Clear-air turbulence in the upper atmosphere

*Robert R. Long **
Department of Mechanics,
The Johns Hopkins University
Baltimore, Maryland

Paper presented at the XIth OSTIV Congress 1968, Leszno, Poland

1. Introductory Remarks

Clear-air turbulence in the upper atmosphere is intimately associated with the presence of density variations, and, paradoxically, primarily with gravitationally stable density variations. In review of gravitational stability in fluids, consider the case of a resting **liquid** with a density decrease with height, associated perhaps with a variable salt content, or a variable temperature field. If the fluid is disturbed slightly, as in Fig. 1, the density lines will oscillate up and down, but the motions will weaken with time as frictional effects come into play. The fluid is gravitationally stable.

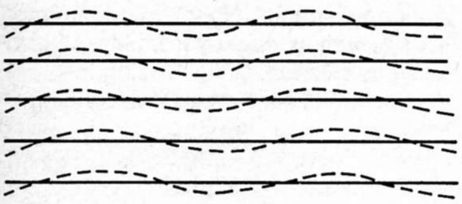


Fig. 1

The atmosphere is a compressible fluid, and its density normally varies with height because of two effects: (1) Pressure and temperature normally decrease with height and, as revealed by the relevant thermodynamic equations, the density also decreases with height except, possibly, in extremely thin layers near the ground. (2) The atmosphere has a variable moisture content and, since water vapor is lighter than dry air, the density will tend to be less where the air is more moist.

The effect of moisture on density variations is a minor one and can be formally eliminated altogether by using, instead of actual temperature T, a virtual temperature T′ defined by

$$T' = \frac{T}{1-\frac{3}{8}\frac{e}{p}} \qquad (1)$$

where e is water-vapor pressure and p is pressure.

The effects of temperature and pressure on the density variation of air may be considered together as follows: For the time periods involved in clear-air turbulence problems, a sufficiently accurate equation is

$$\frac{d}{dt}\left(p/\rho^{\lambda}\right) = 0, \qquad \lambda = 1.4 \qquad (2)$$

which is derivable from the first law of thermodynamics and from the assumption that there is no gain or loss of heat from radiation, conduction, etc. Equation (2) is analogous to the approximation that density itself is conserved in the mechanics of **liquids**. The equation may be integrated to yield p/ρ^{λ} = const for an individual particle, and it is convenient to evaluate the constant by supposing that the air parcel is brought adiabatically to a new reference pressure P, in which state it assumes a new density, $\bar{\rho}$ If we choose the reference pressure to be the same for all parcels, we then speak of $\bar{\rho}$ in the equation

$$p/\rho^{\lambda} = P/\bar{\rho}^{\lambda} \qquad (3)$$

as the **potential density.** It follows that $d\bar{\rho}/dt = 0$, so that for the atmosphere $\bar{\rho}$ is analogous to the density itself in a liquid. It is easy to show that just as $g(\rho_1-\rho_0)/\rho_0$ is the buoyancy force per unit mass acting on a parcel of liquid of density ρ_1 in an environment of density ρ_0, $(\bar{\rho}_1-\bar{\rho}_0)/\bar{\rho}_0$ represents the buoyancy force on a parcel of air of potential density $\bar{\rho}_1$ in an environment of potential density $\bar{\rho}_0$. Thus we obtain the basic aspects of gravitational stability and instability in the atmosphere if we consider variations of **potential density.** There are other effects of compressibility, but these can be shown to be negligible if $g\delta/c^2 \ll 1$, where g is the acceleration of gravity, δ is the vertical amplitude of the disturbances, and c is the speed of sound. This number is less than 0.1 in problems of clear-air turbulence[1]. In summary, if we consider potential density variations in air instead of density variations, we may completely neglect the compressibility of air and regard the atmosphere as a liquid.

With this in mind, we may now enumerate five basic causes of atmospheric turbulence. (1) Mechanical stirring, as in flow of air over trees and houses. (2) Direct heating effects (primarily near the ground) in which the potential density of some of the fluid is decreased by heating. The light parcels rise and are replaced by heavier fluid from above. (3) Vertical accelerations caused by buoyancy as heat is added to a parcel by release of the latent heat of condensation of water vapor. (4) Instability caused by the motion of potentially heavy air over potentially light air. This can be caused by advection processes as layers of heavy air move over layers of light air, or by motions connected with the growth or deformation of gravity waves. (5) Instability of shear, in which energy of mean shearing motion is converted into turbulent energy by the growth of wave disturbances.

