Statistical Aspects of Handling Criteria Research

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Presented at the XIth OSTIV Congress at Leszno, Poland

Abstract
Except in the worst cases of stability problems and spinning the fundamental cause and ultimate control in drawing up handling criteria is pilot opinion. Preferences differ, hence in sampling and interpreting pilot opinions due attention should be given to statistical methods. On the occasion of a public opinion research among sailplane pilots the distribution of the ratings – employing the Cooper-scale – has been found to converge on a binomial-like distribution. Consequently, 90 % confidence limits may be assigned to the mean ratings, giving more exact pilot opinion boundary charts.
We have as yet relatively few measured data for sailplane longitudinal dynamic characteristics (stability, manoeuvrability and sensitivity), it is therefore advisable to rely temporarily on power-plane experience too. Rolling characteristics of sailplanes may be assessed on the basis of rolling angular velocity, aileron force and friction.
Sometimes the probability of pilot-induced oscillations may be computed using vector methods; e. g. in aeroplane tow.

Notation

d	differential sign	
f_{oa}	longitudinal short period undamped natural frequency	[sec⁻¹]
g	gravitational acceleration	[m sec⁻²]
$i = \sqrt{-1}$	imaginary unit	
j	number of votes	
n	normal acceleration/g	
s	Laplace transform variable	
$\bar{s}$	wing span	[m]
t	time	[sec]
x	rating	
$\bar{x}$	mean rating	
y	lateral deviation in aero-tow	[m]
A	constant	
K	constant of proportionality	[m sec⁻²]
M	degree of manoeuvrability (see [7])	[sec⁻¹]
M	degree of manoeuvrability with overcontrol (see [7])	[sec⁻¹]
ΔP_c	aileron friction	[kp]
S	degree of stability (see [7])	[sec⁻¹]
T_h	dead time of pilot	[sec]
T_I	lag time constant in pilot transfer function	[sec]
T_L	lead time constant in pilot transfer function	[sec]
T_N	neuro-muscular time constant in pilot transfer function	[sec]
T_1	time constant of roll mode	[sec]
$Y_e(s)$	pilot transfer function	
ε	phase angle	[rad]
ζ	damping ratio	
ξ, η	coordinates in Weibull plot	
ϑ	error of mean rating	
$\varkappa_x$	wing tip helix angle	[rad]
$\lambda = \mu \pm i\,\omega$	root of stability equation	[sec⁻¹]
μ	damping constant	[sec⁻¹]
μ^x	damping constant with pilot controlling	[sec⁻¹]
φ	roll angle	[rad], [°]
ω	angular natural frequency	[rad sec⁻¹]
ω^x	angular natural frequency with pilot controlling	[rad sec⁻¹]
ω_x	rolling angular velocity	[rad sec⁻¹], [° sec⁻¹]
ω_o	undamped angular natural frequency	[rad sec⁻¹]
$\omega_o{}^x$	undamped angular natural frequency with pilot controlling	[rad sec⁻¹]

Subscripts

a	longitudinal short period
crit	limit value for pilot-induced oscillations
0.2	with 20 % overcontrol
90	of 90 % confidence

Introduction
One of the main problems in handling criteria research is the proper rating of flying qualities. While commenting on the characteristics of airplanes, pilots are using quite often terms like 'safe-dangerous' or 'pleasant–unpleasant', reflecting the common belief that the dual role of handling criteria should be flight safety and ease of control. Unfortunately, in going into details this unity of opinion ceases to exist. In flying quality research one seldom gets unambiguous answers either from flight mechanics, or from pilot opinion. The fundamental nature of the problems makes a deterministic approach to them very difficult, if not impossible. Becoming aware of this situation, it seems advisable to give statistical methods their fair share in flying quality research work. In this paper an attempt has been made in this direction, as far as space and conditions permit, without claim to completeness.

