Dynamic lateral stability of a towed glider in steady horizontal flight

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Introduction

This paper is a continuation of the author's previous works (4, 5, 6) dealing with the longitudinal stability problem of towed gliders. In (4) the simplified longitudinal stability of a glider towed behind a plane of infinite mass has been considered. In (5) the longitudinal stability of a towed system, i. e. towing plane, towing rope and towed glider has been discussed and the results have been compared with the results of (4).

The problem of stability of kites and gliders towed with a rope was considered in (1) by Brown and Sweeting, but the authors in their considerations of the lateral stability derivatives of a rope, did not take into account the weight of the rope and the aerodynamic forces acting on it.

S. Neumark in (7) and (8) considered the stability problems of baloons with unstretching ropes. In (8) he derived the aerodynamic derivatives and defined the geometric configuration of a rope, but without taking into account the full aerodynamic load on the rope.

In the present paper the lateral stability of a towed glider in steady horizontal flight is considered. Before the disturbance the glider is in the same vertical plane as the direction of its flight. The towing rope is assumed to be ideally flexible, longitudinally elastic, heavy, and also loaded by aerodynamic forces. The influence of the rope bending moments and the internal damping is neglected, as for typical towing ropes the ratio of the radius of curvature to diameter is large and the forces acting on the rope are small in steady horizontal flight. The static influence of the rope is considered under the assumption, that the towing plane is of infinite weight and is in steady, rectic linear horizontal flight. The dynamic influence of the rope, and the disturbances caused by the towing plane deviating from the assumed flight path, are neglected.

The problem is treated by the method of small disturbances. The equations of motion are of the form of ordinary second order differential equations with the constant coefficients. Thus, the determination of the coefficients of the characteristic equation, and application of the Routh-Hurwitz stability criteria as well as the determination of the roots of the characteristic equation by Bairstow's method (9) is possible. The problem is solved by the same method as is commonly used in the stability analysis of aircraft in free flight (2, 3, 13, and 14), which makes an easy comparison of the results obtained in the free-flight and towed-flight cases possible and makes the analysis relatively simple.

On the basis of a numerical example, calculated on the electronic digital computer GIER, for a current high performance glider and for a current towing plane, an analysis has been made of stability and the influence of different parameters of the design and handling qualities.

1. Differential equations of motion

The equations of motion of a towed glider are derived by the method of small perturbations; this makes linearization possible and thus leads to a simple form of solution, convenient for further analysis and for comparison with results obtained in the free-flight case.

Before the disturbance the towed glider is in the same vertical plane as the direction of flight. The small perturbations are related to the lateral displacement y, and the changes of angle of yaw ψ and angle of bank φ.

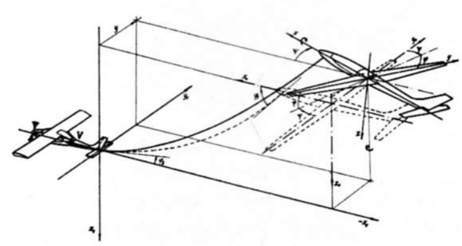

Fig. 1. System of coordinates and geometrical relations between them.

y – a change of a center of gravity position relatively to the system of axis x_1, y_1, z_1 connected with the towing plane,

φ – a change of a glider bank angle, the rotation around the glider longitudinal axis x,

ψ – a change of a glider yaw angle, the rotation around the glider vertical axis z,

v_1 – a change of the glider velocity component in the direction of the towing plane axis y_1,

p – a change of the glider rolling angular velocity,

r – a change of the glider yawing angular velocity.

The equations of asymmetric motions of towed glider relatively to the fixed relative of the towing airplane axis x_1, y_1, z_1, (figure 1) are:

$$\begin{aligned}
&\frac{Q_1}{g}\dot{v}_1 = Y_v v_1 + Y_y y + (Q_1 + Y_\varphi)\varphi + (Y_\psi - Y_v V)\psi,\\
&J_x \dot{p} - J_{xz}\dot{r} = L_v v_1 + L_p p + L_r r + L_y y + L_\varphi \varphi + (L_\psi - L_v V)\psi,\\
&J_z \dot{r} - J_{xz}\dot{p} = N_v v_1 + N_p p + N_r r + N_y y + N_\varphi \varphi + (N_\psi - N_v V)\psi,\\
&v_1 = \dot{y},\\
&p = \dot{\varphi},\\
&r = \dot{\psi},
\end{aligned} \tag{1.1}$$

