Analysis of Problems Concerned with Justification and Regulation of Total Energy Variometer Compensators

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As an introduction to problem, avoiding the considerations of well known general total energy variometer theory, I should like only to recall that the difference between the conventional and total energy variometers depends on the fact that the latter does not show those components of vertical velocity (further conventionally named the components of change) which are equivalent of the change of potential energy of the glider into kinetic energy or vice versa. According to the above, the total energy variometer (TEV) does not record the static pressure variations, due to the change-components.

In earlier arrangements of the TEV, the above effect was obtained in such a way that the elimination of pressure variation of the change-components was carried out at the vent to atmosphere. The variations of dynamic pressure were subtracted from the static pressure by means of a Venturi head having the constant K = 1.

Several difficulties arose, such as icing, finding the correct Venturi head position, variation of K-factor, the additional aerodynamic drag etc.; the operation of a TEV by means of a Venturi head appears to be not recommended in practice. Accordingly, some other solution must to be sought.

The second way of avoiding the sensitivity of the TEV to pressure variations due to change-components is the method of compensating inside the variometer. It is based on the following philosophy: If on both sides of the orifice there exist the same pressure variations (as regards the value and time of appearance) they do not influence the flow through the orifice and consequently the variometer reading.

The above rule is realized in the so-called K-TEV, in which the pressure variations due to the change-components coming from static pressure circuit to one side of the orifice are compensated by the same variations (on the opposite side) of pressure in the compensating capacity.

The device for transmitting the total pressure variations to capacity, and at the same time dividing it by two, is the constant-stiffness capsule or membrane.

The stiffness «C» of the capsule must have such a value for the assumed active area of capsule «F» and capacity «V» to achieve the pressure increment on both sides of the capsule equal to pressure increment (adiabatic pressure) in capacity due to capsule deformation. From deformation conditions:

$$dV = df \cdot F = \frac{dp_m \cdot F^2}{C}$$

whence:

$$dp_m = \frac{dV \cdot C}{F^2}$$

where:

dV — volume deformation of capsule

df — linear deformation of capsule

dp_m — pressure increment involving the capsule deformation

The height of adiabatic pressure in capacity:

$$dp_N = -n \cdot p_s \cdot \frac{dV}{V}$$

where:

n — 1,4 p (ratio of specific heats)

p_N — pressure in capacity

Noting that

$$p_N - p_s \leqslant \Delta p_{max}$$

where

$$\Delta p_{max} \leqslant 40\ mm.\ H_2O - max\ value$$

of pressure difference across the orifice of the variometer.

If p_S = static pressure we can write:

$$dp_N = -n \cdot p_N \cdot \frac{dV}{V}$$

and in turn as mentioned above:

$$dp_m = -dp_N$$

Comparing the right-hand sides of the equations, and simplifying them, we obtain:

$$\frac{C}{F^2} = \frac{n \cdot p_s}{V}$$

This formula, being the condition of complete compensation of the pressure variations corresponding to the change-components, has on the right-hand side the value p_S depending on the altitude of flight (static pressure).

Taking this into account, it is easy to see that the condition of complete compensation can be fulfilled only for the one defined flight altitude. At other altitudes there appears either under- or over-compensation.

In consequence, the variometer will show only part of the change-components. The fraction of change-component recorded by the variometer, due to the p_S value being different from that for complete compensation can be expressed by the following equations:

$$\left(\frac{dp_{sk}}{dt}\right)_r = \frac{2\,dp_{sk}}{dt} \cdot \frac{\frac{n\,p_s\,F\,df}{V}}{\frac{n\,p_s\,F\,df}{V} + \frac{C\,df}{F}}$$

$$-\frac{dp_{sk}}{dt} = \frac{dp_{sk}}{dt}\left(\frac{2}{1 + \frac{CV}{n\,p_s\,F^2}} - 1\right)$$

where:

$\frac{dp_{sk}}{dt}$ — compensated pressure variation

$\left(\frac{dp_{sk}}{dt}\right)_r$ — pressure variation resulting from either under- or over-compensation

