Automatic Recording and Analysis for Glider Performance Testing

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With a mathematical appendix by Miss D. M. Southey

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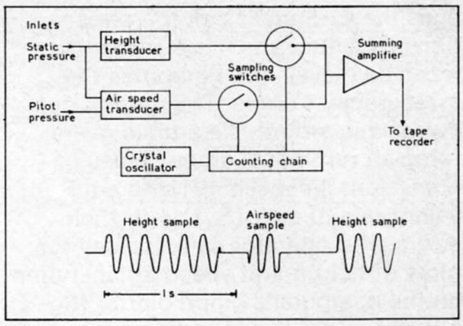

Fig. 1. Block diagram of recording system and example of recorded waveform.

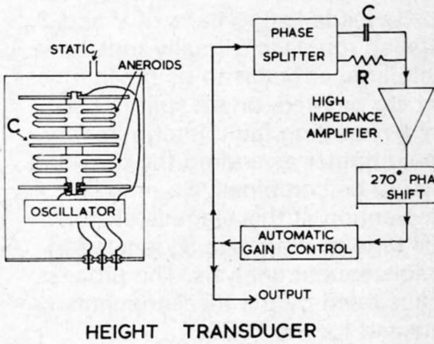

Fig. 2. The height transducer.

Summary

This paper describes an automatic recording system and its use to measure the performance of gliders. The 'partial glide' method is used, recording height and speed as functions of time; the recording is replayed into a digital computer for analysis. Results are presented for four types of glider.

Introduction

The only practical method of measuring the performance of a glider is to measure the rate of sink at a series of constant airspeeds with clock and altimeter. Because of the random vertical motion of the air, many such 'partial glides' – typically more than 100 – have to be measured to establish the polar with reasonable accuracy. The recording methods used so far include pencil and kneepad [1], radio to an observer on the ground and photography of the instrument panel [2]. These all require manual analysis on the ground after the flight. The present research began with a resolution to avoid this manual labour, by using a recording system in the glider that could be played back directly into a digital computer on the ground. All subsequent calculations can be done by the computer. Such a system greatly reduces the time and labour required to measure each glider's polar.

This paper briefly describes the flying programme and the recording apparatus, which is described more fully elsewhere [3]. It concentrates mainly on the analysis techniques used and the results obtained for four types of glider.

The Flying Programme

The flying programme is very similar to that used by previous observers. The glider is towed to a convenient height, usually 9,000 ft, and is flown in a series of straight runs at constant speed. The pilot is given a flight plan consisting of a list of the desired speeds for the runs with the height to be lost at each speed. A significant improvement on earlier work is the high sensitivity (±3 ft) of the height measurement, which allows the use of very short runs. As many as 25 runs, over the entire speed range from 30 to 100 kts, can be obtained from one flights. Six flights, giving between 100 and 150 runs, are sufficient to establish a polar.

The weather must, of course, be carefully chosen to minimise the vertical air movements which are the most serious source of error [2]. All the flights took place in the early morning, before convection started; but standing waves were troublesome, and perhaps only days with little or no wind are really suitable.

Apart from these flights, at least one flight is needed to measure the position error of the pitot and static pressure sources in the glider.

The Recording System

The system (fig. 1), measures pressure height and indicated airspeed (IAS) at intervals of about 1.6 sec. The transducers convert the input quantity into frequency, variable from 4 to 10 kHz, by using standard altimeter or ASI capsules to move the plates of a variable capacitor, which controls the frequency of an R-C oscillator. Figure 2 shows the height transducer, and a block diagram of its oscillator (which is contained in the same case. A crystal oscillator and counting chain is used to generate sampling pulses, which gate samples from the transducers alternately, with gaps between, to the recorder. The samples are of defined duration, so the number of cycles in each sample is a measure of the appropriate quantity.

On the ground, the record is played into a computer, which counts the cycles in each sample. Using a calibration table the computer converts each count to a reading of height or equivalent air speed.

In addition to the data a filtered speech channel (up to 3.5 kHz) is fed to the summing amplifier and recorded. It does not affect the data, which occupy the band 4 to 10 kHz, and is filtered off before feeding the data to the computer; but it allows the pilot to mark the recording, and make comments which the computer operator can use to guide him when playing it back.