The aerodynamic derivatives Y_v, L_v, L_p, L_r, N_v, N_p and N_r appearing in the system of equations 1.1 are derived and discussed in (3) and are not considered here. To solve the system 1.1 it is also necessary to calculate the derivatives Y_y, $Y\varphi$, $Y\psi$, L_y, $L\varphi$, $L\psi$, N_y, $N\varphi$ and $N\psi$, depending on the presence of the towing rope. This is done below.

2. Rope lateral force coefficient. Rope aerodynamic derivative

The influence of a towing rope on the lateral force acting on a glider is derived under the assumption, of linear dependence of the force on the displacements of the rope end.

Following the same method as in the case of aeroplane stability analysis (2, 3, 13), the rope lateral force derivative is defined according to (4, 5) and (8) as

$$Y_y^{\,l} = \frac{\partial Y_1}{\partial y}$$

and is derived below on the base of consideration given in (4, 6) and (8).

It is assumed that before perturbation the glider is in steady, rectilinear horizontal flight, and is in a given position relative to the towing plane.

The infinitely short element dl of the rope of weight q dl, loaded by the axial force T and T + dT, at both ends by the normal aerodynamic force n and by the tangential aerodynamic force t, is considered. The element of the rope is in an airstream of velocity V in the x_1 axis direction (figure 2).

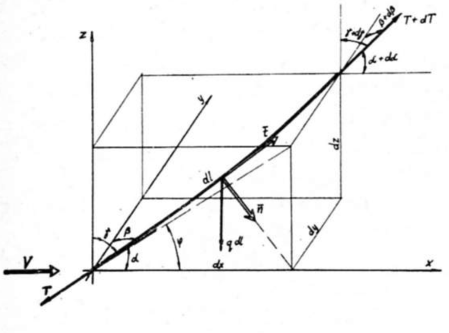

Fig. 2. System of forces acting on the rope element.

The following euqilibrium equations of the rope in the x_1, y_1 and z_1 axis directions are obtained:

$$
\begin{aligned}
&dT\cos\alpha - T\sin\alpha\, d\alpha + n\sin^3\alpha\, dl + t\cos^3\alpha\, dl = 0,\\
2.1\quad &dT\cos\beta - T\sin\beta\, d\beta - n\sin\alpha\cos\alpha\cos\beta\, dl + t\cos^2\alpha\cos\beta\, dl = 0,\\
&dT\cos\gamma - T\sin\gamma\, d\gamma - n\sin\alpha\cos\alpha\cos\gamma\, dl + t\cos^2\alpha\cos\gamma\, dl - q\, dl = 0
\end{aligned}
$$

Following 4, 6 and 8 the rope lateral force derivative with respect to the lateral displacement y of the rope end is derived.

2.2 $$Y_y^l = \frac{1}{\vartheta_1^l - \vartheta_2^l}\sqrt{n\tau_1(n\sin^2\varphi_1 + q\cos\varphi_1)}\,,$$

where

$$\vartheta_1^l = \vartheta^l(\varphi_1)\ ; \quad \vartheta_2^l = \vartheta^l(\varphi_2)\,,$$

and

2.3 $$\vartheta^l(\varphi) = \vartheta'^l(\varphi) + \vartheta''^l(\varphi)\,,$$

2.4 $$\vartheta'^l(\varphi) = \int_0^{\varphi}\sqrt{\frac{n\,\tau(\varphi)}{n\sin^2\varphi + q\cos\varphi}}\cdot d\varphi$$

2.5 $$\vartheta''^l(\varphi) = \lambda\frac{T_1 e^{\eta_1}}{\tau_1}\int_0^{\varphi}\tau(\varphi)\, e^{-\eta(\varphi)}\sqrt{\frac{n\,\tau(\varphi)}{n\sin^2\varphi + q\cos\varphi}}\, d\varphi$$

