Spanwise Distribution of Aerodynamic Torsion on Sailplane Wings for Dive Condition

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Summary
The spanwise distribution of air-load torsion in the dive condition is calculated, for a wing having a parallel center section and trapezoidal outer parts, with twist on the latter only. The effect of wing twist on the magnitude of wing torsion in the vertical dive condition was found to be usually very small.

Wing Lift Distribution in Vertical Dive

For the wing shown in Fig. 1, the chord lengths c_o and c (m) can be expressed as

$$c_o = \frac{2S}{b}\left\{\frac{1}{(1+a)+(1-a)\lambda}\right\}$$
$$c = \frac{2S}{b}\left\{\frac{(1-\eta)+(\eta-a)\lambda}{(1-a^2)+(1-a)^2\lambda}\right\} \quad (1)$$

where $\lambda = c_t/c_o$ = wing taper ratio, S = wing area (m²).

wing tip

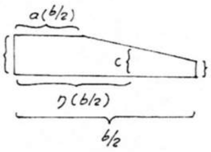

Fig. 1

wing root

The lift distribution for wings with aerodynamic twist may be considered to comprise two parts, the basic lift distribution due to twist, and the additional lift distribution for the wing with no twist. Then the section lift coefficient c_l corresponding to any value of C_L for the entire wing may be expressed as

$c_l = C_L \cdot c_{lal} + c_{lb}$

where c_{lal} is the additional section lift coefficient corresponding to $C_L = 1.0$, c_{lb} the basic section lift coefficient.

In most cases, the total wing lift of a sailplane in vertical dive is very small, and we may assume $C_L = 0$ for this loading condition. Then the section lift in vertical dive will become $c_l = c_{lb}$ which is given by Eq. (12) of reference (1) as follows:

for the rectangular part

$$c\,c_{lb} = 2 k a_o e_t \left(\frac{S}{b}\right)\frac{(1-a)\lambda}{\{(1+a)+(1-a)\lambda\}^2} \quad (2)$$

for the trapezoidal part

$$c\,c_{lb} = 2 k a_o e_t \left(\frac{S}{b}\right)\left[\frac{(1-\eta)+(\eta-a)\lambda}{\{(1+a)+(1-a)\lambda\}^2} - \frac{(\eta-a)}{(1-a)\{(1+a)+(1-a)\lambda\}}\right]\lambda \quad (3)$$

where k = (A–2)/(A+2), A = wing aspect ratio, a_o = 0.11 (deg⁻¹) the slope of the section lift coefficient curve, e_t the aerodynamic twist angle (deg) at the wing tip relative to the wing root.

It is convenient to use the term $c\,c_l$, which multiplied by the dynamic pressure q, will give the air load per unit span.

Wing Torsion in Vertical Dive

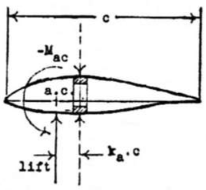

Fig. 2

As shown in Fig. 2, let $k_a \cdot c$ be the distance of the aerodynamic center (a.c.) from wing spar, c the wing chord at section, and C_{mac} the moment coefficient of the wing section about its a.c. (nose up +). Assuming k_a and C_{mac} to be constant along the span, the torsional moment at η section will be:

for the trapezoidal part

$$T_\eta = C_{mac}\, q \left(\frac{b}{2}\right)\int_\eta^1 c^2 d\eta + k_a q \left(\frac{b}{2}\right)\int_\eta^1 c_l c^2 d\eta \quad (4)$$

for the rectangular part

$$T_\eta = T_a + (C_{mac} + k_a c_l)\, q\, c_o^2 \left(\frac{b}{2}\right)\int_\eta^a d\eta \quad (5)$$

where T_a is the torsional moment at η = a section. Or these two equations will become: for trapezoidal part