The whole airborne system occupies about 1 ft^3 and weighs 25 lb with batteries. It records height from 0 to 10,000 ft ICAN with a reading accuracy of ±3 ft and long term accuracy of about ±300 ft, and IAS from 0 to 120 kts, with a reading accuracy of ±0.1 kt and a long term accuracy of ±0.5 kt. The time intervals, controlled by a crystal oscillator, are so accurate that their errors are negligible.

Analysis

The magnetic tape is played back into a PDP-8 computer, which is programmed to count and store the number of cycles in each sample, together with a run number allocated by the operator. All these counts are punched out on paper tape for later analysis on a larger computer.

This reads the data one run at a time, converts each count to ICAN height or EAS (there is no doubt which count is which, since the height count is greater than 3,000 and the speed count is less than 2,500) and performs a least squares fit of a straight line to the height v. time graph. It also averages the EAS makes all the corrections to reduce the measurements to mean sea level, and computes the measurement errors. The normal output of this program is a table giving, for each run, the average values of equivalent air speed (V) and equivalent rate of sink (S_o), with their errors. The operator can also call for plots of height and speed against time on the computer's graph plotter (fig. 3), but this is only used as an occasional check as it takes 4 minutes per run to plot the graphs.

When all the runs have been analysed, these tables are examined and a new tape is made listing pairs of V and S_o for each run. Occasionally runs have such large errors as to be meaningless, and are omitted, or are split in two (by a recording fault, interpreted by the computer as ending the run) and have to be combined. So manual intervention at this stage is useful. This tape of V against S_o is used in all subsequent analysis. The process is illustrated by the measurements obtained for the T53B.

First, every point is plotted by the computer (fig. 4). This shows the general shape of the polar and the amount of scatter. It allows the operator to estimate the speed, V_{min}, at which the glider starts to stall and the polar departs from the theoretical curve

$$S_o = AV^3 + B/V \qquad (1)$$

Next, this theoretical curve is fitted, by a least squares technique, to all the points faster than V_{min}. This gives a fitted polar, with lines at 2 standard deviations above and below it (fig. 5). The values of A and B are also used to estimate C_{DO} and k of equation (2). Then using a table of defining speeds (usually every 5 kts, but closer around V_{min}), the centre of gravity of all the points in each interval is plotted, with the standard deviation of S_o for each (fig. 6). Comparison of this plot of '5-knot means' with the fitted polar will show any significant deviations of the polar from equation (1). None have yet been found except at very low speeds.

Another earlier program, still in use, plots a graph of $C_D v C_L^2$, fits a straight line (by a least squares fit) to all points for speeds above V_{min}, and converts this back to a polar. It also prints out the constants C_{DO} and k in the equation

$$C_D = C_{DO} + kC_L^2/\pi A \qquad (2)$$

and their standard deviations. The polar curve differs little from the later fitted curve, but the newer technique is theoretically better since the scatter of the measurements of S_o is independent of air speed (it is due to random vertical air movements).

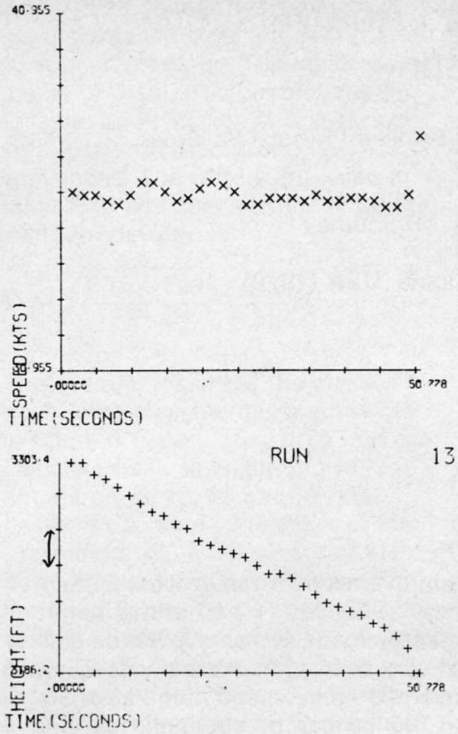

Fig. 3. Height and speed graphs from a typical run.

