Experimental Investigations of Stratified Shearing Flow

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Abstract
A discussion is given of an experiment involving turbulence in a stratified fluid. Whereas turbulence in most fluid systems tends to have a smoothing effect, it often serves to create and maintain density discontinuities in fluids in which density stratification is a basic feature. In the first investigation along these lines, Rouse and Dodu (1955) showed that stirring in one layer of a two-fluid system of fresh and salt water caused the initially diffuse zone of transition between the two layers to develop into a sharp interface. This density discontinuity was maintained as long as the stirring continued. Other experiments along these lines have also shown a tendency for the creation and maintenance by turbulence of density interfaces in stratified systems resembling those found in the oceans and atmosphere.
A new experiment is described in which there is created a stratified, shearing system in turbulent motion which is homogeneous both in time and in distance along the direction of flow. The mass flux in this experiment could be directly measured and a simple relationship was found between a non-dimensionalized form of this flux and the overall Richardson number of the system. This relationship is shown to be consistent with earlier findings.
A related experiment is also described in which a fluid initially at rest and linearly stratified is eroded by a turbulent homogeneous layer. The rate of entrainment follows the same law as found in an experiment by Kato and Phillips (1969) designed to model the development of the upper homogeneous layer of the ocean in the presence of a suddenly applied wind stress.

1. Density Discontinuity in Stratified Fluids

Thin layers with strong vertical variations of density (or potential density) are common in the oceans and atmosphere. They are called inversions in the atmosphere and occur at various levels. An especially pronounced one is found at the tropopause. In the oceans such interfaces are characteristic features of the thermocline, where transistors and salinity sensors reveal successions of layers of nearly uniform density separated by interfaces across which the salinity and/or temperature gradients are very large (Stommel and Fedorov, 1967; Woods, 1968). Woods has also discovered the same phenomenon in lakes. It is surprising that density discontinuities can exist in what are presumably turbulent media, because one would normally expect turbulence to tend to wipe out density variations.

Density interfaces are of great interest in geophysical fluid mechanics. In both the oceans and atmosphere, they are often accompanied by strong vertical shear of the horizontal velocity so that we would expect an important interplay of shear and stratification at these surfaces. As is well known, the shear is a source of energy for disturbances whereas the stratification tends to cause the turbulence to die out. These two effects are apparent from the energy equation (Phillips, 1966) and are embodied in a non-dimensional number called the gradient Richardson number. In its simplest form, this may be written

$$\mathrm{Ri} = \frac{\delta \overline{\varrho}}{\delta z} \Bigg/ \left(\frac{\delta \overline{u}}{\delta z}\right)^2 \tag{1}$$

where u is the horizontal speed, where the overbar represents an average, and where ϱ is the buoyancy and is related to the density ϱ_1 and a representative density ϱ_0 by

$$\varrho = g\left(\frac{\varrho_1 - \varrho_0}{\varrho_0}\right) \tag{2}$$

In the form of Eq. (1), we have neglected any variation of the direction of the horizontal velocity with height. When the Richardson number is large, the stabilizing effect of gravity is dominant and stability is indicated. When the Richardson number is small (less than 0.25), the destabilizing effect of the shear is dominant and instability is indicated (Miles, 1963). Another form is the overall Richardson number for a layer,

$$\mathrm{Ri}^* = \frac{H \Delta \varrho}{(\Delta u)^2} \tag{3}$$

where H is the thickness, Δu is the velocity difference and $\Delta \varrho$ is the buoyancy difference across the layer. Observations in the oceans and atmosphere indicate that interfaces with strong density and velocity gradients are often the source of turbulence and confirm that these turbulent regions have Richardson numbers of order one or less. Woods (1968), for example, has directly observed breaking waves on these surfaces in the ocean. In the atmosphere, ultra-sensitive radar indicates that inversions are often accompanied by very turbulent conditions suggestive of the breaking of internal gravity waves (Hardy, Glover and Ottersten, 1969).

It is tempting to suggest that the presence of density discontinuities and accompanying turbulence are analogous phenomena in the oceans and atmosphere. In fact, however, caution is advised because Turner and Stommel (1964) have shown experimentally that well-mixed layers separated by interfaces can be set up in a way which depends utterly on the presence of both heat and salt in the medium. The great difference between the conductivities of heat and salt are directly responsible for the phenomenon. Thus, there is some chance that the occurrence of well-mixed layers and their interfaces in the oceans stems from the presence of both heat and salt as stratifying elements, although the observation by Woods of similar phenomena in fresh-water lakes is contrary evidence.