Writing further we have:

$$\psi = \frac{\left(\frac{dp_{sk}}{dt}\right)_r}{\frac{dp_{sk}}{dt}} = \left(\frac{2}{1 + \frac{CV}{n\,p_s\,F^2}} - 1\right) = \frac{2}{1+\xi} - 1$$

where:

Ψ — the value of under-compensated pressure variation fraction equal to variometer recorded fraction of change-component

$\xi = \frac{CV}{n\,p_s\,F^2}$ — compensation coefficient (equal to 1 for complete compensation) proportional to such a value of «C» which ensure the complete compensation for the considered p_S

Simplifying: $\psi = \frac{1-\xi}{1+\xi}$

substituting: $\xi = 1 + \delta$

where: δ — the fraction giving the difference between value of «C» and the value of «C» for complete compensation for the considered p_S and knowing that:

$$\psi = -\frac{\delta}{2+\delta} = \frac{-\delta/2 - \delta/2}{4 - \delta^2} = \frac{-2\delta + \delta^2}{4 - \delta^2}$$

for $\delta < 0.1$ we can simplyfy $\psi \approx -\frac{\delta}{2}$

In existing K-TEV the above imperfection is accepted, providing that the complete compensation will be correct for the defined altitude of flight. At other altitudes the variometer will record the change-components according to above equations.
The variometer complete with capacity and capsule should be calibrated in such a manner as to obtain the complete compensation for the defined flight altitude. From the equation of the complete compensation:

$$\frac{C}{F^2} = \frac{n\, p_s}{V_1}$$

it follows that the only parameter, the regulation of which can give complete compensation for the defined altitude, is the stiffness of capsule (change of capacity «V» produces considerable scale errors). The resultant stiffness of the compensating element consists of capsule stiffness and adjustable spring stiffness.
Apart from the possibility of regulation of parameter «C» it is necessary to have a measuring set, which would allow the complete compensation for defined flight altitude to be determined. We can consider such a set in laboratory conditions, remembering that the complete compensation takes place when the variometer records nothing when the pressure variations applied to the total pressure side of the variometer are equal to double those on the static side.
The operation of the set is as follows, referring to the diagram:

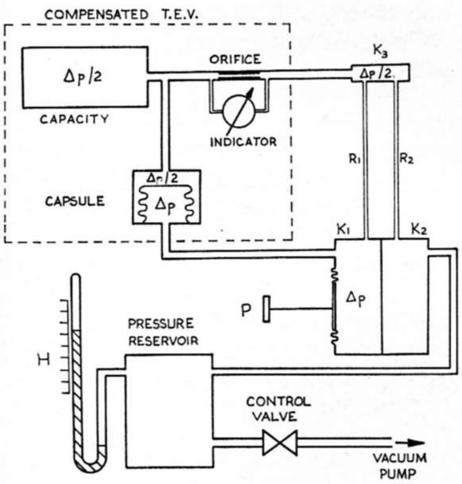

FIG. I SCHEMATIC ARRANGEMENT FOR CALIBRATION OF COMPENSATED TOTAL – ENERGY VARIOMETER

K_1, K_2, K_3 – Capacities. R_1, R_2 – Capillaries. P – Operating knob. H – Altitude scale