1. Flight Safety
The earliest theoretical papers on flying qualities dealt with safety, notably with the stability of the motion of airplanes. Since then considerable progress has been made in this field, but – except in the worst cases of instability and spinning – even now only the dynamics of motions with controls fixed or free can be calculated exactly.
A satisfactory description of the complete pilot-aircraft system is realisable only on statistical terms, hence for the individual case no definite equation of motion can be given.
In analysing safety statistics, the so-called pilot-error type accidents may be divided broadly into two groups. There are accidents occurring in about equal percentage on different types, e. g.: undershoot with a glider equipped with efficient airbrakes. It may be easily shown that in such cases modifications to the plane could not alter the situation. But there are again other types of accidents or incidents happening relatively frequently on some types, whereas other sailplane types – with pilots of equal competence – are almost immune to (e. g.: spinning). It is the absence of this latter type of events over a sufficient long periode of time which characterises really good design work. But as this is good only for proof and not for guidance, in designing for good flying qualities we have to look for other criteria as well.
Of several methods proposed for this purpose so far it is only the systematic analysis of pilot opinions supported by simulator or variable-stability aircraft tests which is sufficiently universal and which has stood the proof of time in modern power-plane practice (e. g.: see [1]).

2. Statistical Treatment of Pilot-Opinions
The role of simulators and variable-stability aircraft in flying quality

research is to make possible the assessment of various characteristics in near-real conditions and – not least – to go beyond the safety limits imposed for actual flight in choosing some parameters for investigation. Sailplane designers are devoid of this possibility because costs would be prohibitive. To get on without them either more prototype work or some more exact form of pilot-opinion interpretation is needed.

Research done recently at the Technical University Budapest, Chair for Aeronautics, was aimed at this latter direction. Against financial handicaps in gliding we have the advantage of a fair number of well trained pilots, ready to collaborate. It was decided therefore to investigate the reliability of pilot-opinions by statistical methods.

Based on preliminary work done a few years ago [2] a public opinion poll, employing a variant of the Cooper rating scale [3], has been conducted. (1 point was assigned to the worst and 10 points to the best rating as against the reverse in the original Cooper scale.) As reported in detail elsewhere [4], analysis of the questionaries of 5 types rated by the greatest number of pilots yielded following results:

Each distribution (separately for each question and for each type) was traced on Weibull probability paper, because points of the binomial distributions – expected to be the analytical distribution nearest to the rating distributions – are lying here with very good approximation on a straight line the gradient of which is a unique function of the mean rating. The only exception to this rule is the point belonging to the greatest rating. The value of all statistical distributions should be here 1 (100 %), lying in infinity on Weibull coordinates. With this graphical method the actual distributions encountered could be classified into following categories:

Cat. 1. All points lying on a straight line (fig. 1). This category may be classified therefore as a binomial distribution – if the gradient is the same as for a binomial distribution of equal mean.

Cat. 2. Distributions involving only the ratings 8, 9 and 10 (fig. 2). As there are only two points (except the 100 % of the rating 10 in infinity) here, the only criterion of similarity is the relationship between gradient and mean.

Cat. 3. Some irregularity at the lowest rating given. Number of votes ranged from 38 to 70 and the worst of the ratings got usually 1–2, so this irregularity is likely to be frequent (fig. 3).

Cat. 4. Double scatter distributions. Rating distributions of some objectionable characteristic could be fitted only to two regression lines (fig. 4). One possible explanation is that some pilots consider the characteristic concerned as essential, while for others it may be only of secondary importance.

Cat. 5. Some of the bad characteristics showed randomly scattered distribution points (fig. 5).

In all 175 rating distributions were traced, the percentages of the respective categories were as follows:

Table I

Percentage distribution of categories

Category	1	2	3	4	5
%	34.28	15.43	38.86	8.00	3.43

88,57 % are perfect distributions or those with minor defects and only 3,43 % may be classified as irregular.