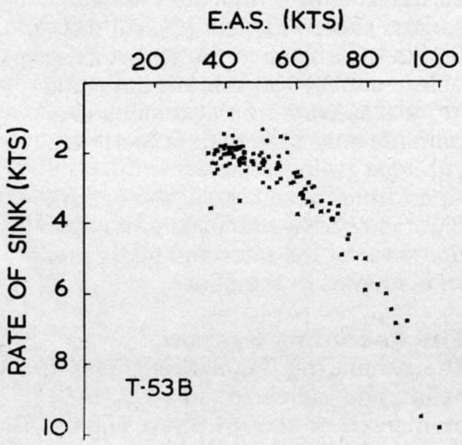

Fig. 4. All the measured points on the polar of the T53B.

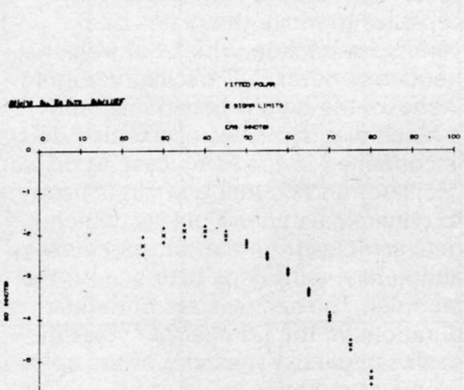

Fig. 5. The computer plot of the fitted polar, with error limits at ± two standard deviations.

The final polar (fig. 7) given for each aircraft is the fitted curve, with its limits at ±2 standard deviations, corrected manually below V_{min}.

Calibrations and Corrections

The height transducer is calibrated every few weeks against a mercury barometer. The airspeed transducer is calibrated daily against a water manometer. Air temperature is measured with a mercury thermometer during the climb.

The position error of the glider is measured, on a separate flight, using a trailing static head and an auxiliary pitot head (an open ended tube, usually mounted through the clear vision panel on the canopy). Two ASI's are used, read manually; one is permanently connected to the aircraft system, the other is switched between aircraft and external pitot and static. Since the glider is sinking, air is flowing from the static head up the long tube to the ASI case; this causes a small error in the measured airspeed,

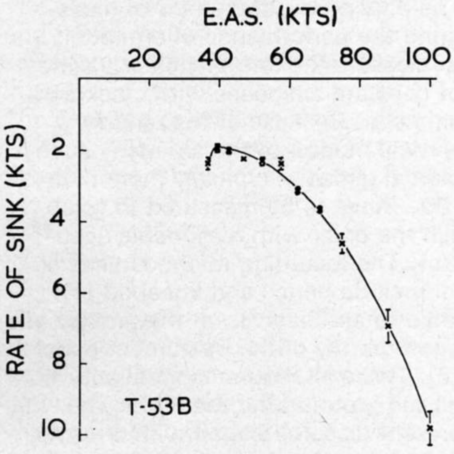

Fig. 6. The 'five-knot means', with their standard deviations, together with the fitted polar.

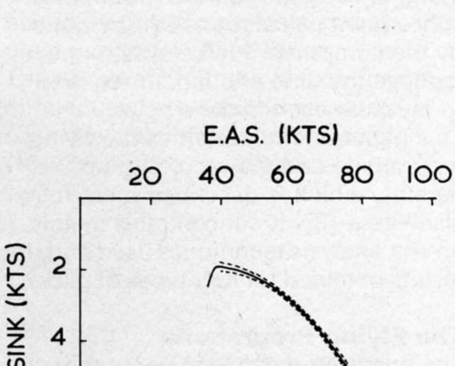

Fig. 7. The fitted polar of the T53B, with error limits at ± two standard deviations.

depending on the rate of sink of the glider [4]. This error, typically 1%, has been allowed for.
All the calibrations, and corrections for air density are applied by the computer.

Measurements of Four Gliders[1]

Dart 15 R
This was measured in March and April, 1967, primarily to test the equipment, before the analysis programs had been written. The aircraft, a late metal sparred Dart 15, with low incidence wing and manually operated retractable undercarriage was in good condition, but was not specially polished. It was ballasted to 780 lb AUW for each flight.
Twelfe measuring flights were made, giving 150 usable runs. The only runs rejected were a few which were unreadable, and a group made with the undercarriage down by mistake. The best estimate of the polar is shown in figure 8, with error limits at ±2 standard deviations, and summarised in table I.
There are few other measurements of Darts. Figure 9 shows all I can find, a Dart 17R from Cranfield [5] and the prototype Dart 15 measured by Zacher [6].