By means of a vacuum pump the pressure inside the set is lowered to the static pressure for the altitude, for which complete compensation is to be achieved. Pressing the knob «P» causes the deformation of the membrane «M» giving in consequence the pressure difference between chambers «K_1» and «K_2», and the flow through pipes «R_1» and «R_2». Since the flow resistances in pipes «R_1» and «R_2» are equal (being so arranged) the pressure increments applied to the variometer orifice will be half the increments applied to the capsule. If there exists complete compensation the variometer should record nothing; this indicates that the capsule stiffness is suitable.
If, however, the variometer records «climb» (the pressure in chamber «K_1» increased and in chamber «K_2» decreased) it shows that the adiabatic pressure in the capacity is too high. This leads to the conclusion that the capsule is absorbing too little a part of the pressure increment, being of unsatisfactory stiffness. The adjustment in such a case consists of increasing the spring stiffness until the variometer gives no response on pressing knob «P». If the reaction of the variometer is the opposite of the above, the adjustment required is to decrease the spring stiffness.
This suggests the idea of arranging the hand regulation of the capsule stiffness, ensuring complete compensation for all flight altitudes. There arise, however, two problems.
The first is whether the continuous regulation of capsule stiffness corresponding to changing altitude will not cause so great and rapid changes of capacity (including the capacity of capsule) that the indicator's errors will be too high to be accepted in practice.
The second problem is connected with the value and duration of the error caused by excessive regulation of capsule stiffness.
According to the first problem it is necessary to remember that the reading errors of the variometer are proportional to capsule deflections corresponding to its stiffness and inversely proportional to the time during which deflection take place. Moreover the capsule deflection corresponding to its stiffness during the constant pressure difference is proportional to this difference. Using the appropriate mathematical formula, the error ΔW is to be expressed as a function of extra flow ΔQ in variometer leak, as a consequence of capsule deformation corresponding to its stiffness change.
From the equation connecting leak flow, capacity, air temperature and the value «W» of indicated rate of climb, we can write:

$$\Delta W = \frac{RT\,\Delta Q}{V}$$

where:
T – air temperature on defined height
V – capacity
R – gas constant.

The value of Q can be expressed as the derivate of capsule volume:

$$\Delta Q = \frac{dV}{dt} = F\frac{df}{dt}$$

where:

F – capsule active area

$\frac{df}{dt}$ – derivative of capsule deflection with respect to time, which can be expressed as the product of two derivatives

$$\frac{df}{dt} = \frac{df}{dC}\cdot\frac{dC}{dt}$$

The first derivative being the deflection change with respect to capsule stiffness variation. It can be expressed therefore by differentiating both sides of the equation:

$$f = \frac{F\cdot\Delta p_m}{C} \quad \text{and hence:} \quad \frac{df}{dC} = -\frac{F\cdot\Delta p_m}{C^2}$$

The second derivative gives the capsule stiffness variation with respect to time as a result of automatic regulation of capsule stiffness assuming the steady preservation of complete compensation conditions. In respect of the above the second derivative can be expressed by differentiating the equation for complete compensation conditions. Therefore:

$$C = \frac{n F^2}{V}\cdot p_H$$

Differentiating produces:

$$\frac{dC}{dt} = \frac{nF^2}{V}\cdot\frac{dp_H}{dt} = \frac{nF^2}{V}\cdot\frac{dp_s}{dt}$$
$$= \frac{nF^2}{V}\cdot\left(-\frac{dH}{dt}\cdot\frac{p_s}{RT}\right) = -\frac{nF^2}{V}\cdot\frac{Wp_s}{RT}$$

Substituting the above expressions in the reading error formula we have:

$$\Delta W = \frac{RT}{V}\cdot F\cdot\frac{df}{dC} = \frac{RT}{V}\cdot F\left(-\frac{F\,p_m}{C^2}\right)$$
$$\cdot\left(-\frac{nF^2}{V}\cdot\frac{Wp_s}{RT}\right) = \frac{n F^4 p_m W p_s}{V^2 C^2}$$