The constants η_1, τ_1 and n in 2.4 and 2.5 are:

$$\eta_1 = \eta(\varphi_1)\ ; \quad \tau_1 = \tau(\varphi_1)\ ; \quad n = \tfrac{1}{2}\varrho V^2 d\, C_n$$

Functions $\tau(\varphi)$ and $\eta(\varphi)$ are derived in 4 and 6, and they are:

2.6 $$\tau(\varphi) = \left(\frac{\cot\psi - \cos\varphi}{\tan\psi + \cos\varphi}\right)^{\cos 2\psi}$$

2.7 $$\eta(\varphi) = \frac{C_t}{C_q}\cos 2\psi\left[-\frac{2\varphi}{\sin 2\psi} + \frac{\tan^2\psi}{\sqrt{1-\tan^2\psi}}\tan^{-1}\left(\sqrt{\frac{1-\tan\psi}{1+\tan\psi}}\tan\frac{\varphi}{2}\right) + \frac{2\cot^2\psi}{\sqrt{\cot^2\psi - 1}}\tan^{-1}\left(\sqrt{\frac{\cot\psi - 1}{\cot\psi + 1}}\tan\frac{\varphi}{2}\right)\right],$$

where

$$\cot 2\psi = \frac{q}{2n}\ ; \quad C_q = \frac{2q}{\varrho d V^2}$$

and, according to 4 and 6: $C_n = 1{,}15$ and $C_t = 0{,}035$.

3. Rope lateral stability derivatives of a towed glider

The rope derivatives of lateral force Y, rolling moment L and yawing moment N, relatively to horizontal displacement y, angle of bank φ and angle of yaw ψ are determined as follows:

3.1 $$Y_\varphi = \frac{\partial Y}{\partial\varphi}\ ; \quad N_\psi = \frac{\partial N}{\partial\psi}\ ;$$

and thus a change of the lateral force acting on the glider towing hook is:

3.2 $$dY = Y_y\, dy + Y_\varphi\, d\varphi + Y_\psi\, d\psi$$

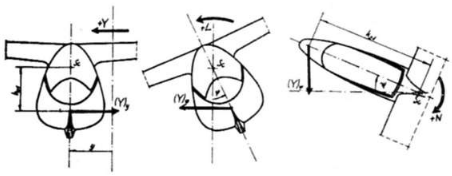

Fig. 3. Changes of lateral force caused by lateral displacements, banking and yawing of a glider.

Introducing 2.2 and geometrical relations shown in figure 3, the change of lateral force is

3.3 $$dY = -Y_y^l\, dy + Y_y^l h_{z1}\, d\varphi - Y_y^l k_{z1}\, d\psi\,.$$

Making the coefficients at dy, dφ and dψ equal in 3.2 and 3.3 the rope derivatives of glider lateral force

are 3.4 $$Y_y = -Y_y^l\,,\quad Y_\varphi = Y_y^l h_{z1}\,,\quad Y_\psi = -Y_y^l k_{z1}\,,$$

Similarly the rope derivatives of glider rolling moment are:

3.5 $$L_y = Y_y^l h_{z1}\,,\quad L_\varphi = -(Y_y^l h_{z1} + Z_1)h_{z1}\,,\quad L_\psi = Y_y^l h_{z1} k_{z1}\,,$$

and of glider yawing moment are

3.6 $$N_y = -Y_y^l k_{z1}\,,\quad N_\varphi = Y_y^l h_{z1} k_{z1}\,,\quad N_\psi = -(Y_y^l k_{z1} + X_1)k_{z1}\,,$$

where

3.7 $$Z_1 = X_1\tan\varphi_1\,,$$

and

3.8 $$X_1 = P_{x1} = \tfrac{1}{2}\varrho S_1 V^2 C_{x1}$$

is drag force of the towed glider.
Now, when we have the rope derivatives 3.4, 3.5 and 3.6, we can solve the system of equation 1.1 and analyse the lateral stability of the towed glider.