Fig. 2

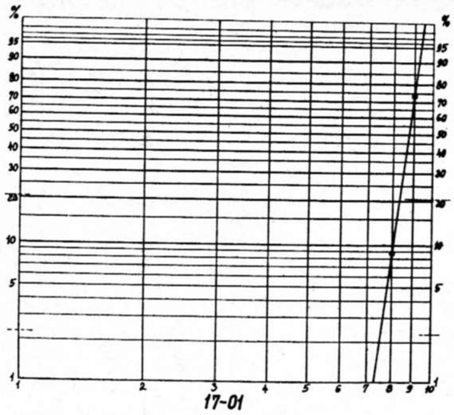


Fig. 3

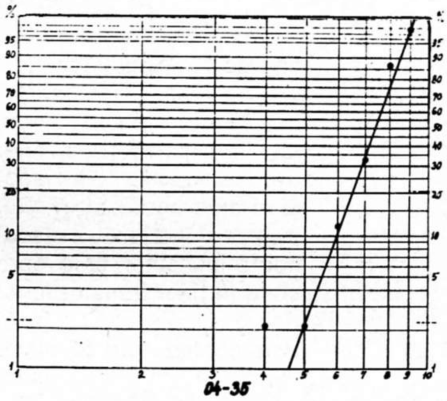


Fig. 4

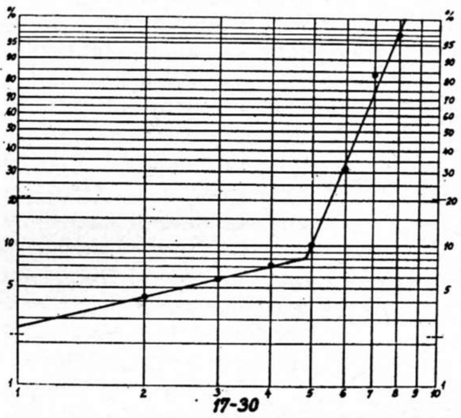


Fig. 6

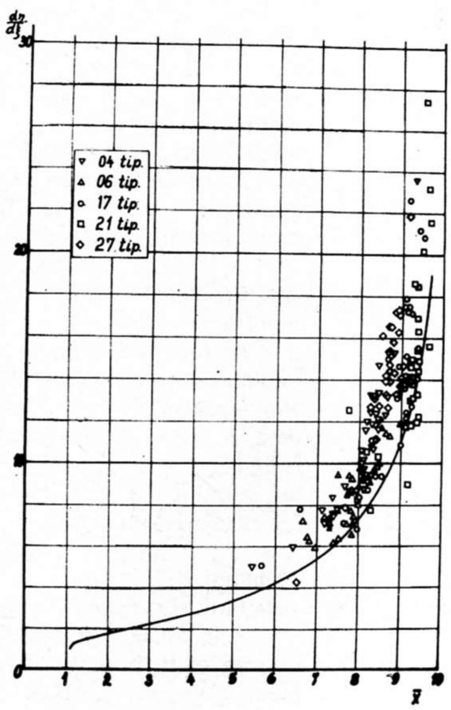


Fig. 1

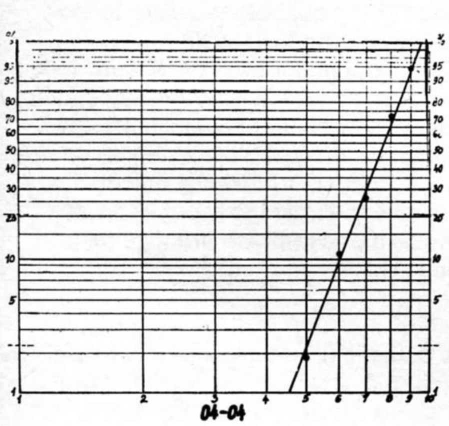
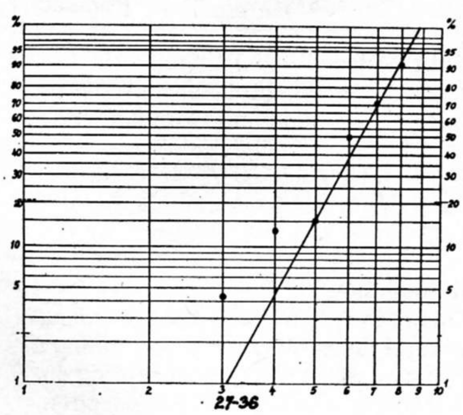


Fig. 5



Plotting the gradients of distributions against the means $\bar{x}$ (fig. 6) showed most of them being above the solid line of the gradient-mean relationship for binomial distributions indicating a percentage of the rating 10 slightly below normal.
From this it can be stated that the distribution of the Cooper-ratings converges to a binomial-like distribution with increasing number of pilots interrogated.
From this follows the ultimate goal of these investigations, the determination of confidence limits for mean ratings, to be attainable. Proceeding from the deviation formula for binomial distributions and making a statistical error survey of the 175 rating distributions (fig. 7) a formula for the calculation of 90 % confidence limits can be set up. For practical purposes:

$$\vartheta_{90} \cong \frac{0.5}{\sqrt{j}} \sqrt{(\bar{x}-1)(10-\bar{x})} \qquad (1)$$