Skylark 4
This was measured in the summer of 1968. It was a Club aircraft in normal use, clean but not specially polished. The polar is shown in figure 10, and compared with that of R. H. Johnson [7] in figure 11.

Bocian
This was also measured in 1968. It was an old aircraft, with paint cracking off the leading edge in places. The polar is shown in figure 12, and compared in figure 13 with those of Merklein and Zacher [8] and Torode (private communication).

T53B Phoenix
This is an aircraft used for meteorological research. Externally it is a standard T53, except for a short badly streamlined nose boom carrying a pitot-static head.
Five flights gave 110 runs, an average of 22 runs per flight. The polar is shown in figure 7.

[1] All the polars in this paper are given in equivalent rate of sink and equivalent air speed, i.e. the values that would be obtained at sea level with perfect instruments and zero position error. They have been corrected where necessary to the AUW given in table I.

Table I. Summary of Results

Type	Wing area (ft^2)	AUW (lb)	Aspect ratio	Minimum sinking speed at (kts)	at (kts)	Best gliding angle	at (kts)	C_{DO}	k
Dart 15R	134.8	780	18.0	1.47±0.07	46.5	31.7±1.2	50	0.0113±0.0006	1.25±0.09
Skylark 4	173	840	20.5	1.27±0.07	40	31.5±1.5	40	0.0142±0.0006	1.14±0.1
Bocian	215.3	1,160	16.2	1.76±0.09	40	24.8±0.8	50	0.0131±0.001	1.58±0.13
T53	194	1,190	15.9	1.91±0.06	40	22.6±0.5	50	0.0167±0.001	1.465±0.08

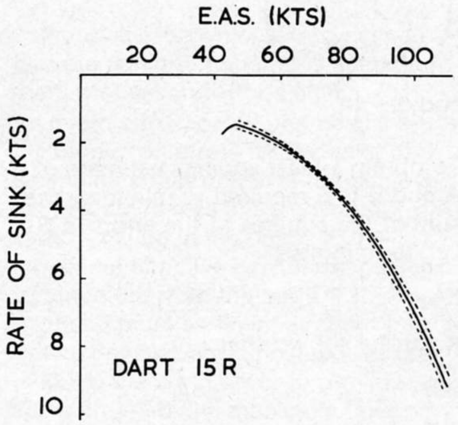

Fig. 8. The fitted polar of the Dart 15R.

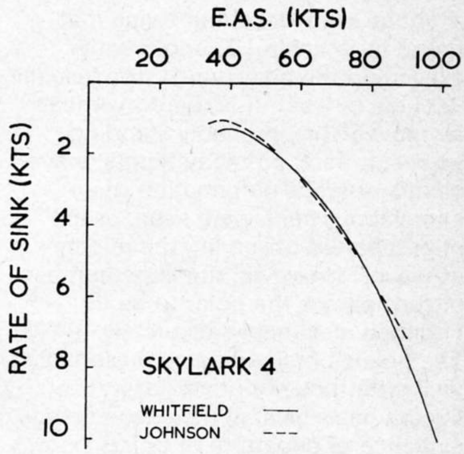

Fig. 11. Collected polars of the Skylark 4.

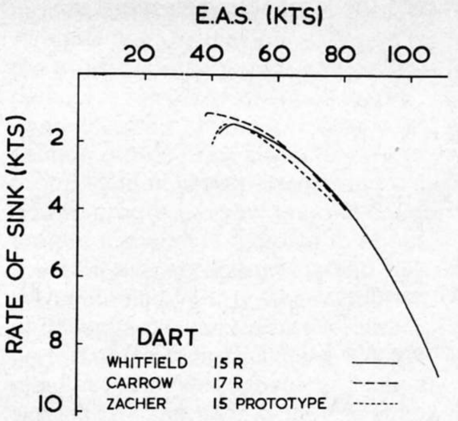

Fig. 9. Collected polars of the Dart.

