An Advanced Variometer System

Wim Toutenhoofd, National Center for Atmospheric Research[1], and
Richard H. Ball, Ball Engineering Co., Boulder, Colorado

1. Introduction

Approximately one half year ago one of the authors of this article (W. T.) decided to buy an electric variometer for his Ka-6. He chose to buy a Ball variometer because he was interested in this instrument and because the designer and manufacturer of this instrument (the co-author of this paper) lived close by and was interested in discussing the instrument and willing to make some minor modifications to make some simple experiments possible. These discussions and experiments resulted in the present article.
Sailplanes are sometimes used for atmospheric research; one advantage over powered aircraft being the relative ease with which vertical motions of air can be measured. The magnitudes of vertical motions of the air are of very great importance in many atmospheric studies.
The ideal variometer will give a direct read-out of the vertical speed of the air (not of the sailplane!) regardless of the attitude, speed or acceleration of the sailplane. In order to consider the problems involved, let us write down the sailplane equation of motion

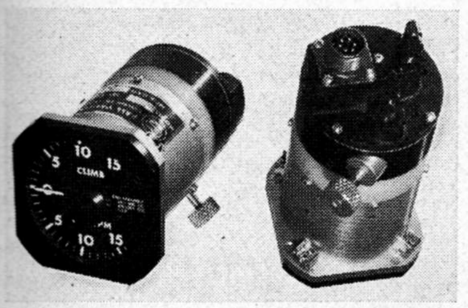
$$m\vec{g} + \vec{L} + \vec{D} = m\frac{d\vec{u}}{dt} \qquad 1.1$$

where m is the mass of the sailplane, $\vec{g}$ the acceleration due to gravity, $\vec{L}$ and $\vec{D}$ are the total aerodynamic lift and drag forces respectively, and $\vec{u}$ is the velocity of the sailplane. We also have

$$\vec{u} = \vec{v} + \vec{w} \qquad 1.2$$

where $\vec{v}$ is the velocity of the sailplane with respect to the air and $\vec{w}$ the velocity of the air (wind) with respect to the earth ($\vec{w}$ includes the vertical component of the wind). By forming the scalar product of eq. 1.1 with $\vec{v}$ one easily derives

Fig. 1. The Ball variometer.



$$w_z = u_z + \frac{Dv}{mg} + \frac{1}{2g}\frac{d}{dt}v^2 + \frac{1}{g}\vec{v}\cdot\frac{d\vec{w}}{dt} \qquad 1.3$$

where $v = \vec{v}$. The vertical speed of the air, w_z can thus be obtained by adding the four terms in the right hand member of eq. 1.3 of which the first term represents the actual vertical speed of the sailplane, the second term the absolute value of the vertical speed the sailplane would have if w_z, $\frac{dv}{dt}$ and $\frac{d\vec{w}}{dt}$ were zero (steady flight in still air), the third term the 'correction' for airspeed variations (usually referred to as 'total energy compensation') and the last term a contribution due to 'turbulence'.
'Normal' variometers measure only the first term u_z and 'total-energy' variometers measure the two terms

$$u_z + \frac{1}{2g}\frac{d}{dt}v^2 = \frac{1}{mg}\frac{dE}{dt} \qquad 1.4$$

where

$$E = mg\,z + \frac{1}{2}mv^2 \qquad 1.5$$

which is the total energy, z being the altitude.
The measured airspeed v is affected by fluctuations in the wind component along the flight path causing fluctuations in fast responding variometers which are usually considered undesirable. These fluctuations can be removed in principle, by means of the last term in eq. 1.3. Unfortunately it is not easy to obtain independent measurements for that term.
The undesirable turbulence induced fluctuations in the total energy compensation $\frac{1}{2g}\frac{d}{dt}v^2$ are often filtered out either electrically or by means of capillaries. If possible (like with total energy diaphragms) one should filter the term $\frac{1}{2g}\frac{d}{dt}v^2$, independently.
With total energy venturis one cannot avoid filtering u_z at the same time thus causing the variometer to become slower in response.