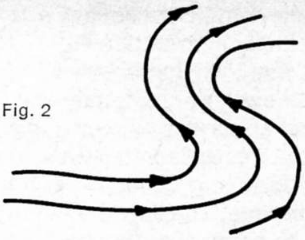
The first two causes mentioned above are not operative to any extent at high levels in the atmosphere. The third is relevant to turbulence in clouds and therefore does not concern us directly in this paper, although it may cause disturbances in the clear air near clouds. The remaining two causes of turbulence, namely gravitational instability and the instability of shear, are obviously the basic causes of clear-air turbulence. The advection of heavy air over lighter air is occasionally mentioned by meteorologists as a basic cause of meteorological phenomena, for example, tornadoes. It is difficult to see, however, how this can happen, except near the ground where, as is well known, a «nose» of a cold front can protrude over lighter air below it as the lighter air is retarded by surface friction. On the other hand wave motion in a stable atmosphere can lead to deformations of density surfaces in which heavy air intrudes above lighter air. The waves will then break and cause turbulence. In the absence of shear, there is no local energy source to cause these deformations, and it is likely, therefore, that

* *The research was supported in part by the Boeing Scientific Research Laboratories, Seattle, Washington*

[1] The Mach number is always very small in meteorology (except possibly in tornadoes), and is not an important criterion for judging the effects of compressibility.

the work[1] done by mountains and hills on the moving air currents is the basic energy source. When there is shear, kinetic energy can be supplied to the disturbances at the expense of the mean motion (Richardson, 1920). We divide the discussion into two parts. In the first we consider the turbulence in mountain waves. In the second we consider the interplay of shear and density gradient.

Fig. 2



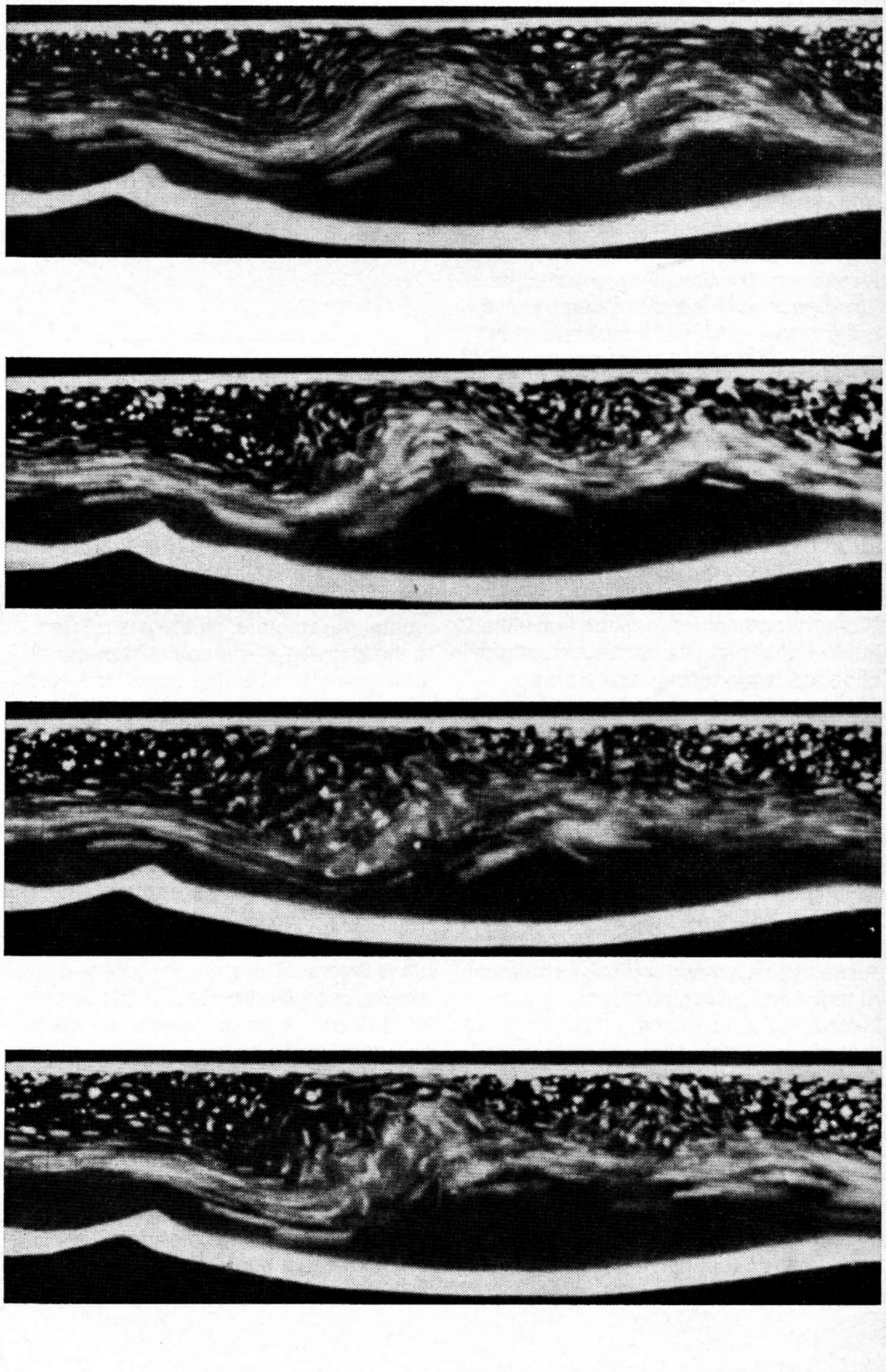
2. Clear-Air Turbulence in Mountain Waves