2. Earlier Experiments

The maintenance and formation of density discontinuities are apparently related to the peculiar nature of turbulence in stratified fluids. The first work illustrating this was by Rouse and Dodu (1955). They experimented with a cylindrical container with fresh water in the upper part and salt water below separated by an initially diffuse interface. They then began to stir the upper layer to create turbulence. When this is done, the interface becomes sharper instead of more diffuse and moves downward at a definite rate of speed called the entrainment velocity, u_e. No turbulence extends into the lower fluid. Its density remains the same while the upper fluid becomes gradually more dense and the density discontinuity weakens. Turbulence at the interface is in the form of wisps and streamers of salt water lifted from the lower fluid and transported into the upper fluid in the form of thin sheets. In this way the molecular diffu-

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sivity can do its work and cause the final mixing. If one stops the stirring, the wisps fall back and give a more diffuse interface, but as long as the stirring goes on the interface is sharp. Rouse and Dodu attempted to be quantitative about their observations and obtained a result in which u_e depended on a Richardson number and a Reynolds number. The scatter was large, however, and the results inconclusive by their own admission. Fortunately, a recent experiment by Turner (1968) represents a refinement of this experiment and provides some fairly definite quantitative information. Turner used a similar container. In his first type of experiment, he stirred the bottom layer. This caused the interface to rise. To avoid this, he removed fluid from the bottom to keep the interface at a fixed level. The upper surface was a floating lid which continued to lower as the experiment continued. However, since the upper fluid is motionless, unless the lid is closer to the interface than some distance of the order of the eddy size, its presence will not be felt. In a second type of experiment, Turner stirred both layers. The interface then had a fixed position midway between the two stirrers. He used both saline water and heated water to provide two experiments with very different molecular diffusivities.

Turner used an arbitrary length unit l (which he later identified with the grid spacing of his stirrer) and obtained a velocity of the form nl, where n is the frequency of his stirrer. He formed a Richardson number,

$$\mathrm{Ri}_n = \frac{g\Delta\varrho_1}{\varrho_0 l n^2} \qquad (4)$$

and plotted u_e/nl against Ri_n on log-log paper. For both salt and heat and at values of Ri_n of one or less, he obtained a straight line with a slope of −1 indicating a relationship,

$$E = \frac{u_e}{nl} \propto \mathrm{Ri}_n^{-1} \qquad (5)$$

The results diverged for larger values of Ri_n, giving Ri_n^{-1} for heat throughout but $\mathrm{Ri}_n^{-\frac{3}{2}}$ for salt at high values of Ri_n. The experiments of Turner and of Rouse and Dodu are important for our eventual understanding of the presence and the maintenance of inversion surfaces in the atmosphere although we are still uncertain precisely how the turbulence causes a concentration of density or potential density. We are sure, however, of the basic importance of velocity gradients or shear in atmospheric and oceanic turbulence and this is absent in these experiments.

One experiment involving shear is described in a recent paper by Kato and Phillips (1969). Their experiment was an attempt to reproduce the development of an upper homogeneous layer of fluid when the wind begins to blow (a surface stress is imposed) on the surface of the ocean. The ocean is assumed to be initially linearly stratified down to considerable depths. As a model, Kato and Phillips used an annular tank with linearly stratified salt water. A movable lid is put on the surface and the lid is rotated by imposing a constant-stress force on it in a circumferential direction.

As expected from the stirring experiments, the motion of the lid (the effect of the stress) caused mixing in the upper level of thickness D(t) and a sharp interface at the transition to the linearly stratified fluid below. The lower fluid was undisturbed. Kato and Phillips found an empirical relationship similar to Turner's for the entrainment velocity as a function of the overall Richardson number namely,

$$E = \frac{u_e}{u_*} = \frac{C}{\mathrm{Ri}^*} \qquad (6)$$

where C = 2.5 approximately and u_* is the friction velocity.

3. Present Experiment

A new experiment (Moore and Long, 1970) in which both shear and stratification are basic effects was designed to be as close as possible to the idealized experiment of fluid between two plates moving in opposite directions at speeds $-\Delta u$ and Δu for the lower and upper plates and with fixed buoyancies $\Delta\varrho$ and $-\Delta\varrho$. If we neglect molecular coefficients[2], the only pertinent non-dimensional parameter is Ri* of Eq. (3).