A rough approximation of the term $\frac{Dv}{mg}$ of eq. 1.3 for the case that the sailplane is flown level (zero bank angle) is the sinking speed of the sailplane in still air at the speed v as obtained from the polar curve (appropriate corrections for the air density must be made). It is possible to build a small computer based on this idea. Such variometer systems are in principle superior to the standard total energy variometers although considerable errors can still occur, especially when the sailplane is banked or when it is pulling up from a high speed run on entering a thermal since then the drag D is apt to be much more than would follow from the above mentioned approximation. The computer will require more sophisticated input data like angles of attack pitch and bank to solve the equations of motion under less severe assumptions. In the rest of this article we will ignore the term $\frac{Dv}{mg}$ and restrict our considerations to total-energy variometers.

2. The Ball Variometer

The Ball variometer operates on the following principle (fig. 2): when the sailplane gains (or loses) altitude, air flows out from (or into) a reservoir through the basic capillary to (or from) the static pressure ports of the sailplane. The pressure difference that is established across the capillary is a measure of the vertical speed u_z of the sailplane. The pressure difference is measured by means of a diaphragm-type pressure sensor, the deflection of the diaphragm being measured electrically. The total energy 'compensation' is obtained by means of a capsule that is mounted inside the reservoir and is connected to the pitot via the pitot damping capillary. The total energy compensation can be adjusted by changing the relative effect of a volume change of the capsule which can be achieved by changing the volume of the reservoir by means of a piston. The plastic diaphragm has a soft-iron patch attached to it in the center. The two coils that are mounted, one on each side of the diaphragm form part of an AC bridge. Any deflection of the diaphragm unbalances the bridge. The block diagram of the electrical circuit is shown in figure 3. The input voltage is controlled by means of a voltage regulator allowing for proper operation of the variometer with battery supply voltages in the range of 10–20 volts. The bridge operates on a frequency of 3 kHz. The output signal is fed into an amplifier having a gain which is adjustable with a screwdriver without opening the housing of the instrument. The output of the amplifier drives a milliammeter having a scale which can easily be calibrated according to the wish of the user. Six different scales are available to choose from.
The basic time constant τ of the instrument is that of the instrument with the damping capillaries (fig. 2) removed and assuming a negligible time con-

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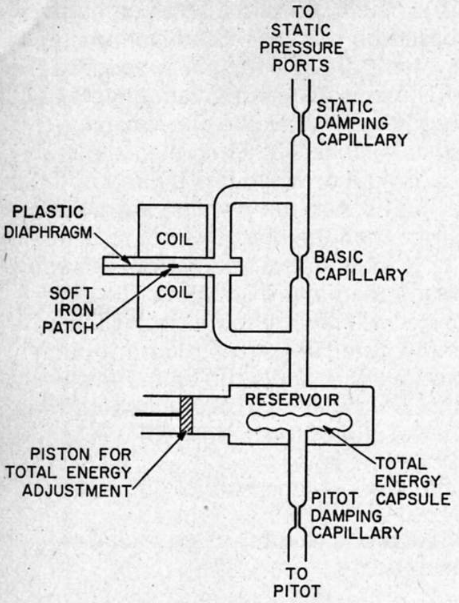

Fig. 2. Principle of operation of the Ball variometer.

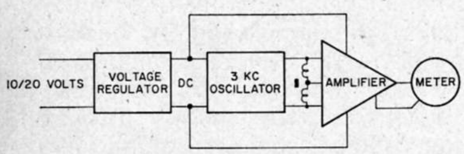

Fig. 3. Block diagram of the Ball variometer electronics.

stant of the milliammeter. This time constant is determined by the volume of the reservoir, the dimensions of the basic capillary and the temperature. For the Ball variometer $\tau \approx 0.1$ sec. The milliammeter is electrically damped with a circuit of which the time constant is approximately 0.5 sec. A pitot damping capillary is usually built in the Ball variometer to filter undesirable, turbulence induced fluctuations out, as was discussed in section 1. The capillary can easily be replaced by the user of the instrument by different capillaries. A more detailed discussion on time constants is given in the next section. A static damping capillary (fig. 2) is not usually provided but can, of course, be added by the user, if so desired.
A schematic sectional view of the instrument is given in figure 4.

