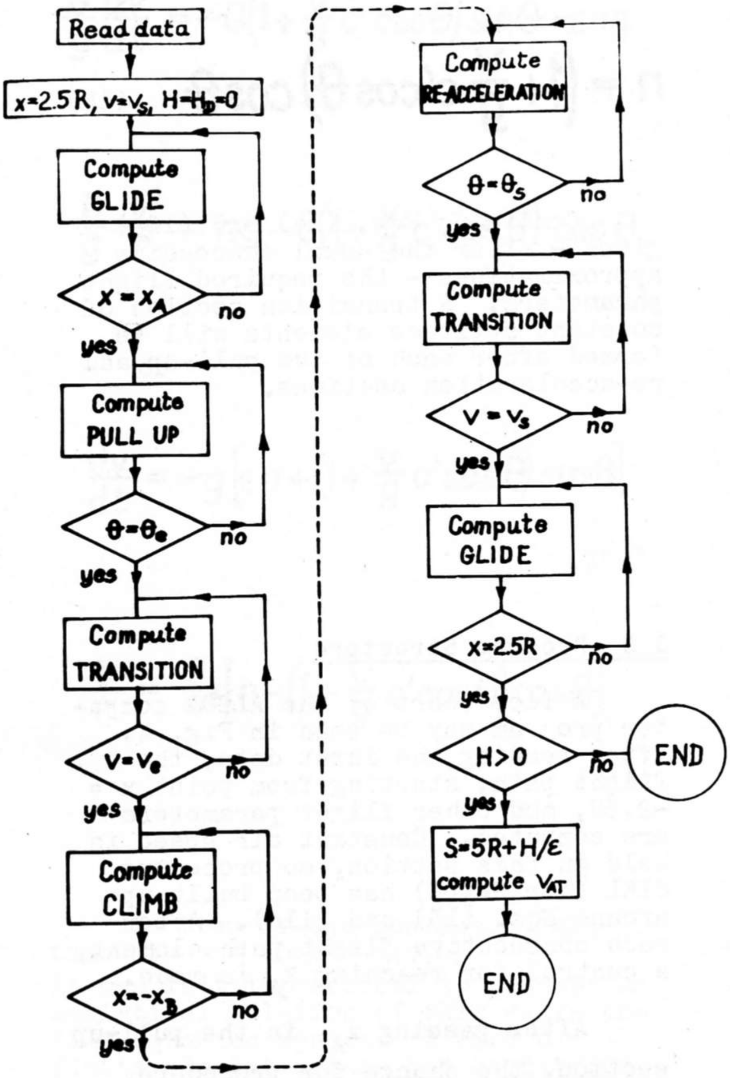

Figure 4. Flowchart of the Computer Program

The constant-speed climb section is working on the same lines as the first glide section.

The re-acceleration seems to be the most difficult part of the computation and without a subsequent transition section a satisfactory pull-up to the correct gliding speed v_s cannot be taken for granted in every case within the given aerodynamic boundaries. Insertion of a transition section is, therefore, a necessity at this point.

The subsequent glide to x = 2.5R is again a routine constant-speed operation.

The momentary values of eleven flight parameters are printed out during the whole computation, normally at the end of each tenth path element.

If, at the end of the last flight section, the sailplane is below the starting height, the combination thermal - flight mode is considered to be unsuitable for sustained dolphin flight. If a net height gain has been found at the end of the sink belt, then the process is completed by computing the glide distance to height zero and the cross-country or mean speed v_{AT}.

4. PRELIMINARY RESULTS

The first question to be settled was whether the outlined computation gives a sufficiently accurate and detailed picture of the dolphin flight process or not. A glance at Fig. 5, showing some of the results from a program test run, may be of informative value.

Computations have been made using the ASW-15 polar converted to the height of 1500 metres. Thermal data: maximum updraft c_o=3 m/sec, nominal radius R = 200 m. Glide speed v_s = 139.88 km/h, climbing speed v_e = 101.09 km/h. The pull-up was made with a normal acceleration n = 1.8 up to $\theta_h = 15^o$, pushing thereafter with n = 0.75 to $\theta = 3^o$ and going finally with n = 1 to $\theta_e = -1.53^o$. The push-down involved n = 0.75 to $\theta_h = -15^o$, then pulling n = 1.8 to about $\theta = -6.2^o$, ending with n = 1 to reach again v_s.