The idealized experiment mentioned above was not attempted. Instead (fig. 1 and 2), we constructed an annular tank with slits along the bottom and top to inject salty and fresh water in opposite directions at nearly horizontal trajectories to create a shear of a stably stratified fluid. Fluid was simultaneously withdrawn from numerous holes along the bottom and top to insure a zero vertical velocity. The water withdrawn at the bottom was slightly less salty than the water injected at the bottom, and the water withdrawn at the top was slightly more salty than the fresh water injected at the top. Thus, we had a flux of salt from bottom and top. Neglecting the molecular diffusion, the buoyancy flux can be represented as

$$q = \overline{w'\varrho'} \qquad (7)$$

In addition, there is a momentum flux from top to bottom of

$$\tau = \overline{u'w'} \qquad (8)$$

We obtained good horizontal homogeneity and, since the motion was quite steady and $\overline{w}$ was close to zero, these two fluxes should be constant in space and time.

The experimental set-up had a brine-storage tank for the lower injection and withdrawal. The fluid withdrawn at the bottom was pumped back into the tank. Since the withdrawn fluid is less salty, this required the continuous addition of salt to the storage tank to maintain it at a fixed density. The rate of addition of the salt mass, when divided by the horizontal cross-sectional area of the tank, A, yielded the mass flux, and this was converted to the buoyancy flux by multiplying by g/ϱ_0. Thus we could measure q in our experiment directly, and one of our fundamental results expresses $Q = q/2\Delta u\Delta\varrho$ as a function of Ri*.

Mean velocity was measured photographically by observing hydrogen bubbles generated along vertical wires. These observations yielded a $\overline{u}$ with

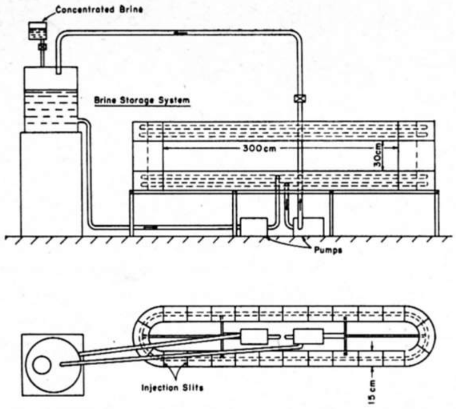


Fig. 1. Overall diagram of experimental tank showing salt-water circulation. Fresh water is circulated through the upper section of the tank in a similar manner.

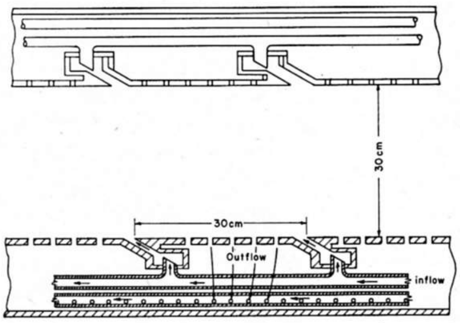


Fig. 2. Sketch of injection-withdrawal detail for fluid circulation.

[2] Two different types of experiments using salt and heat revealed no measurable velocity differences for the same values of Ri*. Since the diffusion coefficient, K, is very different in the two cases, this tends to confirm the assumption that K, at least, is an unimportant parameter. This differs from the findings of Turner who found two different laws for the entrainment velocity for experiments with salt and heat. This may be because of the absence of shear in Turner's experiment.

approximately 10% error. Root-mean-square velocities were measured by a hot film anemometer. Only a few of these were obtained in the range $\Delta u = 4$–5 cm sec^{-1}. In this range the results were very close to

$$\overline{(u'^2)}^{\frac{1}{2}} \simeq \frac{1}{7}\overline{u} \qquad (9)$$

Density was measured by a conductivity probe and calibrated by measurements of density on a specific gravity balance from withdrawn samples at top and bottom. The density measurements were accurate to approximately 10^{-4} gm cm^{-3}. The conductivity probe normally yields mean density, although it could be operated at high sensitivity to measure turbulent fluctuations. In this way we could detect fluctuations as small as 10^{-5} gm cm^{-3}, and this was the order of the density fluctuations in layers of weak mean density gradient.

A typical experiment involved, initially, a layer of salt water in the lower portion of the tank and a layer of fresh water above with an intermediate curved profile of density caused by molecular diffusion. On occasion, an initial linear density profile was used. The pumps were then started. Typical results, after a steady-state was achieved, are shown in figure 3 and may be described as follows:

When $Ri^* > 2$–3 and there is an initial density gradient between two nearly homogeneous layers, the turbulence in the upper and lower layers causes an initially diffuse gradient to become sharper until the entire density difference occurs across a thin layer, which becomes thinner and thinner as Ri^* becomes larger and larger. If the initial density gradient from top to bottom is a linear one, the injection and withdrawal causes an erosion of the interior region in a manner similar to the interface experiment of Kato and Phillips. The process continues until the entire density gradient is again confined to a very thin layer at the center of the channel:

When $Ri^* < 2$–3, the transition layer thickens and when $Ri^* < 1$, the density variation extends over most of the channel depth.