3. Properties of Reservoir Capillary Systems

Figure 5 shows a reservoir-capillary system. We define the time constant τ of this system as follows

$$\tau = -\frac{p_{in} - p_{out}}{\frac{dp_{in}}{dt}} \qquad 3.1$$

where p_{in} is the air pressure inside the reservoir and p_{out} the ambient (static) pressure.
From the theory of laminar flow through capillaries (see for instance [1]) we obtain for the volume of air flowing through, per unit of time

$$\frac{d}{dt}\,(\text{volume of air}) =$$

$$\frac{\pi d^4}{128\,\mu}\,\frac{p_{in} - p_{out}}{l} \qquad 3.2$$

where d and l are the inside diameter and the length of the capillary (fig. 5) and μ is the dynamic viscosity of air. Assuming the temperature to be constant and the air to behave as an ideal gas ($p_{in}V$ = const, where V is the reservoir volume) one easily derives for the time constant

$$\tau = RV \qquad 3.3$$

where

$$R = \frac{128}{\pi}\,\frac{\mu}{p}\,\frac{l}{d^4} \qquad 3.4$$

is the 'resistance' of the capillary. It makes little difference whether one uses p_{in} or p_{out} for p. There is an obvious analogy between the electrical time constant RC of a capacitor-resistance system and the pneumatic time constant RV of a reservoir-capillary system. Equations 3.3 and 3.4 were found to be very helpful in calculations of the dimensions of capillary diameters and lengths in cases where a reservoir volume and a desired time constant were given. One interesting conclusion from 3.3 and 3.4 is that this type of variometer becomes 'slower' with increasing altitude (decreasing p). At the 500 mb level (approximately 5.5 km) the time constant is twice the 1,000 mb time constant (approximately sea level). Since

$$\frac{dp}{dt} = \varrho_{out}\, g u_z \qquad 3.5$$

(where ϱ_{out} is the density of the ambient air) and $p = \varrho_{out} R' T_{out}$ (where R' is the gas constant and T_{out} is the absolute temperature of the ambient air) we obtain

$$\text{meter reading} \propto p_{in} - p_{out} = C_1\,(T_{out})\,u_z \qquad 3.6$$

if we assume that the reading of the meter (calibrated in vertical speed) is proportional to $p_{in} - p_{out}$, where

$$C_1\,(T_{out}) = \frac{128}{\pi}\,\frac{1}{gR'}\,\frac{\mu}{T_{out}}\,\frac{lV}{d^4} \qquad 3.7$$

is a 'constant' which, for a given reservoir-capillary system, depends on the temperature only, through $\frac{\mu}{T_{out}}$.
Since μ decreases with decreasing temperature, $\frac{\mu}{T}$ varies only slowly with altitude. In a 'standard' atmosphere [2] $\frac{\mu}{T}$ at 10 km is only 5% larger than at sea level. Thus the sensitivity of this type of variometer varies only slowly

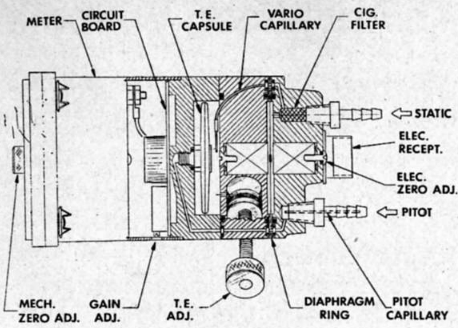

Fig. 4. Sectional view of the Ball variometer.

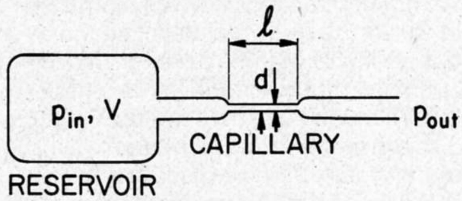

Fig. 5

with altitude. The sensitivity of electric variometers of this type is probably more strongly affected by the temperature effects on the diaphragm than by the variation in $\frac{\mu}{T_{out}}$ with temperature.