4. Solution of the differential equations of motion and stability analysis

Dividing the force equations in system 1.1 by $\varrho S_1 V^2$ and the moment equations by $\varrho S_1 V^2 \frac{b_A}{2}$, and making the following substitutions:

$\hat{t} = \frac{Q_1}{\varrho S_1 V g}$ — aerodynamic time,

$\mu_2 = \frac{2Q_1}{\varrho S_1 g b_1}$ — glider relative density,

$\bar{t} = \frac{t}{\hat{t}}$ — dimensionless time,

$j_x = \frac{2 J_x g}{Q_1 b_1}$ — dimensionless moment of inertia of glider around the longitudinal axis,

$j_z = \frac{2 J_z g}{Q_1 b_1}$ — dimensionless moment of inertia of glider around the vertical axis,

$j_{xz} = \frac{2 J_{xz} g}{Q_1 b_1}$ — dimensionless product of inertia,

$\bar{V}_1 = \frac{V_1}{V}$ — dimensionless velocity of glider,

$\bar{p} = p\hat{t}_1,\ \bar{r} = r\hat{t}$ — dimensionless angular rolling velocity and yawing velocity respectively.

It is assumed that direction of velocity V agrees with the longitudinal axis of the glider before disturbance, and that the directions of glider axes x, y and z are very close to the directions of glider principal axis of inertia, so that it is possible to neglect terms $\frac{j_{xz}}{j_x} = 0$ and $\frac{j_{xz}}{j_z} = 0$

Now, the system 1.1 has the following dimensionless form:

$$
\begin{aligned}
&\ddot{\bar{y}} - y_v\dot{\bar{y}} - y_y\bar{y} - (y_\varphi + y_{\varphi l})\varphi - (y_\psi - y_v)\psi = 0,\\
4.1\quad &\ddot{\varphi} + \bar{l}_p\dot{\varphi} + \bar{l}_r\dot{\psi} + \bar{l}_v\dot{\bar{y}} + \bar{l}_\varphi\varphi + (\bar{l}_\psi - \bar{l}_v)\psi + \bar{l}_y\bar{y} = 0,\\
&\ddot{\psi} + \bar{n}_p\dot{\varphi} + \bar{n}_r\dot{\psi} + \bar{n}_v\dot{\bar{y}} + \bar{n}_\varphi\varphi + (\bar{n}_\psi - \bar{n}_v)\psi + \bar{n}_y\bar{y} = 0,
\end{aligned}
$$

where

$$
\begin{aligned}
&y_y = \frac{Y_y}{\varrho V^2 S_1}\,,\quad y_{\varphi l} = \frac{Y_\varphi}{\varrho V^2 S_1}\,,\quad y_\psi = \frac{Y_\psi}{\varrho V^2 S_1}\\
4.2\quad &\bar{l}_y = -\frac{\mu_2}{j_x}\frac{L_y}{\varrho V^2 S_1}\,,\quad \bar{l}_\varphi = -\frac{\mu_2}{j_x}\frac{2L_\varphi}{\varrho V^2 S_1 b_1}\,,\quad \bar{l}_\psi = -\frac{\mu_2}{j_x}\frac{2L_\psi}{\varrho V^2 S_1 b_1}\,,\\
&\bar{n}_y = -\frac{\mu_2}{j_z}\frac{N_y}{\varrho V^2 S_1}\,,\quad \bar{n}_\varphi = -\frac{\mu_2}{j_z}\frac{2N_\varphi}{\varrho V^2 S_1 b_1}\,,\quad \bar{n}_\psi = -\frac{\mu_2}{j_z}\frac{2N_\psi}{\varrho V^2 S_1 b_1}
\end{aligned}
$$

These dimensionless aerodynamic derivatives are derived in (3).
The general solutions of the system of second order differential equations with constant coefficients have the form:

$$\bar{y} = y_0 e^{\bar{\lambda}\bar{t}}\,,\quad \varphi = \varphi_0 e^{\bar{\lambda}\bar{t}}\,,\quad \psi = \psi_0 e^{\bar{\lambda}\bar{t}}$$

After substituting the above relations to the system 4.1, dividing by $e^{\bar{\lambda}\bar{t}}$, and putting in order with respect to y_o, φ_o and ψ_o, the system of homogeneous algebraic equations is obtained. To get the non-zero values for the solutions, the characteristic determinant of the system has to be equal to zero, which leads to the following characteristic equation of the system:

4.3 $$\bar{\lambda}^6 + B\bar{\lambda}^5 + C\bar{\lambda}^4 + D\bar{\lambda}^3 + E\bar{\lambda}^2 + F\bar{\lambda} + G = 0$$