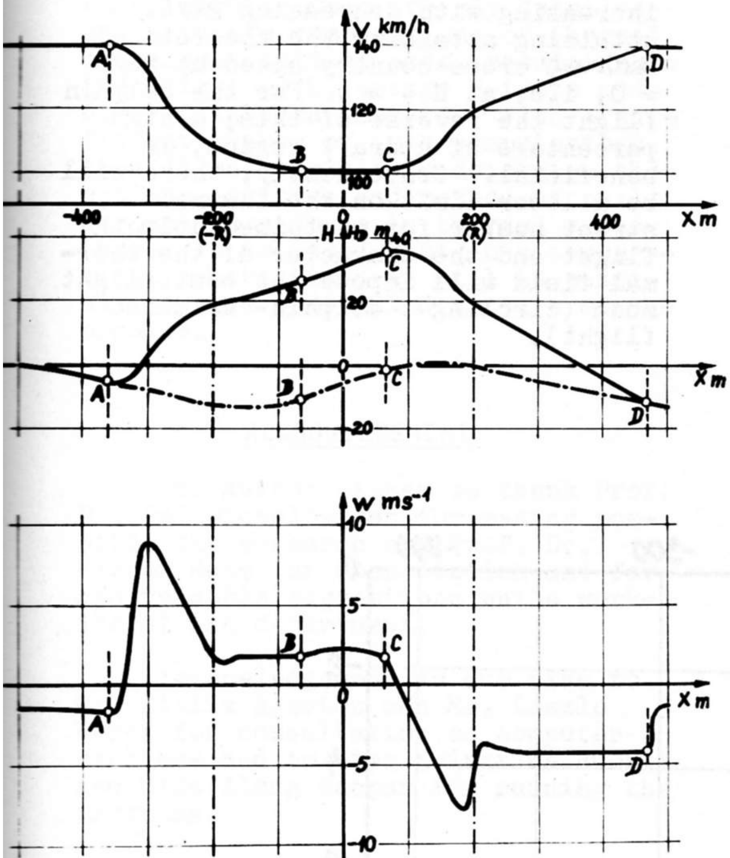

Figure 5. Selections from the Results of a Computer Run

On the upper diagram showing air speed against distance covered the first half to x = 0 may be described as normal. In the second half of the crossing two anomalies are to be found:

a. re-acceleration is much longer than pull-up

b. there is a fairly sharp speed discontinuity at transition to glide.

The first problem is not a serious one since there is no need to make the pull-up and push-down strictly symmetrical and since the process may be accelerated at will.

But the speed graph "corner" i.e. acceleration peak at the beginning of the glide should be eliminated, for this kind of ham-fisted handling (e.g. pulling more than 2 g-s for some tenth of seconds) by experienced pilots cannot be accepted. Insertion of a transition section as mentioned above and as shown in Fig. 4, seems, therefore, a must.

The dual height/energy height diagram is instructive by offering a comparison with observed thermal crossings (H) and a rapid insight into the energy management (H_D). The upper branch of the curve (H) seems to be in fair agreement with the usual flight path profiles. The lower branch (H_D) is showing quite clearly that there is no energy gain except in the inner parts of the thermal, about 25 to 30 metres inwards from the lift boundary. Spectacular as the zoom up may be, it is contributing only indirectly to the net height gain, by making possible a longer stay in the buoyant thermal core supplying energy.

The imperfect preliminary computer runs indicated no absolute optima so far, nevertheless, they gave some practical hints.

The best climbing speed v_e to be arrived at after the pull-up has been clear previously from common-sense and as stated by (1). In the opinion of the author, it does not make much difference whether the lift is crossed at, say, the speed for minimum sink or at the theoretical MacCready speed, some one to three kilometres per hour lower and changing constantly. Hence, we can concentrate on finding the optimum point for starting the pull-up and the best load factor (or, more correctly, the best sequence of load factors) to execute it.

Fig. 5 shows the distance covered during the pull-up to be of the same order of magnitude as the thermal radius. Should the slow-down be initiated at the border of the thermal core, then most - if not the whole- of the lift would be traversed at high speed, much above the optimum. So the best place to start the pull-up is at some distance <u>before</u> entering the updraft region. Fig. 6 shows energy height (H_D) at x = 0 over x_A for five computer runs. For increasing lead distances definite improvement has been registered as against circling thermal cross-country tactics where the downdraft belt is crossed best at the appropriate MacCready speed commencing the pull-up only after entering the lift region.