4. Total Energy Compensation

Combining 1.4, 3.5 and $q = \frac{1}{2}\varrho v^2$ where q is the dynamic pressure one obtains for the reading of a total energy variometer

$$\frac{1}{mg}\,\frac{dE}{dt} = u_z + \frac{1}{2g}\,\frac{d}{dt}\,v^2 =$$

$$\frac{1}{\varrho g}\,\frac{d}{dt}\,(p - q). \qquad 4.1$$

The advantages of measuring $\frac{d}{dt}(p - q)$ rather than $\frac{dp}{dt}$ were first discussed by Arthur Kantrowitz [3]. Connecting a venturi, especially designed to supply the pressure $p - q$, to the static connection of a conventional variometer converts that instrument into a 'total energy variometer'.
Vögeli [4] proposed the compensation mechanism in the Ball variometer (see capsule in figure 2) that avoids the aerodynamic drag of a venturi. Irving [5] derived for this mechanism (assuming $p_{in}\,V^n$ = constant where n = 1 for isothermal behavior of the air in the reservoir and 1.4 for adiabatic behavior, see section 5)

$$\varDelta V_c = c_2 q \qquad 4.2$$

where $\varDelta V_c$ is the change of capsule volume and c_2 is the capsule constant:

$$c_2 = \frac{V}{np} \qquad 4.3$$

V being the reservoir volume. The capsule constant can only be made correct for one value of p. A capsule calibrated for sea level operations will 'undercompensat' at 500 mb (5.5 km) by a factor of two. The authors live in an area where altitudes of 7 km MSL are regularly obtained in sailplanes (air traffic control regulations making it difficult to climb to higher altitudes) and were thus interested in a system in which the capsule constant c_2 is adjustable in flight.

Ball variometers are sometimes mounted in sailplanes in such a way that the T. E. adjustment can be performed in flight. This is the best method to obtain the correct adjustment for a given pressure level. The gain should be adjusted afterwards. It is possible to use the adjustment for in-flight variation of the capsule constant when flying much higher than the pressure level for which the instrument is calibrated but, because V will then be reduced, the readings of the instrument will be less than the true rate of climb.

When we were searching for a satisfactory system that would easily allow adjustment of the total energy compensation with varying pressure we did a laboratory experiment with a T. E. diaphragm (trade name: PZL) and a control knob (trade name: Thermaline). The purpose was to determine if eq. 4.2 would be satisfied by this diaphragm and if c_2 would be adjustable over a wide enough range. It was found (fig. 6) that the dependence of the displaced volume on the applied pressure was very close to linear, as required by eq. 4.2, when the diaphragm was used with c_2 equal to the value recommended by the factory (0.35 cm³/cm of water). However the further we move away from the factory recommended setting the more non-linear the dependence of ΔV_c on q becomes.

This is not an exhaustive study of T. E. compensation devices but it seems likely that venturis are more useful than capsules or diaphragms when the sailplane is operated through a wide range of altitudes. Once a total energy venturi is made to provide the correct pressure $p-q$ it will not require any further in-flight adjustments with change of altitude. The price is the aerodynamic drag of the venturi.

The ideal solution is probably the use of a small electronic computer that calculates the rate of change of $p-q$.

5. The Heat Transfer Error

We believe that there is little documentation on an error that occurs in most variometers that we shall call the heat transfer error.

One of the authors (W. T.) learned about this error in the mid 1950's from Ilbert de Boer, a competition pilot who represented Holland in early World Soaring Championships. The error is mentioned by Moore [6] and Teuling [7].

The heat transfer error occurs during a rapid change in vertical speed and is most apparent immediately after release from a winch launch when, for a period that cannot be accounted for by the normal time constant of the variometer, a rate of climb is indicated which is not real. The error is caused by the adiabatic cooling of the air in the reservoir during the rapid climb. After the climb is finished the air in the reservoir is colder than the walls of the reservoir. The temperature of the air will gradually rise to the temperature of the walls. The resulting rise of pressure will cause a false reading of the variometer.

The old remedy is to fill the reservoir with steelwool resulting in a much faster heat exchange between the air and the steel wool than there is otherwise between air and the walls. The temperature will then essentially remain constant and the error is eliminated.

We conducted an experiment to convince ourselves of the effectiveness of the steel wool and to obtain some quantitative information on the heat transfer error. A ½ liter thermos flask was used as a reservoir and connected through a valve to a much larger reservoir in which the pressure was a little below the ambient pressure (approximately 830 mb in our laboratory at 1,800 m MSL). The pressure in the thermos bottle was recorded when the valve was momentarily opened so that the pressure very suddenly dropped a few mb. The chart recording of the pressure is shown in figure 7. As can be seen the heating of the air, after the adiabatic expansion, caused a gradual rise in pressure. In this case the time constant τ for heat transfer between bottle and air appeared to be approximately 6 sec. Figure 7 also shows the result of the experiment repeated with 57 gram of steel wool (7.3 cm³) in the flask. There is no apparent heating of the air in the second experiment indicating the effectiveness of the steel wool.