The coefficients of the equation 4.3 are divided into two groups: those which correspond to free-flight B_2, C_2, D_2 and E_2 (3), and those which correspond to the presence of the towing rope C_2^1, D_2^1, E_2^1, F_2^1 and G_2^1:

4.4 $$\begin{aligned} B &= B_2, \\ C &= C_2 + C_2^l, \\ D &= D_2 + D_2^l, \\ E &= E_2 + E_2^l, \\ F &= F_2^l, \\ G &= G_2^l. \end{aligned}$$

The free-flight coefficients of the characteristic equation are (3):

$$\begin{aligned} B_2 &= \bar{l}_p + \bar{n}_r - y_v, \\ C_2 &= (\bar{n}_r - y_v)\bar{l}_p - (1 + y_v)\bar{n}_r - \bar{l}_r \bar{n}_p, \\ D_2 &= (\bar{n}_p + y_\varphi)\bar{l}_v + (\bar{l}_r \bar{n}_p - \bar{l}_p \bar{n}_r) y_v - \bar{l}_p \bar{n}_v, \\ E_2 &= (\bar{l}_v \bar{n}_r - \bar{l}_r \bar{n}_v) y_\varphi. \end{aligned}$$

The rope terms of the characteristic coefficients of the equation are:

$$\begin{aligned} C_2^l &= \bar{l}_\varphi + \bar{n}_\psi - y_y \\ D_2^l &= -(\bar{l}_p + \bar{n}_r) y_y - (\bar{n}_\psi + \bar{l}_\varphi) y_v + \bar{l}_\varphi \bar{n}_r - \bar{l}_r \bar{n}_\varphi + \\ &\quad + \bar{l}_p \bar{n}_\psi - \bar{l}_\psi \bar{n}_p + \bar{l}_v y_{\varphi l} + \bar{n}_v y_\psi \\ E_2^l &= (\bar{n}_v - \bar{l}_\varphi - \bar{n}_\psi) y_y + y_\varphi \bar{l}_y + (\bar{n}_\psi - \bar{n}_v)\bar{l}_\varphi - \bar{l}_\psi \bar{n}_\psi + \bar{n}_\varphi \bar{l}_v + \\ &\quad + (y_\psi - y_v)\bar{n}_y + (\bar{n}_p \bar{l}_\psi - \bar{n}_r \bar{l}_\varphi) y_v + (\bar{n}_r y_{\varphi l} - \bar{n}_p y_\psi)\bar{l}_v + \\ &\quad + (\bar{n}_v y_\psi - \bar{n}_\psi y_v - \bar{n}_r y_y)\bar{l}_p + (\bar{n}_\varphi y_v - \bar{n}_v y_{\varphi l})\bar{l}_r \\ F_2^l &= [(\bar{n}_v - \bar{n}_\psi)\bar{l}_p + (\bar{l}_\psi - \bar{l}_v)\bar{n}_p - \bar{l}_\varphi \bar{n}_r + \bar{l}_r \bar{n}_\varphi] y_y + \bar{l}_y y_{\varphi l} + \\ &\quad + (y_\varphi + y_{\varphi l})(\bar{l}_y \bar{n}_r - \bar{l}_\psi \bar{n}_v + \bar{l}_v \bar{n}_\psi) + y_\psi (\bar{l}_\varphi \bar{n}_v - \bar{l}_v \bar{n}_\varphi + \bar{l}_p \bar{n}_y + \\ &\quad - \bar{l}_y \bar{n}_p) + (\bar{l}_y \bar{n}_p - \bar{l}_p \bar{n}_y + \bar{l}_\varphi \bar{n}_v + \bar{l}_\psi \bar{n}_\varphi) y_v - \bar{l}_\varphi \bar{n}_\psi \\ G_2^l &= [(\bar{n}_v - \bar{n}_\psi)\bar{l}_\varphi + (\bar{l}_\psi - \bar{l}_v)\bar{n}_\varphi] y_y + (\bar{l}_\varphi \bar{n}_y - \bar{l}_y \bar{n}_\varphi) y_\psi + \\ &\quad + (y_\varphi + y_{\varphi l})[(\bar{n}_\psi - \bar{n}_v)\bar{l}_y + (\bar{l}_v - \bar{l}_\psi)\bar{n}_y - \bar{l}_r \bar{n}_y] + \\ &\quad + [\bar{l}_y \bar{n}_\varphi - (\bar{n}_y + \bar{n}_v)\bar{l}_\varphi] y_v. \end{aligned}$$