Aircraft reports show that clear-air turbulence is frequent in mountainous areas, and this suggests the basic importance of the breaking of gravity waves set up in airflow over mountains. It is well known that waves are possible in the air flowing over mountains and hills because of the buoyancy forces which exist in the normally stable atmosphere. Until a few years ago, attention given to the phenomenon was confined to an analysis of waves of infinitesimal amplitude. Such an analysis is particularly unsuitable for our problem because small disturbances in a uniform current are stable and cannot lead to the growth of turbulence. More recently, however, it became apparent from theory and experiment (Long, 1955) that disturbances can be sufficiently large to lead to local gravitational instability and turbulence. This comes about in one of two days. If a mountain wave gets large enough, it can create within itself large vertical shears. If the shear becomes great enough, local disturbances can become unstable and develop into turbulence. The other cause of turbulence in large mountain waves comes from an overturning instability as the wave amplitude gets so large that the local density variation with height reverses, yielding potentially heavy air over lighter air. One occurrence is an S-shaped streamline pattern as shown in Fig. 2 (Long, 1955). The exact breakdown of these waves requires more study. As indicated in the four photographs of Fig. 3, the breakdown into turbulence is an explosive occurrence. These pictures were taken 1/16 of a second apart. Obviously a careful investigation will require high-speed photography to follow the process. In the atmosphere the characteristic time is about 100 times that of the experiment so that a similar breakdown will take 5–10 seconds. The turbulent velocities that occur in the photographs are about one-third of the speed of the stream. If the highest stream velocities that can occur in the atmosphere are approximately 200 mph in the jet stream, this leads to a maximum vertical velocity in clear-air turbulence associated with mountains, of about 75 mph or 100 fps.

In the atmosphere, perhaps the most violent mountain waves in the world occur over the Owens Valley in the lee of the Sierra-Nevada Range. Observations in the UCLA project (Holmboe and Klieforth, 1954), indicated vertical velocities in the breaking waves with a maximum of about 60 fps. On the basis of the above discussion, this is a reasonable figure and indicates that the maximum figure of 100 fps will be experienced very infrequently.

The laminar waves in flow over mountains are so long that present aircraft can maintain reasonably uniform altitude in flying through them. Supersonic aircraft, however, may have difficulty, so that the wave motion itself may become a cause for concern in the future.

Fig. 3



[2] In a coordinate system moving with the fluid.