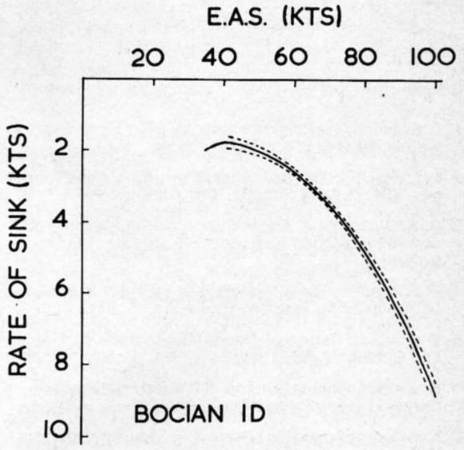

Fig. 12. The fitted polar of the Bocian 1D.

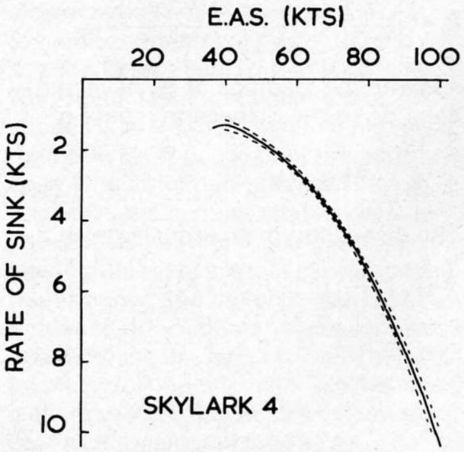

Fig. 10. The fitted polar of the Skylark 4.

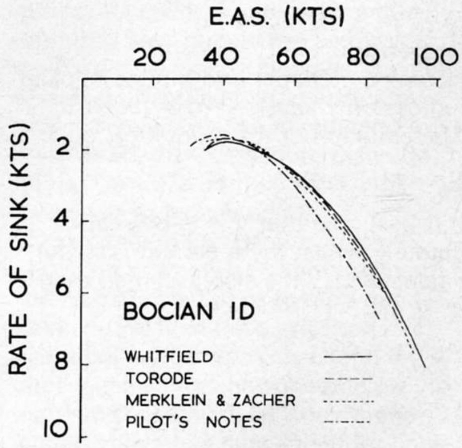

Fig 13. Collected polars of the Bocian 1D.

Conclusions

The new method has been shown to work and to produce acceptable polars. The minimum effort required is about 8 flights of 1 hour each (6 measuring, 1 position error and 1 spare), followed by 6 hours replaying the tapes into the computer, 6 hours manual work on calibrations and a further 6 hours computing.

The points (fig. 4) show considerable scatter. The standard deviation of S_o is about ± 0.5 kts, about twice that found by Machin [2] and greatly exceeding the measuring error (usually ± 0.1 kts or less). It is due to vertical air movements, probably standing waves; in fact, on many flights, wave clouds or wave deformation of air liner vapour trails were seen, or the pilot reported observing the effects of waves. However, the large number of runs allows the polar to be determined reasonably accurately.

The polars obtained agree reasonably well with those of other observers. Except near the stall there is very little evidence of departure of polars from the theoretical law of equation 1.

Copies of the computer programs used may be obtained from the author.

References

1 F. G. Irving: Performance tests by BGA flight test groups. Gliding 2: 7 (1961).

2 K. E. Machin: The performance testing of the Slingsby Sky. J. Roy. Aero. Soc. 58: 470 (1954).

3 G. R. Whitfield: A simple data recording system with computer analysis. Radio and Electronic Engineer (in press).

4 K. E. Machin: The effect of lag in pitot-static systems. BGA No. 2, Flt. Test Gp., Rep. No. 5 (1954).

5 D. Carrow: 'Guinea pig' performance trials. Sailplane and Gliding 17: 86 (1966).

6 H. Zacher: Flugmessungen mit Standard-Segelflugzeugen. OSTIV Publication IX (1965).

7 K. H. Doetsch and D. Lampard: Performance polars. Sailplane and Gliding 16: 44 (1965).

8 H. J. Merklein and H. Zacher: Flight performance measurements on twelve sailplanes. OSTIV Publication VII (1963).

Appendix

Reference: Draper and Smith: Applied regression analysis (Wiley 1966).