Should one deem it necessary to adhere to theoretical values one could write:

$$\vartheta_{90} \cong \frac{0.548}{\sqrt{j}} \sqrt{(\bar{x}-1)(10-\bar{x})} \qquad (1a)$$

Thus significant differences in ratings may be recognized and in general the design of pilot-opinion charts may be improved.
Collecting and processing pilot-ratings on different types constitutes the raw material for development work on handling criteria. For the interpretation of the results of pilot-opinion polls they should be viewed against flight test results and against principles deduced from theoretical work. In the following some examples should be given of how this may be attempted with statistical methods.

Fig. 7

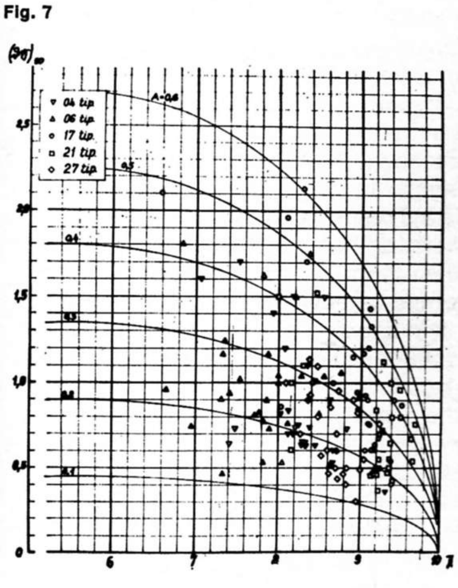


Fig. 8

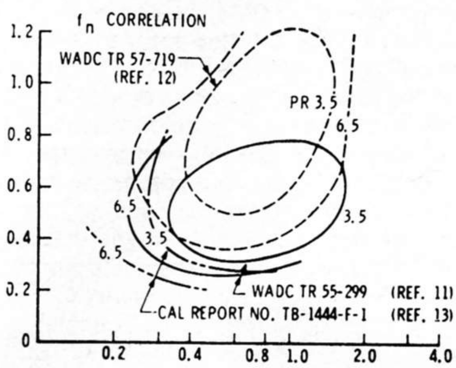
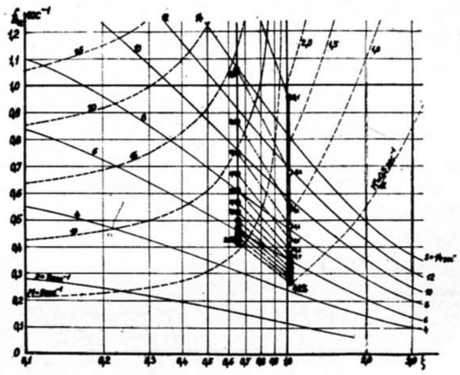


Fig. 9



3. Longitudinal Short Period Dynamics

For sailplanes, short period dynamical parameters are the most important characteristics in longitudinal handling. Some good theoretical work was done in this domain by Lehmann [5] and Morelli [6], with the author also contributing [7], but flight tests are badly lacking. The few done so far are nowhere near enough for drawing pilot-opinion boundaries for this essentially two- (or perhaps three-) parameter problem. It is therefore advisable to rely temporarily on charts constructed for power-planes. But as requirements for sailplanes might differ from those of other aircraft a check against calculated parameter values and practical experience seems to be indicated.
Stability and manoeuvrability in this mode are determined by the undamped natural frequency (ω_{oa} or f_{oa}) and by the damping ratio (ζ) [1, 5, 7] et al., fig. 8, from the work of Shomber and Gertsen [8], is showing iso-opinion boundaries from various sources (rating numbers are as in the original Cooper-scale). On fig. 9 lines of constant stability [7] are drawn and constant manoeuvrability lines also. $f_{oa} - \zeta$ points of a typical sailplane calculated for several lift coefficients and for the foremost and rearmost center of gravity positions are also shown.
Comparing the two figures the following conclusions may be drawn:
a) Sailplanes are operating under widely varying conditions. As it is not possible to have optimal parameter values for all speeds and for all pilot weights, some compromise must be sought. Even a crude form of statistics can give the center of gravity and lift coefficient ranges most common in everyday use. These should be optimized if possible at all, while otherwise compliance with only primary safety standards may be acceptable.
b) Different comments by pilots may be explained partially by allowing for the differences in C. G. positions and eventually for the speeds in the manoeuvres that were commented on.
c) On the whole, this kind of treatment seems to be well suited to our needs and calculated parameters of sailplane types rated well by pilots are lying mostly in areas marked as satisfactory on the graphs. There is more concern about the differences in regions of satisfactory and acceptable characteristics as reported by several authors. To put an end to this inconsistency Shomber and Gertsen [8] suggested new criteria applying well to jet transport category airplanes. Their proposal differs from the previous method in taking into account sensitivity type parameters too — a different one for high- and for low lift coefficients.
Space available prevents us from going into details, but sailplane characteristics do not seem to meet the proposed requirements (fig. 10 and 11). It remains therefore to adhere to the $f_{oa} - \zeta$ plots — at least until availability of flight test results in sufficient numbers will make a detailed statistical analysis practicable.