3. Turbulence in Shearing Currents

The other basic source of clear-air turbulence, as mentioned above, comes from the energy of the shear in a current in which the velocity varies with height. It is paradoxical that such regions occur where the density[3] variation with height is strongly stable. The reason is clear, however. Shear is a destabilizing effect in fluids. If there is no counteracting stability, the shear will break down immediately and disappear. Strong turbulence comes from strong shears, but strong shears owe their existence to stable density distributions. This reasoning is borne out by recent radar observations by Isadore Katz[4], showing that the most disturbed regions of the clear air are regions of strong stability.

The quantity of basic importance in relation to the interplay of shear and rotation is the Richardson number

$$Ri = \frac{\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}}{\left(\frac{d\bar{U}}{dz}\right)^2} \qquad (4)$$

where ρ_0 is a reference density, and $\bar{\rho}$ and $\bar{U}$ are the mean density and velocity distributions.

A review of the literature reveals that turbulence at Richardson numbers of order one is fairly well understood in so far as its basic characteristics are concerned. Then stability and shear are of equal importance, and the turbulence does not differ in a major way from turbulence in homogeneous fluids except for a tendency for the eddies to be flattened by a numerical factor of three or four (Ellison, 1962). The same literature reveals a general feeling that motions at high values of the Richardson number must be wavelike, indeed, that high Richardson numbers cannot sustain turbulence. Some believe, in fact, that this was «proved» by Richardson himself in his original work. What Richardson showed was that the Richardson number must be less than the ratio of turbulent momentum conduction to turbulent heat conduction. If these two are assumed to be equal, the result follows, but this is an assumption that is not supported by close argument.

A second argument against turbulence at high Richardson numbers has been based on a theorem by Miles (1963) that all small disturbances are stable when the Richardson number exceeds $1/4$. It is well known, however, that finite waves easily become unstable when the Richardson number is high, for then even modest waves cause local shears to increase considerably, reducing the **local** Richardson number and creating local instability (Philips, 1966).

There is another point. If the mean density and velocity distributions in a layer are such that the Richardson number is large everywhere, it may still happen that sublayers with strong shear (and local Richardson number below $1/4$) may form and be maintained for some time. These will be unstable even to small disturbances and therefore will tend to break up and produce turbulence. Recent evidence in the oceans (Woods, 1968) indicates that the thermocline can contain numerous sublayers of strong shear that produce patches of turbulence as they break up locally.

The question of turbulence at Richardson numbers of the order of one or larger may also be approached by considering an idealized experiment in which two infinite, horizontal, parallel flat plates move in opposite directions and are heated above and cooled below to produce a gravitationally stable, shearing current in the fluid contained between the plates (Fig. 4).

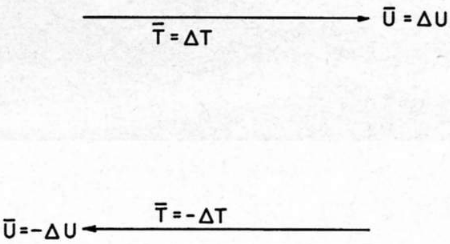


Fig. 4. Idealized experiment of shearing stratified flow between two moving plates.

For the sake of argument, we assume that the coefficients of viscosity ν, and heat diffusion K, are arbitrarily small, whereas ΔU, ΔT and H are finite. In our discussion it is convenient to use $\Delta\rho$ instead of ΔT as one of the fundamental parameters, where ρ is related to the density ρ_1 and some representative density ρ_0, by the equation[5]