$$\begin{aligned}T_\eta = {} & K_{cm}\frac{1}{\{(1-a^2)+(1-a)^2\lambda\}^2}\Big\{(1-a\lambda)^2(1-\eta) - \\ & -(1-a\lambda)(1-\lambda)(1-\eta^2)+\tfrac{1}{3}(1-\lambda)^2(1-\eta^3)\Big\} + \\ & + K_l \frac{\lambda}{\{(1-a^2)+(1-a)^2\lambda\}^2}\Bigg[\left\{\frac{(1-a)^2(1-a\lambda)^2}{(1-a^2)+(1+a^2\lambda} + \right. \\ & \left. + a(1-a\lambda)\right\}(1-\eta) - \left\{\frac{(1-a)^2(1-a\lambda)(1-\lambda)}{(1-a^2)+(1-a)^2\lambda} + \right. \\ & \left. + \tfrac{1}{2}(1+a-2a\lambda)\right\}(1-\eta^2) + \\ & + \left\{\frac{(1-a)^2(1-\lambda)^2}{(1-a^2)+(1-a)^2\lambda} + (1-\lambda)\right\}\tfrac{1}{3}(1-\eta^3)\Bigg]\end{aligned} \quad (6)$$

for rectangular part

$$T_\eta = T_a + \left[K_{cm} + K_l\left\{\frac{(1+a)\lambda}{(1+a)+(1-a)\lambda}\right\}\right]\frac{(a-\eta)}{\{(1+a)+(1-a)\lambda\}^2} \quad (7)$$

where

$$\left.\begin{aligned} K_{cm} &= 2 C_{mac}\, q \frac{S^2}{b} \\ K_l &= 2 a_o e_t k \cdot k_a q \frac{S^2}{b}\end{aligned}\right\} \quad (8)$$

Torsional Moment on Skylark-4 Wing in Vertical Dive

The wing area, S = 16.1 m², the wing span, b = 18.2 m, the aspect ratio, A = 20.5, the gross weight, W = 376 kg, the moment coefficient of the airfoil about a.c. C_{mac} = –0.1. According to OSTIV Airworthiness Requirements for Sailplanes, the design diving speed V_D for normal category sailplane V_D = 3.25(W/S) + 150 = 226 km/h (63 m/s).

The dynamic pressure, q in the diving condition, q = (63)²/16 = 248 km/m².

From Eq. (8) K_{cm} = –705 kg.m.

The slope of the section lift coefficient curve, a_o = 0.11 (deg⁻¹), the twist angle at the wing tip relative to the untwisted rectangular part of the wing, e_t = 3.0 (deg), k = (A–2)/(A+2) = 0.82, coefficient giving the distance of a.c. from the wing spar, k_a = 0.1.

From Eq. (8) K_l = 192 kg.m.

The coefficient giving the length of the rectangular part of the wing, a = 0.33, the wing taper ratio, λ = 0.5.

For the trapezoidal part, the torsional moment at η section may be found from Eq. (6) as

$T = -397(1-\eta)+238(1-\eta^2)-47.6(1-\eta^3)$
$+43.1(1-\eta)-51.8(1-\eta^2)+15.5(1-\eta^3)$

At η = a = 0.33

T_a = –100 –2 = –102 (kg.m)

where (–) means nose down, the term (–2) is the torsional moment resulting from the twist angle (e_t).

For the rectangular part, the torsional moment at η section may be found from Eq. (7) as

$T_\eta = -102 + (-254 + 13.9)(0.33 - \eta)$

The torsional moment at the wing root (η = 0)

To = –102 + (–84.7 + 4.6) = –186.7 + 4.6 = –182.1 (kg.m)

The term (+4.6) is the torsional moment resulting in the twist angle (e_t).

For the wing without twist

T_o = –100 –84.7 = –184.7 (kg.m)

In the calculation of the wing torsional moment for the vertical dive condition, the following Eqs. (9) and (10) may be used, which are obtained, putting e_t = 0, or K_l = 0, in Eqs. (6), (7) and (8).

The torsional moment at η section will be: for the trapezoidal part

$$T_\eta = \frac{K_{cm}}{\{(1-a^2)+(1-a)^2\lambda\}}\Big\{(1-a\lambda)^2(1-\eta) - (1-a\lambda)(1-\lambda)(1-\eta^2)+\tfrac{1}{3}(1-\lambda)^2(1-\eta^3)\Big\} \quad (9)$$

for the rectangular part

$$T_\eta = T_a + \frac{K_{cm}(a-\eta)}{\{(1+a)+(1-a)\lambda\}^2} \quad (10)$$

where T_a will be obtained from Eq. (9) putting η = a.

References
(1) Hiroshi Sato: Spanwise Distribution of Aerodynamic Shear and Bending-Moment on Cantilever Tapered Wings. Aero Revue, April 1967.
(2) OSTIV: Airworthiness Requirements for Sailplanes. July 1964.
(3) D. J. Peery: Aircraft Structures. McGraw-Hill, 1950.