In all cases a linear velocity gradient exists over the central region roughly of the thickness of the region of the density variation but generally somewhat larger (fig. 4). The velocity is nearly uniform over the homogeneous layers but with wall jets at the extreme top and bottom. The gradient Richardson number was impossible to measure in the homogeneous layers since the mean density gradient could not be detected, but, since they are definitely turbulent layers, we can safely assume that the Richardson number is

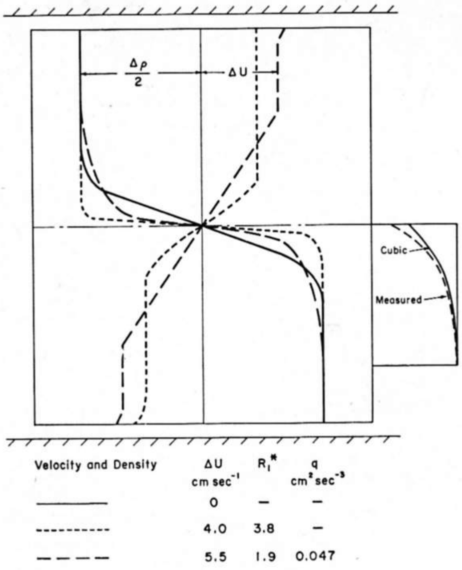
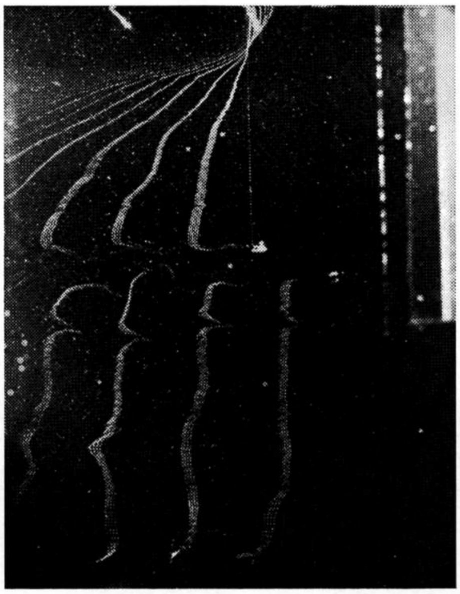


Fig. 3. Mean velocity and mean buoyancy profiles with $\Delta\varrho = 3.2$ cm sec^{-2}.

Fig. 4. Photograph of velocity profile at low value of Ri^* ($\simeq 2$). The bubble «time lines» are shown for the lower half of the channel. The level of zero velocity is at the top of the photograph. Note the linearity of the velocity gradient in the central region.



of order one or less. The transition region in the center adjusts itself to yield a Richardson number close to but somewhat less than one in all cases.

Thus the gradient Richardson number, Ri, is of the order one everywhere even when the overall Richardson number, Ri^*, was as high as 60 or so. This empirical result has an interesting consequence[3]:

The velocity difference between the two layers occurs over a distance L. The density difference occurs over a distance of the order L but generally somewhat less. Therefore the gradient Richardson number, Ri, based on L and the overall Richardson number (based on the distance H) are related by

$$\frac{L}{H} \propto \frac{Ri}{Ri^*}$$

As we have seen Ri is always close to one so that we get an estimate for the thickness of the transition region knowing only the thickness of the whole layer and its overall Richardson number, namely,

$$L \propto H\,(Ri^*)^{-1}$$

Observations of L tend to confirm this relationship.

At high Ri^*, an interface could always be clearly seen. Waves existed on the interface, and these resulted in wisps or streamers of salty water drawn off from the crests and carried into the upper fluid. This is apparently the process by which mass is transferred across the interface. When the transition region is thicker, i.e. when $Ri^* < 2$–3, the turbulence in the central layer is more in the form of highly irregular rollers and breakers.

One of the most important results of the experiment was the empirical relationship between the non-dimensional flux of buoyancy and the overall Richardson number. As seen in figure 5, it is given quite accurately over the whole range of overall Richardson numbers by the equation

$$Q = \frac{q}{2\Delta u \Delta\varrho} = \frac{C_1}{Ri^*} \qquad (10)$$

This result is of the form of the result by Turner in his heating experiments and by Kato and Phillips in their entrainment experiments except with Q replaced by the entrainment velocity. It can be shown, however, that these two quantities are essentially the same (Moore and Long, 1970).