Fig. 10

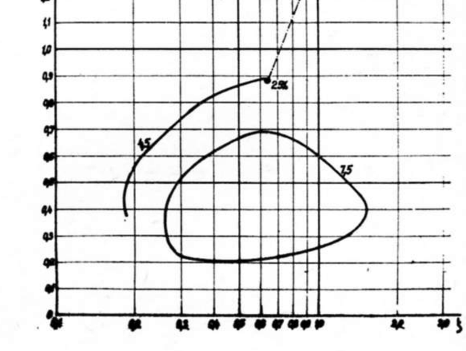
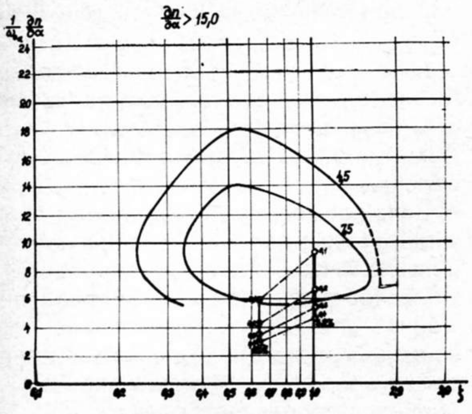


Fig. 11



4. Rolling Characteristics

Problems inherent in rolling control are fundamentally different from those in the longitudinal mode, hence an example from this domain would not be useless here. From the general equations of undisturbed lateral motion the rolling mode may be separated (in first order approximation) in the form:

$$\ddot{\varphi} + \frac{1}{T_1}\dot{\varphi} = 0 \qquad (2)$$

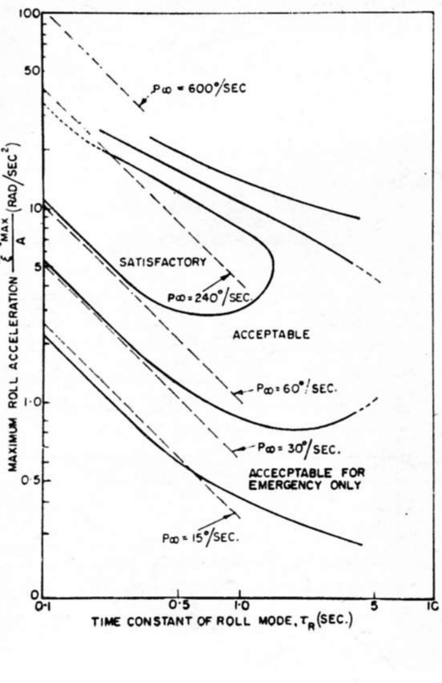
By applying the definition given in [7] the degree of stability may be easily shown to be:

$$S = \frac{1}{T_1} \quad [\sec^{-1}] \qquad (3)$$

With values of T_1 from 0.06 to 0.15 sec for sailplanes, pilots never complain of instability, all the more of insufficient aileron power.

