A NOTE ON THE MEASUREMENT OF THE INDUCED DRAG FACTOR (k) OF A GLIDER

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INTRODUCTION

The coefficient of induced drag on a wing which has a perfect elliptical spanwise loading distribution is given by

$$C_{di} = \frac{C_l^2}{\pi A} \qquad (1)$$

where C_l is the lift coefficient and A is the aspect ratio.

Unfortunately, such practical factors as non-elliptic planform, wing twist, part span flaps, and the presence of a fuselage all detract from the perfect spanwise loading and result in a higher C_{di} than the theoretical minimum. In practice, therefore,

$$C_{di} = \frac{kC_l^2}{\pi A} \qquad (2)$$

where k, termed the induced drag factor, is always greater than 1.0 and typically of the order of 1.04 to 1.06 in high-performance gliders.

FLIGHT TESTS

There is no easy way to measure C_{di} directly, and the normal practice is to make flight measurements of the performance of the glider over a wide speed range. From these results the relative values of C_d and C_l can be determined over the range covered.

It is usual then to assume that the overall C_d of the glider is made up of two components such that

$$C_d = C_{do} + \frac{K\,C_l^2}{\pi A} \qquad (3)$$

where C_{do} is the profile drag coefficient of the whole glider at (or extrapolated to) zero lift, and the second term covers all the contributions to drag which appear to vary with lift. In general, these drag contributions are found to be proportional to C_l^2 over the larger part of the C_l range; hence they can all be wrapped up in the one C_l^2 term using the constant K. The induced drag factor k (Eq. 2) is simply one contribution towards the overall value of K.

From the flight test measured results, C_d can be plotted against C_l^2 and the result should be a substantially straight line, at least over the lower C_l range. At high C_l's, it is normally found that C_d increases even more rapidly.

The slope of the best line through C_d/C_l^2 points then gives the value of $K/\pi A$ and the value of C_d when $C_l = 0$ is the C_{do} value. Thus the two unknowns in Eq. 3 have been determined.

ACTUAL RESULTS

When careful and accurate flight measurements have been made and the foregoing method has been used to determine K, the result has usually been to find a value very much in excess of the calculated value for k. The indications are that the other contributions to drag variation with lift are much more significant than non-elliptic lift distribution.

CALCULATED PERFORMANCE

With the availability of good wind tunnel results on the Wortmann wing sections and of a computer program for calculating glider performance in fine detail, it has been possible to produce calculated performances for many gliders. To do this, assumptions have been made about the value of k and of the drag contribution of miscellaneous drag sources other than the wing, tail surfaces, and induced drag. Where good measured performance results have been available, it has been found that with suitable values of miscellaneous drag coefficient and of k the calculated performance is extremely close to the measured result. But whereas, on gliders of reasonable planform, the value of k used in the calculations has always been of the order of 1.05, the values of K found by the C_d/C_l^2 method are frequently in the range 1.3-1.5 or even higher.

In order to investigate this difference, a C_d/C_l^2 plot was made (see Fig. 1), from the calculated performance of Sigma. This showed the "text-book" shape being substantially straight up to $C_l = 1$ and with C_d increasing at a greater rate with higher C_l's. The calculated performance was based on the following data:

wing section	FXW/Sigma/136-017
aspect ratio	36.2
induced drag factor (k)	1.04

Line A represents the best straight-line fit to the points below $C_l = 1$. The equation of this line is:

$$C_d = 0.00825 + 1.47 \frac{C_l^2}{\pi A} \qquad (4)$$

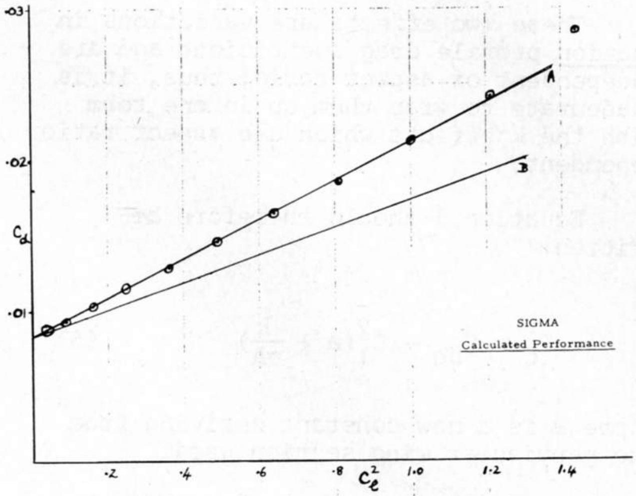

FIGURE 1

Thus, whereas the performance was calculated on the basis of k = 1.04, the overall result shows a value of K of 1.47.