4. Entrainment Experiment

An experiment, closely connected to the investigation of Kato and Phillips, was run in the apparatus of figures 1 and 2. The tank was filled with a fluid stratified linearly, and the pumps were turned on either at the top or bottom to begin the injection and withdrawal. A homogeneous layer began to develop and to erode the linearly stratified fluid. Measurements were made of the depth, D, of the homogeneous layer as a function of time. The results in figure 6 indicate

$$D^3 \propto t \qquad (11)$$

in full agreement with the observations of Kato and Phillips.

[3] I am indebted to Dr. Joachim Kuettner for pointing this out.

5. Applications to the Atmosphere

Clear-air turbulence in the atmosphere seems to be of two distinct types: one is associated with mountains and hills and involves, essentially, the breaking of lee waves. In this case a growing lee wave can reach a state in which heavier air instantaneously lies above lighter air[4] and leads immediately to gravitational instability, or the wave can create locally strong shears which break down into shorter unstable waves superimposed on the longer lee waves. The mountain-wave turbulence is reasonably well understood from observations of similar phenomena in laboratory experiments (Long, 1955). The second type seems to be associated with inversion layers in the atmosphere (layers with large vertical potential temperature gradients) which have an accompanying large shear of the horizontal wind. This interplay of stability and shear is the main subject of this paper.

In the first place, the very presence of inversion layers in the atmosphere is somewhat surprising. The atmosphere is an enormous body of fluid and therefore one would expect it to be generally turbulent. However, our classical concept of turbulence is that it has a diffusive effect and so should tend to disperse strong gradients of shear and potential temperature. Instead, according to experiments described above, turbulence in a stratified fluid like the atmosphere tends to cause a formation and maintenance of these gradients. The experiments referred to do not really explain this phenomenon; they merely show that it exists and that it is related to the stable density stratification of the fluid. A detailed understanding of the mechanism is still needed.

There are two possibilities for explaining the turbulence on inversion surfaces. One is that the surface has a build-up of potential temperature gradient and shear, presumably through the action of turbulence[5] which continues in such a way that shear increases faster than temperature gradient. Then when the gradient Richardson number falls below the critical value of 1/4, small waves are able to grow and develop into turbulence. A process resembling this is actually observed in the growth and breaking of waves occasionally seen in cloud formations and frequently seen on ultra-sensitive radar. These atmospheric waves are very much like the waves observed in experiments by Thorpe (1968) on the interface of two liquids of different density and velocity. Thorpe's waves are obviously of the Kelvin-Helmholtz type. A second possibility for clear-air turbulence is that it is a steady-state type of turbulence such as is encountered in the experiment reported on in this paper. As mentioned above, the eddies are not regular as in Kelvin-Helmholtz instability. This turbulence exists indefinitely and does not build up and die away as one would expect if the shear and instability develop as functions of time. It is an unanswered question which of the two possibilities mentioned above account for the bulk of the clear-air turbulence in non-mountainous regions.

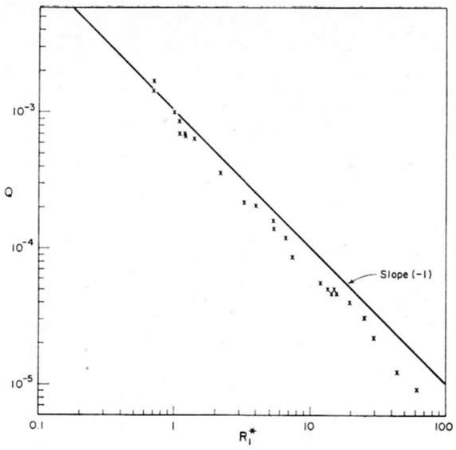


Fig. 5. Plot of the non-dimensional buoyancy flux, Q, against the overall Richardson number, Ri*.

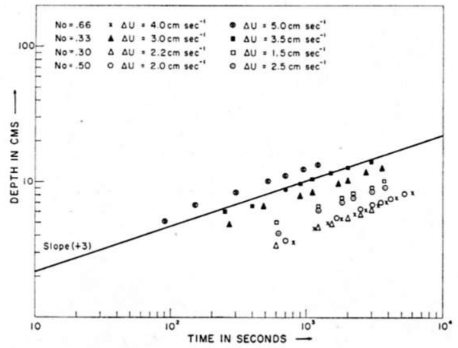


Fig. 6. Plot of the entrained depth of the homogeneous fluid against time in the erosion experiments. N_0 is the initial Brunt-Väisällä frequency.

[4] More accurately air with lower potential temperature lies above air of higher potential temperature.

[5] Another mechanism for the build-up of the gradients is the superposition of two layers of air of different geographical origin. If they come from different regions, they could have quite different properties. This is an advective mechanism.

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