The equation of the theoretical polar is

$$S_o = AV^3 + BV^{-1}. \qquad (1)$$

It is assumed that V is known accurately whilst there are errors in the values of S_o. It is also assumed that the errors in S_o are independent of the flying speed V.

Suppose n sets of readings are taken. Let $\underline{S}_o$ be the vector of the n sets of readings of S_o. Then the set of n equations becomes the matrix equation

$$\underline{S}_o = \underline{R}\,\underline{\theta} \qquad (3)$$

where $\underline{R}$ is the n × 2 matrix

$$\begin{pmatrix} V_1{}^3 & V_1{}^{-1} \\ V_2{}^3 & V_2{}^{-1} \\ \vdots & \vdots \\ V_n{}^3 & V_n{}^{-1} \end{pmatrix}$$

and $\underline{\theta} = \begin{pmatrix} A \\ B \end{pmatrix}$

To obtain a least squares estimate of A and B it is required to minimise the sum of the squares of the errors in S_o, i.e., to minimise

$$S = (\underline{S}_o - \underline{R}\,\underline{\theta})' \; (\underline{S}_o - \underline{R}\,\underline{\theta}) \qquad (4)$$

It can be shown that

$$\frac{\partial S}{\partial \theta} = 0, \text{ if}$$

$$\underline{R}' \; (\underline{S}_o - \underline{R}\,\underline{\theta}) = 0, \qquad (5)$$

i.e.,

$$\underline{R}'\underline{S}_o = \underline{R}'\underline{R}\,\underline{\theta} = \underline{C}\,\underline{\theta}, \text{ where } \underline{C} = \underline{R}'\underline{R}.$$

Hence the least squares estimate

$\hat{\underline{\theta}} = \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix}$ of $\underline{\theta}$ is given by

$$\hat{\underline{\theta}} = \underline{C}^{-1}\,\underline{R}'\underline{S}_o. \qquad (6)$$

$$\underline{C} = \underline{R}'\underline{R} = \begin{pmatrix} \Sigma V^6 & \Sigma V^2 \\ \Sigma V^2 & \Sigma V^{-2} \end{pmatrix} \qquad (7)$$

$$\text{so } \underline{C}^{-1} = \frac{1}{\Delta} \begin{pmatrix} \Sigma V^{-2} & -\Sigma V^2 \\ -\Sigma V^2 & \Sigma V^6 \end{pmatrix} \qquad (8)$$

$$\text{where } \Delta = \Sigma V^{-2}\;\Sigma V^6 - (\Sigma V^2)^2$$

$$\hat{\underline{\theta}} = \frac{1}{\Delta} \begin{pmatrix} \Sigma V^{-2} & -\Sigma V^2 \\ -\Sigma V^2 & \Sigma V^6 \end{pmatrix} \begin{pmatrix} \Sigma V^3 & S_o \\ \Sigma V^{-1} & S_o \end{pmatrix} \qquad (9)$$

Hence

$$\hat{A} = \frac{\Sigma V^{-2}\;\Sigma V^3\; S_o - \Sigma V^2\;\Sigma V^{-1}\; S_o}{\Sigma V^{-2}\;\Sigma V^6 - (\Sigma V^2)^2}$$

$$\hat{B} = \frac{\Sigma V^6\;\Sigma V^{-1}\; S_o - \Sigma V^2\;\Sigma V^3\; S_o}{\Sigma V^{-2}\;\Sigma V^6 - (\Sigma V^2)^2} \qquad (10$$

Assume the readings of S_o have variance σ^2 so the dispersion matrix of $\underline{S}_o$ is

$$D\;(\underline{S}_o) = \sigma^2\,\underline{I}. \qquad (11)$$

The dispersion matrix of $\hat{\theta}$ is given by

$$V\;(\hat{\theta}) = E\;[(\hat{\theta} - \theta)\;(\hat{\theta} - \theta)]. \qquad (12)$$

But $\hat{\underline{\theta}} - \underline{\theta} = \underline{C}^{-1}\,\underline{R}'\underline{S}_o - \underline{C}^{-1}\,\underline{C}\,\underline{\theta}$.