CONTRIBUTIONS TO K

The calculated performance is based on wind-tunnel measurements of section profile drag at appropriate Reynolds Numbers. Examination of the wind-tunnel results shows that profile drag coefficient increases at higher C_l's at any one Reynolds Number; also profile drag coefficient increases as Reynolds Number decreases and this too produces an apparent dependence on C_l. Figure 2 shows the wind-tunnel data for FX 66-S-196 which demonstrates both these effects.

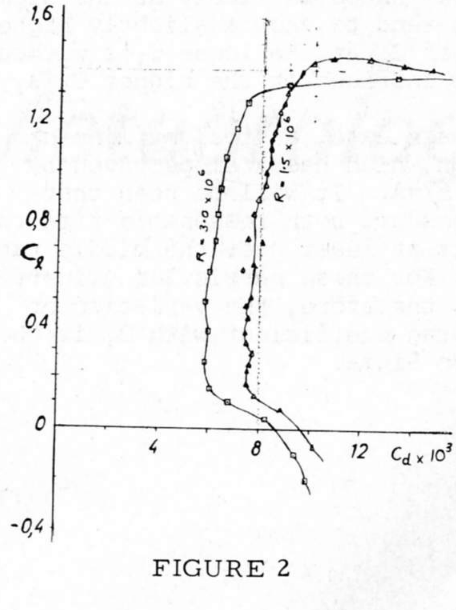

FIGURE 2

These two effects are variations in section profile drag coefficient and are independent of aspect ratio; thus, it is inaccurate to wrap them up in one term with the k effects which are aspect ratio dependent.

Equation 3 should therefore be written:

$$C_d = C_{do} + C_l^2(a + \frac{k}{\pi A}) \qquad (5)$$

where a is a new constant deriving from the particular wing section used.

The equation for line A in Fig. 1 is:

$$C_d = 0.00825 + C_l^2\ (0.0038 + \frac{1.04}{\pi A}) \qquad (6)$$

In order to emphasize the effect of this constant a, line B has been drawn on Fig. 1 having the equation:

$$C_d = 0.00825 + 1.04\ \frac{C_l^2}{\pi A} \qquad (7)$$

OTHER RESULTS

Figure 3 shows C_d/C_l^2 plots based on calculated performances of other gliders having Wortmann laminar flow wing sections. These curves do not show the "text-book" shape as clarly as the Sigma curve but tend to show a slightly higher increase of C_d at the lower C_l's without the rapid increase at the higher C_l's.

In each case, a straight line has been drawn which has a slope given by 0.0038 + k/πA. It will be seen that these lines are both reasonable fits to the points at least over the middle range of C_l's. For these particular gliders at any rate, therefore, the variation of profile drag coefficient with C_l is the same as on Sigma.

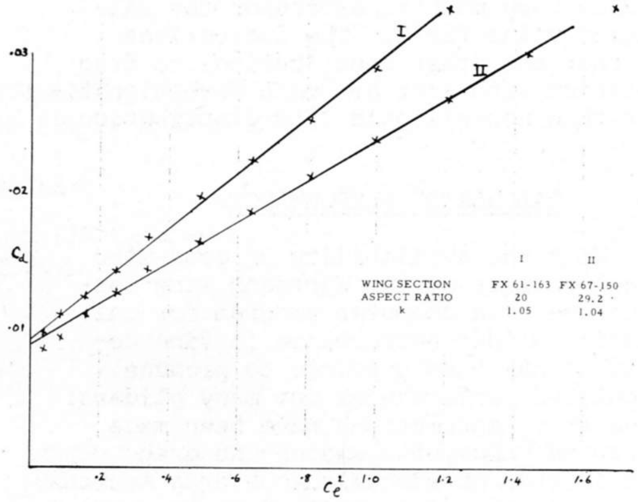

FIGURE 3

CONCLUSION

There appears to be a good case for treating lift dependent drag as being composed of two separate terms varying as C_l^2. One is purely the induced drag (with a k value to cover non-ellipticity of lift distribution) which is inversely proportional to aspect ratio. The other is the variation of wing profile drag coefficient and is independent of aspect ratio.

In the Reynolds Number range normally used by gliders and for Wortmann wing sections, it appears that this variation of wing profile drag coefficient contributes a slope of about 0.0038 to the C_d/C_l^2 plot.