$$\rho_1 = \rho_0 + \frac{\rho_0}{g}\rho \qquad (5)$$

There are three possibilities for the resulting flow in this experiment if we consider averages over arbitrarily long periods of time. They are: (1) laminar flow everywhere, (2) turbulent flow in some layers and laminar flow in the other layers, (3) turbulence everywhere. Because the Reynolds number is arbitrarily large, we would not expect the flow to be fully laminar, although it must be acknowledged that the laminar solution is stable for all infinitesimal disturbances. In any event, we may create turbulence in the vicinity of the boundaries by roughening the boundaries. We therefore dismiss the fully laminar motion from consideration. If the motion is fully turbulent, on the other hand, the vertical fluxes of momentum and heat, τ and q will be independent of the molecular coefficients and, therefore, will have finite values. Finally, let us consider the possibility that the motion is turbulent in the regions near the boundaries and laminar in a layer which may occupy part or all of the interior region. If the laminar layer or layers have finite thicknesses the transport of heat through them must be by molecular conduction[6] so that the heat transfer, q, will be infinitesimal in these layers. On the other hand, for statistically steady conditions, the heat transfer must be the same at all levels, so that q will be arbitrarily small in the turbulent layers as well. This means that the variation of $\bar{\rho}$ across the turbulent boundary layers will be vanishingly small. In the transition layer between the turbulent and laminar regions, there must be wave disturbances caused by the turbulent agitations. Certainly, however, some of these waves will be breaking. This will transfer heat and momentum of finite magnitudes, and our supposition that q is arbitrarily small cannot be correct. We conclude that there cannot be layers of finite thickness in which the motion is **completely** laminar. There may, however, be thin layers of laminar motion with large variation of $\bar{U}$ and $\bar{\rho}$ across them, such that q and τ will be finite in these thin layers and therefore everywhere.

Let us now consider in detail the turbulent flow in this experiment. We assume, on the basis of the above discussion, that the transfers

$$q = \overline{w'\rho'} \qquad (6)$$

$$\tau = \overline{u'w'} \qquad (7)$$

are finite. We assume also that the quantity

$$\frac{H\Delta\rho}{(\Delta U)^2} \qquad (8)$$

is large. (8) is of the form of a Richardson number based on the externally imposed velocities and densities. Although we assume (8) is large, the case of Ri $\sim$ 1 appears as a limiting case.

[3] For brevity we will henceforth use the word «density» instead of «potential density».

[4] Private communication.

[5] We employ the Boussinesq approximation so that the acceleration of gravity, g, is eliminated as one of the constants of the problem. The phenomena of this experiment depend, therefore, only on ν, K, Δ U, Δ ρ, and H.

[6] It is possible that momentum transfer can be effected by wave motions.

Since H is large in the sense that the Richardson number in (8) is large, we conclude that it is not an important parameter in determining conditions near the boundary. As a result, we may write down from dimensional analysis conditions that exist in the laminar boundary layer[7] (in the case where the boundaries are not rough) and in the turbulent boundary layer above.

(a) **Laminar Boundary Layer.**

$$z \sim \nu/\tau^{\frac{1}{2}} = l_1$$

$$\bar{U}_z = \frac{\tau}{\nu} f_1\left(\frac{z}{l_1}\right), \qquad \bar{\rho}_z = \frac{q}{\nu} g_1\left(\frac{z}{l_1}\right), \qquad Ri = \frac{z}{l_m}$$

$$u' \sim v' \sim w' \sim \tau^{\frac{1}{2}}, \qquad \rho' \sim \frac{q}{\tau^{\frac{1}{2}}}, \qquad l_x \sim l_y \sim l_z \sim l_1 \tag{9}$$

where $l_m = \tau^{\frac{3}{2}}/q$.

(b) **Turbulent Boundary Layer.**

$$z \sim l_m$$

$$\bar{U}_z = \frac{q}{\tau} f_2\left(\frac{z}{l_m}\right), \qquad \bar{\rho}_z = \frac{q^2}{\tau^2} g_2\left(\frac{z}{l_m}\right), \qquad Ri \sim 1$$

$$u' \sim v' \sim w' \sim \tau^{\frac{1}{2}}, \qquad \rho' \sim \frac{q}{\tau^{\frac{1}{2}}}, \qquad l_x \sim l_y \sim l_z \sim l_m \tag{10}$$

In writing down (9) we have assumed that $l_1 \ll l_m$ (τ and q finite). In writing down (10) we have made the high Reynolds number assumption that the molecular coefficients may be neglected. The results for the turbulent boundary layer are in agreement with those of Monin and Obukhov (1954).
The assumption (Millikan, 1938) that the solutions in (a) and (b) have overlapping validity in the region $l_1 \ll z \ll l_m$ leads to the logarithmic boundary layer:

(c) **Logarithmic Boundary Layer.**

$$l_1 \ll z \ll l_m$$

$$\bar{U}_z \sim \frac{\tau^{\frac{1}{2}}}{z}, \qquad \bar{\rho}_z \sim \frac{q}{\tau^{\frac{1}{2}} z}, \qquad Ri \sim \frac{z}{l_m}$$

$$u' \sim v' \sim w' \sim \tau^{\frac{1}{2}}, \qquad \rho' \sim \frac{q}{\tau^{\frac{1}{2}}}, \qquad l_x \sim l_y \sim l_z \sim z \tag{11}$$

The discussion of the interior region may be based on a paper by the author (Long, 1968). It is shown that the local time rate of change of the **total** disturbance energy (kinetic energy plus available potential energy) is given by a number of terms. One is an energy increase from the release of energy in the mean shear. Another is an increase of (potential) energy equal to the heat flux, q. A third term represents a decrease of energy and is given by the expression

$$-\frac{\overline{w'\rho'^2}}{2} \frac{\bar{\rho}_{zz}}{\bar{\rho}_z^2} \tag{12}$$

The last two terms in the energy equation are an energy loss through dissipation and an energy gain (or loss) by advection into the region in question. If we adopt arguments used originally by Richardson, we expect an approximate balance of the first three terms in the energy equation. Since the first two are positive, the term in (12) must be negative and of equal order of magnitude. The surprising importance of the curvature term in the energy balance may be explained physically. We expect the interior region to be one of laminar wave motion together with turbulence, perhaps in isolated patches, as indicated in certain observations in the oceans (Woods, 1968). If the turbulence is caused by wave breaking in a region, the mixing process will cause a slight rise in the center of gravity of the patch when the density has a negative curvature (Fig. 5). The patch will rise, transporting heat upward. When the curvature is negative, mixing causes the patch of slightly cooler fluid to sink, again yielding heat transport upward. This appears to be the essential process, since wave motion is incapable of transferring heat.

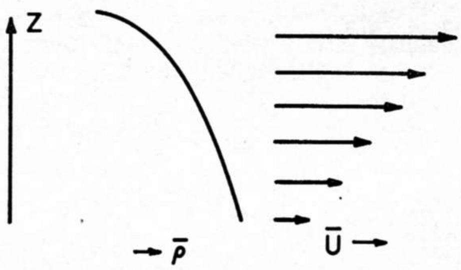


Fig. 5. A curved density profile and a linear velocity profile.

With this model in mind, quantitative estimates may be made of various quantities such as heat transfer q, momentum transfer τ, the representative dimension of the patch l_p, the distance of rise (or fall) of the patch L_p, the velocity of rise w_p, the ratio R of the volume of the turbulent patches to a given volume of fluid, and other quantities of interest. Some of the directly interesting results are that the mean velocity increases linearly above the turbulent boundary layer, and the mean density decreases as z^3. The wave length and amplitude of the wave disturbances are proportional to the Monin-Obukhov length l_m.

4. Final Remarks

Much of the above discussion relates to basic questions regarding turbulence in stratified shearing currents, rather than the specific problem of clear-air turbulence. It is obvious, however, that these basic considerations are essential. The emphasis in the discussion of turbulence in shearing currents has been on a quasi-steady model. There are indications, however, especially in the oceans, that the most severe turbulence may be associated with the local build-up of strong shears and their subsequent breakdown. This process may be considered a steady one over arbitrarily long periods of time, but as a practical matter the time periods of the build-up may be quite long. Some understanding of the physical process involved in the build-up of strong shears comes from a recent paper by Booker and Bretherton (1967). They show that waves moving through a stratified shearing current are strongly attenuated, when the Richardson number is large, at critical levels where the mean velocity equals the horizontal component of the wave speed. The lost wave energy goes into the mean field and builds up the mean velocity locally.
The problem of clear-air turbulence is complicated by its occurrence in the free atmosphere far from solid boundaries. The environmental layers are moving in unknown ways, and the turbulence problem is by no means a determinate one in the sense of turbulence between two moving, heated, parallel plates, for example. Nevertheless, considerations contained in Section 3 of this paper may have important applications to the clear-air turbulence problem. As an example, consider the energy spectra of the waves and turbulence. The spectrum of the waves may be obtained by noticing that $E(k) \sim u'^2 k^{-1}$.
But $u' \sim \bar{\rho}_z^{-\frac{1}{2}} l_z$ or $u'^2 \sim \bar{\rho}_z k^{-2}$, so that