The problem of choosing slow against abrupt pull-up cannot be decided as yet. Each of them has its merits and disadvantages; the final choice will only be possible after much more analysis.

Some comparative calculations of circling cross-country flight performance have also been made, but there is no point in giving details before finishing the dolphin flight study. The non-dimensional parameters φ and $\varkappa$ proposed in item 1 turned out to be usable, the former allowing a fair approximation of the optimum rate of climb in the thermal and the latter (or φ and 2R/S) indicating the greatest cross-country speed possible. In circling cross-country flight, the mean speed is increasing with decreasing 2R/S, attaining attaining the theoretical MacCready cross-country speed at 2R/S = 0, i.e. at H = ∞. For the dolphin flight the reverse of this, a high percentage of updraft region, is beneficial. Consequently, there will be a lower limit on the thermal street number for sustained dolphin flight and the character of the thermal field will impose the best flight mode (circling-, dolphin- or mixed flight).

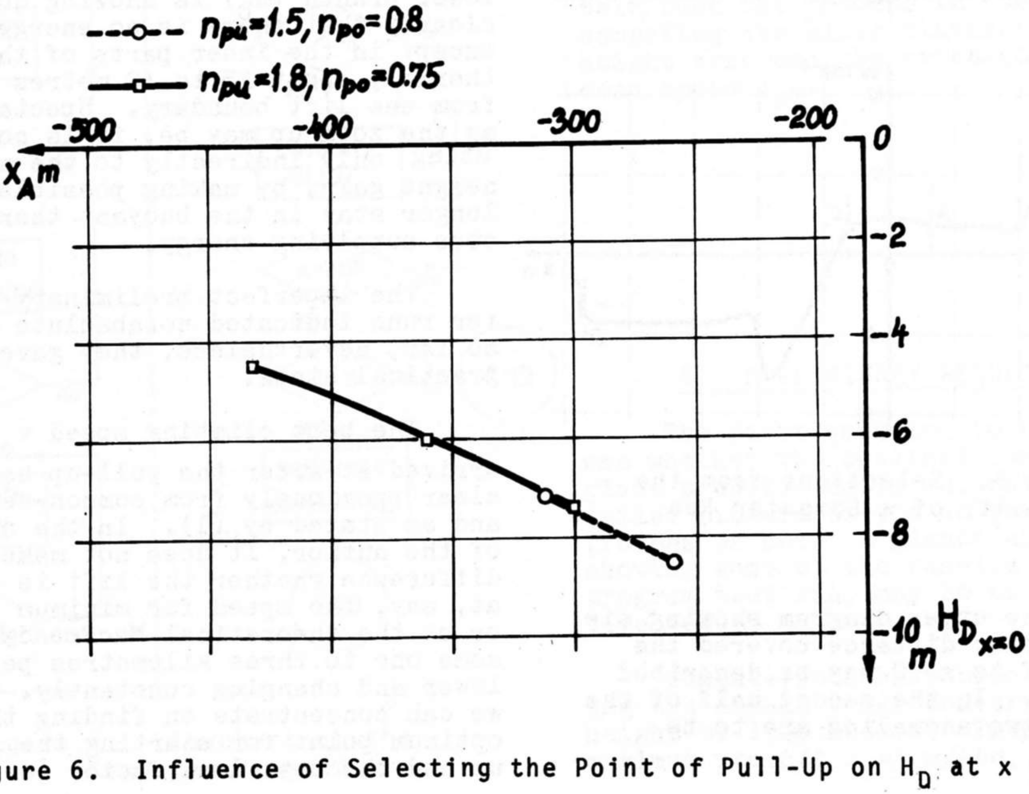

Figure 6. Influence of Selecting the Point of Pull-Up on H_D at x = 0

SUMMARY

A computer study of dolphin-style sustained thermal cross-country flight has been started using a variant of the so-called finite-element method. The thermal model adapted allows for the appropriate downdraft belt surrounding the thermal core. The equations of motion have been solved for short path- (and time-) elements in a wind co-ordinate system accelerating according to the thermal gradient and for three flight modes: constant speed, constant normal acceleration and constant attitude flight. Thus, fairly detailed and accurate computations of thermal crossings could be made, resulting in the flight path, energy heights and other flight parameters. With a sufficient number of such computations, it will be possible to determine the optimum position and method of pull-ups and push-downs in thermal crossings. It is hoped that in a comparatively short time, it will be possible to give a detailed picture of the problem.

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