Pilot opinion boundaries for manoeuvrability of fightertype airplanes in the

Fig. 12



roll mode, as worked out by Creer et al. on ground simulators were given by O'Hara [1] (fig. 12). It is apparent that – according to this – for the low T_1 (noted on the graph as τ_R) values of sailplanes the maximum rate of roll with full aileron (ω, noted P_∞ on the graph) should be the standard parameter for the rating of roll manoeuvrability. This is in perfect agreement with the theoretical definition given by the author, but for sailplanes it is still customary to think in terms of wing tip helix angle:

$$\chi_x = \frac{\bar{s}\, W_x}{2V} \qquad (4)$$

From the pilots point of view, this latter parameter is giving the space necessary for roll manoeuvres, whereas ω_x the time required for them. In order to find out the aspect more consistent with pilot's needs, pilot ratings on roll manoeuvrability of 10 types were plotted as a function of $\varkappa_x$ and ω_x resp., determined from flight tests done by the author (fig. 13 and 14).

Fig. 13

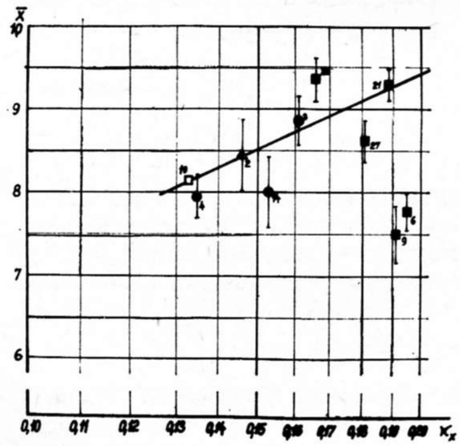
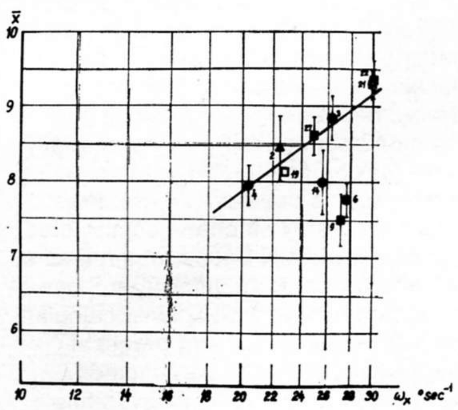


Fig. 14

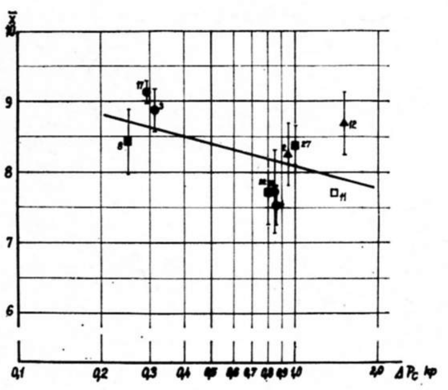


On both graphs types 6, 9 and 14 are significantly out of line from the main sequence. This may be explained by aileron ineffectiveness on the ground and excessive aileron forces. Calculation of correlation coefficients for the remaining 7 gives 0.9737 for $\varkappa_x$ as against 0.9948 for ω_x with gradients of 7.337 and 7.500, respectively. Rolling angular velocity appears therefore to be a slightly better roll parameter, but wing tip helix angle, thus far more accustomed to by sailplane designers, is not bad either.

The problem of aileron forces may be treated in a similar way; space available prevents us from doing this here in detail.

Aileron friction is also part of the aileron sensitivity picture. As friction in control runs could not be regarded but as an inconvenience – although an inevitable one – upper limits may be given only. Some hints on trends in pilot opinions may be obtained from fig. 15.

Fig. 15



5. Pilot-Induced Oscillations in Aero-Tow

There are certain aircraft motions fairly easily amenable to calculation provided that the control displacements producing them are known. As pilots are not alike, we are here up against another sort of statistical problem. Pilot induced oscillations (e. g. lateral oscillations in aero-tow) belong to this category and may be chosen to illustrate its peculiarities.

Lateral oscillations in aero-tow are essentially of sinusoidal character, similar to Duch-roll in free flight. As it is a 3 degree of freedom motion with none of them negligable with respect to the others, a full theoretical treatment would be obviously out of reach for sailplane designers. Fortunately, phase difference between the single freedoms is constant as they are oscillating with the same frequency. This makes it possible to treat the problem – from the pilot's control point of view – as a single degree of freedom motion, e. g. with the lateral displacement from the plane of symmetry of the tow plane (y).