$$E(k) \sim \bar{\rho}_z k^{-3} \tag{13}$$

This agrees with a deduction by Lumley (1964), although its derivation here does not concern dissipation at all and simply reflects that the frequency of these waves is independent of wave number.
The above theory may be used to explain the interesting observations in the atmosphere by Reiter and Burns (1966) that the kinetic energy spectrum consists of a decrease for larger wave numbers except for a «hump» in the spectrum at a wave length of 2000 ft. Since the «bumpiness» or «cobblestones» seem to be associated with large wave numbers, it is likely that the «humps» come from a dumping of energy into turbulence from the wave components in the turbulent patches.

[7] For simplicity we now assume that ν = K.

The highest wave numbers should reveal a $k^{-\frac{5}{3}}$ behavior, and this was clearly indicated by the data. On the other hand, the slopes of the curves had no consistent behavior in the wave region and a k^{-3} law could not be inferred. Other data by Shur (see Phillips, 1966) indicate a k^{-3} behavior at smaller k and $k^{-\frac{5}{3}}$ at larger k, but the hump was missing. It is possible that Shur's data was taken at smaller Richardson numbers than those of Reiter and Burns.

REFERENCES

Booker, J. R. and Bretherton, F. P., 1967: The critical layer for internal gravity waves in a shear flow. **J. Fluid Mech., 27**, p. 513.

Ellison, T. H., 1962: Laboratory measurements of turbulent diffusion in stratified flow. **J. Geophys. Res., 67**, p. 3029.

Holmboe, J. and Klieforth, H., 1954: **Sierra Wave Project**. Final Report of Contract AF19 (122)–263, University of California at Los Angeles.

Long, R. R., 1955: Some aspects of the flow of stratified fluids. III. Continuous density gradients. **Tellus, 7**, p. 341.

1968: Turbulent stratified flow. (To be submitted to the **J. Fluid Mech.**).

Lumley, J. L., 1964: The spectrum of nearly inertial turbulence in a stably stratified fluid. **J. Atmos. Sci., 21**, p. 99.

Miles, J. W., 1963: On the stability of heterogeneous shear flows. Part 2. **J. Fluid Mech., 16**, p. 209.

Millikan, C. B., 1938: A critical discussion of turbulent flows in channels and circular tubes. **Proc. 5th Int. Congr. Appl. Mech.**, Cambridge, Mass., p. 386.

Monin, A. S. and Obukhov, A. M., 1954: Basic regularity in turbulent mixing in the surface layer of the atmosphere. **Trudy Geophys. Inst. Akad. Nauk, SSSR, 24**, p. 163.

Phillips, O. M., 1966: **The Dynamics of the Upper Ocean**. Cambr. Univ. Press, England.

Reiter, E. R. and Burns, A., 1966: The structure of clear-air turbulence derived from «TOPCAT» aircraft measurements. **J. Atmos. Sci., 23**, p. 206.

Richardson, L. F., 1920: The supply of energy to and from atmospheric eddies. **Proc. Roy. Soc., A, 97**, p. 354.

Woods, J. D., 1968: Wave-induced shear instability in the summer thermocline. **J. Fluid Mech., 32**, p. 791.

Zusammenfassung

Die verschiedenen Möglichkeiten der Erklärung von «Clear-Air Turbulence» (CAT) werden erörtert und die Leewellen- und Scherungsinstabilitäten als wahrscheinlichste Ursache von CAT bezeichnet. Der explosive Uebergang von laminarer zu turbulenter Leewellenströmung wird im Experiment gezeigt (Fig. 3a–d). Als wesentliche Eigenschaft der Scherungslabilität findet der Verfasser ein gekrümmtes Dichteprofil (in der Vertikalen), das zu beschränkten Gebieten («patches») von CAT führt und stabile Schichtung mit vertikaler Scherung voraussetzt. Kuettner