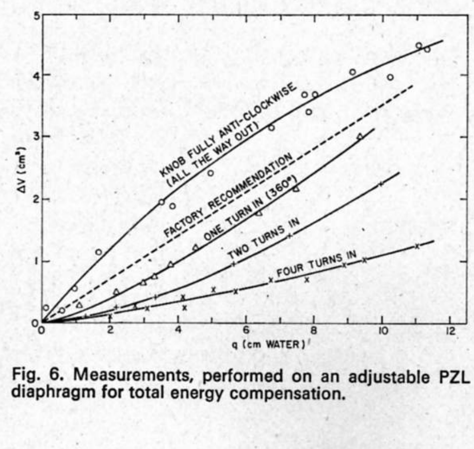

Fig. 6. Measurements, performed on an adjustable PZL diaphragm for total energy compensation.

The magnitude of the heat transfer error can be estimated theoretically. Let T_{air} and T_{wall} be the absolute temperatures of the air in the reservoir and of its walls. For a change in T_{air} we may write

$$dT_{air} = \gamma dz + \frac{T_{wall} - T_{air}}{\tau} dt \qquad 5.1$$

where the first term in the right hand member results from adiabatic temperature changes (γ is the dry adiabatic lapse rate) and the second term from heat flow between the walls and the air in the flask. From 5.1 we obtain for the rate of change of T_{air}:

$$\frac{dT_{air}}{dt} = \gamma \dot{u}_z \tau \left(l - e^{-t/\tau}\right) \qquad 5.2$$

where it is assumed that the sailplane accelerates with a constant vertical acceleration $\dot{u}_z$ and furthermore that at $t = 0$ $T_{air} = T_{wall}$ and $u_z = 0$.

From 5.2 one obtains for the heat transfer error

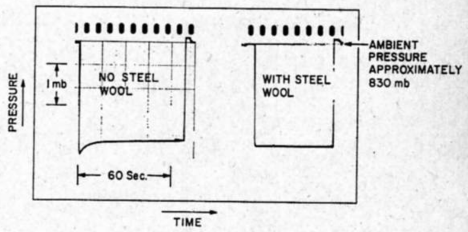

Fig. 7. Measurements, performed on a vacuum flask, as described in text.

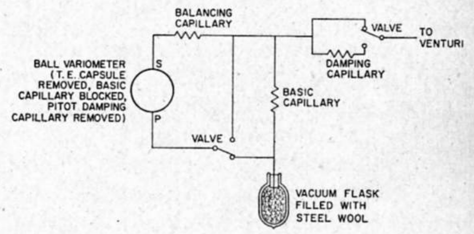

Fig. 8. Variometer system on Ka-6 N7481.

$$\Delta u = \frac{R'\gamma}{g} \dot{u}_z \tau \left(l - e^{-t/\tau}\right) \qquad 5.3$$

where R′ is the gas constant for dry air; $R' \gamma / g = 0.29$ (dimensionless).

As an example: we obtain from 5.3 that in a sustained vertical acceleration of 1 m/sec², using a reservoir with a heat transfer time constant of 6 sec the heat transfer error will eventually reach a value of 1.7 m/sec. If the time constant is reduced to 0.1 sec the same error will be reduced to 0.03 m/sec.

6. The Variometer System of Ka-6 N7481

Figure 8 shows the variometer system of the Ka-6, owned by one of the authors (W. T.). A standard Ball variometer was used with a switch for 3 ranges of full scale deflection. A few very small modifications were made: the total energy capsule was removed,

the basic capillary was blocked and the pilot damping capillary removed. The valve between the vacuum flask and the variometer as shown in figure 8 was included for in-flight zero checks. (We never detected noticeable drift in flight.) The balancing capillary shown in figure 8 is theoretically necessary to make the time constants for the flow of air to each side of the diaphragm equal. We found during laboratory tests that the error in the zero check caused by not using a balancing capillary is extremely small.
The damping capillary used provided for a time constant of approximately 2 sec. This variometer was found to function extremely satisfactorily.

Acknowledgements

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