but from the complete compensation equation it results that

$$\frac{nF^4}{V^2C^2} = \frac{1}{n\,p_s^2}$$

whence $\Delta W = \frac{\Delta p_m \cdot W}{n\cdot p_s}$

and the relative error is $\frac{\Delta W}{W} = \frac{\Delta p_m}{n\cdot p_s}$

Since Δp_m the pressure difference on capsule has a maximum value of

$$q_{max} + \Delta p_{max} = 330 \text{ mm } H_2O$$

it follows that

$$\frac{\Delta W}{W} = \frac{330}{1{\cdot}4 \cdot 10063} = 0{\cdot}023$$

where $\Delta p_{max.}$ = the pressure difference across the orifice corresponding to the maximum reading of the variometer.
It shows, that the error of continuous regulation is very small and can be neglected. For the extreme case of flight with a speed of V_1 = 300 km/h its value is not quite 2 per cent (at sea level). For increasing altitude it grows in inverse proportion to the value of p_S. To appreciate the results of excessive regulation we can consider the extreme case of gliding from level H_{max} to level H_{min} with no change of capsule stiffness, the regulation taking place rapidly when the level H_{min} is achieved.
Assuming that static pressure p_1 corresponds to hight H_{max}, and p_2 corresponds to H_{min} it is possible to find the capsule stiffness constants C_1 and C_2 from the equation of complete compensation conditions in a form:

$$C_1 = -\frac{n F^2}{V} p_1 \qquad C_2 = -\frac{n F^2}{V} p_2$$

Assuming further that the pressure difference Δp (corresponding to height decrement of $H_{max} - H_{min}$) is of constant value, and knowing that the elastic force of capsule is also constant, the deflection of the capsule is constant and equal to:

$$f_1 = \frac{\Delta p_m \cdot F}{C_1}$$

When at H_{min} level the constant C_1 rapidly changes into C_2 (when $C_2 > C_1$ according to $p_2 > p_1$) there appears no balanced elastic force increment on capsule deflection f_1

As a consequence the deflection of the capsule begins to change, producing a decrement of un-balanced elastic force and simultaneous increment of pressure difference on the capsule equal to the pressure decrement in the capacity as a consequence of capsule deformation. The above process persists until the pressure differences on the capsule balances the excess of elastic force. Denoting by:

Δf – change of capsule deflection
$\Delta\Delta p_{Nr}$ – increment of pressure difference on capsule the forces equilibrium equation is:

$$(C_2 - C_1) f_1 - C_2 \cdot \Delta f = \Delta\Delta p_{N_r} \cdot F$$

For $\Delta\Delta p_{Nr}$ being the function of capsule deflection

$$\Delta\Delta p_{N_r} = \frac{n\, p_2\, dV}{V} = \frac{n\, p_2\, F\, \Delta f}{V}$$

and: $$f_1 = \frac{\Delta p_m \cdot F}{C_1}$$

taking into account the equation of complete compensation we obtain:

$$\Delta p_m \cdot F \cdot \left(\frac{C_2 - C_1}{C_1}\right) - C_2\, \Delta f = \frac{n\, p_2\, F^2 f}{V} = C_2\, \Delta f$$

hence: $$\Delta f = \frac{\Delta p_m\, F\, (C_2 - C_1)}{2\, C_1 C_2}$$

and $$\Delta\Delta p_{N_r} = \frac{n\, p_2\, F^2\, \Delta p_m}{V} \cdot \frac{(C_2 - C_1)}{2 C_1 C_2}$$

simplifying and remembering that:

$\frac{C_2}{C_1} = \frac{p_2}{p_1}$ we obtain: $\Delta\Delta p_{N_r} = p_m \cdot \frac{p_2 - p_1}{2\, p_1}$

Since $\Delta p_m = 10\Delta p_{max}$ (where Δp_{max} is the pressure difference corresponding to max. variometer reading)

therefore if:

H_{max} = 3500 (p_1 = 0,6714 kg/cm²)

and

H_{min} = 500 m (p_2 = 0,9734 kg/cm²)

we obtain:

$$\Delta\Delta p_{N_r} = 10 \cdot \frac{0{\cdot}3}{1{\cdot}34} \cdot \Delta p_{max} = 22{\cdot}4\ \Delta p_{max}$$

Hence we reach the conclusion that the error appears to be 22.4 times greater than the maximum reading, but after doubling the time delay in reading the record the value of this error drops 20^2 = 400 times which gives the value 0.056 of the maximum reading namely about 6 per cent.
Thus the error is great initially but rapidly vanishes. Taking into account the above considerations the hand adjustment of TEV according to the altitude of flight appears to be real and profitable.