The transfer function of the pilot, as given by several authors, is:

$$Y_e = \frac{K(T_L s+1)e^{-T_h s}}{(T_I s+1)(T_N s+1)} \quad (5)$$

where the «human constants» can be determined only as statistical quantities. For the given case it can be simplified into:

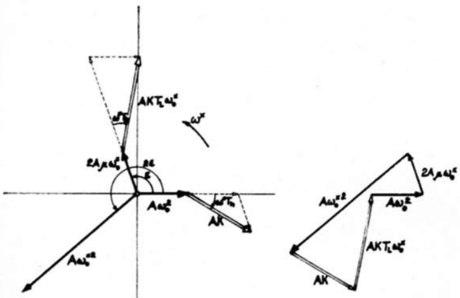
$$Y_e \cong K(T_L s+1)e^{-T_h s} \quad (6)$$

With this the equation of motion should be:

$$\ddot{y}(t)+2\mu\dot{y}(t)+\omega_0^2 y(t) = -KT_L\dot{y}(t-T_h)-Ky(t-T_h) \quad (7)$$

ω_0 and μ may be calculated or determined by flight tests with controls fixed. K may be determined from maximum rudder power.
This problem was briefly touched upon in [7], with slightly different notation. For stability analysis of the motion use may be made of the rotating vector diagram representation (fig. 16). From the condition of a closed vector poligon ε may be constructed or calculated; $\varepsilon > \frac{\pi}{2}$ giving stable damped and $\varepsilon < \frac{\pi}{2}$ instable divergent oscillations.

Fig. 16



As we are interested primarily in the degree of difficulty in controlling the motion, further simplification is allowable by supposing the pilot to control only for deviations and not for the time derivatives of deviations ($T_L = 0$) and calculating the stability boundary ($\varepsilon = \frac{\pi}{2}$), as shown in fig. 17. Then the pilot dead time giving $\varepsilon = \frac{\pi}{2}$ may be calculated from the following equations:

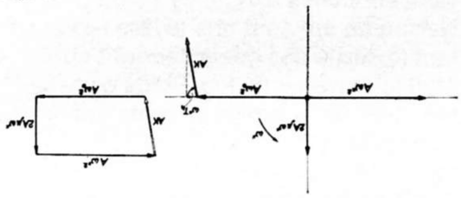
$$\left(\frac{W^x}{W_0}\right)^2 = 1 + \frac{K}{W_0^2}\cos W^x T_h \quad (8)$$

$$2\}\frac{W^x}{W_0} - \frac{K}{W_0^2}\sin w^x T_h = 0 \quad (9)$$

From these the value of $T_{h\ crit}$ giving

$$\varepsilon = \frac{\pi}{2} \text{ i.e.:} \qquad tg\,\varepsilon = -\frac{W^x}{\mu^x} = -\infty \quad (10)$$

Fig. 17



may be found by iteration as in [7]. From $T_{h\ crit}$ the likelyhood of pilot-induced oscillations can be estimated. For instance from our experience $T_{h\ crit} = 2$ sec may be regarded as a safe value, with an extremely remote possibility of overcontrol by very inexperienced pilots only. For $T_{h\ crit} < 0.5$ sec stability can be maintained only by the pilot correcting for $\dot{y}$ too (T_L!), hence the machine can be regarded to be safe for experienced pilots only. When enough flight test data on different types becomes available, a graph of $\overline{x}$ for ease of handling in aero-tow as function of $T_{h\ crit}$ may be drawn.

Summary

Except in the worst cases of stability problems and spinning the fundamental cause and ultimate control in drawing up handling criteria is pilot opinion. Preferences differ, hence in sampling and interpreting pilot opinions due attention should be given to statistical methods. On the occasion of a public opinion research among sailplane pilots the distribution of the ratings – employing the Cooper-scale – has been found to converge on a binomiale-like distribution. Consequently, 90 % confidence limits may be assigned to the mean ratings, giving more exact pilot opinion boundary charts.
The paper gives some examples of the usefulness of statistical methods in flying quality research interpretation of safety statistics, appraisal of pilot-opinion polls, short period longitudinal dynamics, roll manoeuvrability and pilot-induced oscillations in aero-tow. It seems to be worth trying these methods because of the possibility of improving significantly our knowledge of the human aspects of handling criteria.

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