$$\begin{aligned} \text{Hence } V\;(\hat{\theta}) &= E\;[\underline{C}^{-1}\,\underline{R}'\,(\underline{S}_o - \underline{R}\,\underline{\theta}) \\ &\quad (\underline{S}_o - \underline{R}\,\underline{\theta})'\,\underline{R}\,\underline{C}^{-1}] \qquad (13) \\ &= \underline{C}^{-1}\,\underline{R}'\,E\;[(\underline{S}_o - \underline{R}\,\underline{\theta})\;(\underline{S}_o - \underline{R}\,\underline{\theta})']\;\underline{R}\,\underline{C}^{-1} \\ &= \underline{C}^{-1}\;\underline{R}'\;\sigma^2\;\mathrm{I}\;\underline{R}\;\underline{C}^{-1} \end{aligned}$$

$$V\;(\hat{\theta}) = \sigma^2\;C^{-1} \qquad (14)$$

Therefore the variances of $\hat{A}$ and $\hat{B}$ are given by

$$\text{var}\;(\hat{A}) = \frac{\sigma^2}{\Delta}\;\Sigma\,V^{-2}$$

$$\text{var}\;(\hat{B}) = \frac{\sigma^2}{\Delta}\;\Sigma\,V^6 \qquad (15)$$

$$\text{and cov}\;(\hat{A}\,\hat{B}) = \frac{-\sigma^2}{\Delta}\;\Sigma\,V^2. \qquad (16)$$

The residual sum of squares is

$$\begin{aligned} S_r &= (\underline{S}_o - \underline{R}\,\hat{\underline{\theta}})'\;(\underline{S}_o - \underline{R}\,\hat{\underline{\theta}}) \qquad (17) \\ &= \underline{S}_o{}'\underline{S}_o - \underline{S}_o{}'\underline{R}\,\hat{\underline{\theta}} \\ &= \Sigma S_o{}^2 - A\,\Sigma V^3\,S_o - \hat{B}\,\Sigma V^{-1}\,S_o. \qquad (18) \end{aligned}$$

A standard result of statistics is that the expected value of S_r is $\sigma^2\;(n-2)$, so σ^2 can be estimated by $S_r/(n-2)$.

Hence $\sigma^2 \approx$

$$\frac{\Sigma S_o{}^2 - \hat{A}\,\Sigma V^3\,S_o - \hat{B}\,\Sigma V^{-1}\,S_o.}{n-2} \qquad (19)$$

Estimates of the variances and covariance of $\hat{A}$ and $\hat{B}$ can be obtained by substituting this estimate of σ^2 in equations 15 and 16.

Let $\hat{S}_o$ be the estimated value of S_o, i.e., $\hat{\underline{S}}_o = \underline{R}\,\hat{\underline{\theta}}$. (20)

Then var $(\hat{S}_o)$ =

$$E\;(\hat{S}_o{}^2) - [E\;(\hat{S}_o)]^2 \qquad (21)$$

$$\begin{aligned} &= E\,[(\hat{A}V^3 + \hat{B}\,V^{-1})^2] - [E(\hat{A}V^3 + \hat{B}V^{-1})]^2 \\ &= V^6 E(\hat{A}^2) + 2V^2 E(\hat{A}\hat{B}) + V^{-2} E(\hat{B}^2) \\ &\quad - V^6\,[E(A)]^2 - 2V^2 E(\hat{A}) E(\hat{B}) - V^{-2} \\ &\quad [E(B)]^2 \\ &= V^6 \{E(\hat{A}^2) - [E(\hat{A}^2)]^2\} \\ &\quad + V^{-2} \{E(\hat{B}^2) - [E(\hat{B})]^2\} \\ &\quad + 2V^2 \{E(\hat{A}\hat{B}) - E(\hat{A})\;E(\hat{B})\} \\ &= V^6\;\text{var}\;(\hat{A}) + V^{-2}\;\text{var}\;(\hat{B}) + \\ &\quad 2V^2\;\text{cov}\;(\hat{A}\,\hat{B}). \qquad (22) \end{aligned}$$

Substituting the estimates of the variances and covariance of $\hat{A}$ and $\hat{B}$ already found into this equation gives an estimate of the variance of $\hat{S}_o$. The square root of this is the standard deviation of $\hat{S}_o$.

The computer program uses equations 1 and 10 to calculate the best fit to the polar; and equations 20, 15, 16 and 22 to calculate the error of the fitted polar.