During the sailplanes free flight the above terms are equal to zero and the characteristic equation becomes a fourth order algebraic equation of the form

4.5 $$\bar{\lambda}^4 + B_2\bar{\lambda}^3 + C_2\bar{\lambda}^2 + D_2\bar{\lambda} + E_2 = 0.$$

If the location of the towing hook coincides with the glider center gravity, distances $h_{z1} = k_{z1} = 0$, $G_2^1 = 0$ and the characteristic equation 4.3 becomes the fifth order equation:

4.6 $$\bar{\lambda}^5 + B\bar{\lambda}^4 + C\bar{\lambda}^3 + D\bar{\lambda}^2 + E\bar{\lambda} + F = 0.$$

The solution of equations 4.3 and 4.5 were obtained by Bairstow's numerical method (9).
The roots of the characteristic equation are of complex form:

4.7 $$\bar{\lambda}_K = \bar{\xi}_K \pm i\bar{\eta}_K$$

where

$\bar{\xi}_K = \xi_K \hat{t}$ – dimensionless damping coefficient
$\bar{\eta}_K = \eta_K \hat{t}$ – dimensionless oscillation frequency.

For a stable glider, the real parts of 4.7 have to be negative. This leads to the following in equalities:

4.8 $$B, C, D, E, F, G > 0,$$

and

4.9 $$R = \Delta_0 \Delta_2 - \Delta_1^2 > 0,$$

where

$$\Delta_0 = \begin{vmatrix} B & 1 & 0 \\ D & C & B \\ F & E & D \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} B & 1 & 0 \\ D & C & B \\ 0 & G & F \end{vmatrix}, \quad \Delta_2 = \begin{vmatrix} B & 1 & 0 \\ F & E & D \\ 0 & G & F \end{vmatrix},$$

The terms 4.8 and 4.9 are the same as Lienard's and Chipart's criteria (11, 12).
On the base of the above considerations the example numerical calculations and the analysis of the towed glider stability have been performed for a chosen glider.

5. Numerical results and conclusions

Numerical calculations have been performed for the chosen example of a high performance glider. The influence of the following parameters has been analysed: tow velocity, position of the glider with respect to the towing aeroplane, location of the towing hook with respect to the glider center of gravity and towing rope length. Similar calculations for the free-flight case have also been performed.
The roots of equation 4.3 are marked as $\bar{\lambda}^h$ and the roots of equation 4.5 are marked as $\bar{\lambda}$.

The roots of the same subscripts k in both cases free and towed flight correspond to the same modes of the glider. There exist two real roots $\bar{\lambda}_1^h = \bar{\xi}_1^h$ $(\bar{\lambda}_1 = \bar{\xi}_1)$ and $\bar{\lambda}_2^h = \bar{\xi}_2^h$ $(\bar{\lambda}_2 = \bar{\xi}_2)$ corresponding to aperiodic motion and two or one pairs of complex roots $\bar{\lambda}_{3,4}^h = \bar{\xi}_{3,4}^h \pm i\bar{\eta}_{3,4}^h$ $(\bar{\lambda}_{3,4} = \bar{\xi}_{3,4} \pm i\bar{\eta}_{3,4})$ and $\bar{\lambda}_{5,6}^h = \bar{\xi}_{5,6}^h \pm i\bar{\eta}_{5,6}^h$ corresponding to periodic motion.
The real roots $\bar{\lambda}_1^h = \bar{\lambda}_1 < 0$ correspond to the glider rolling with angular velocity p around the longitudinal axis x and the roots $\bar{\lambda}_2^h$ $(\bar{\lambda}_2)$ correspond to spiral movements. The pair of complex roots $\bar{\lambda}_{3,4}^h$ $(\bar{\lambda}_{3,4})$ corresponds to the so-called lateral oscillations (2, 3).

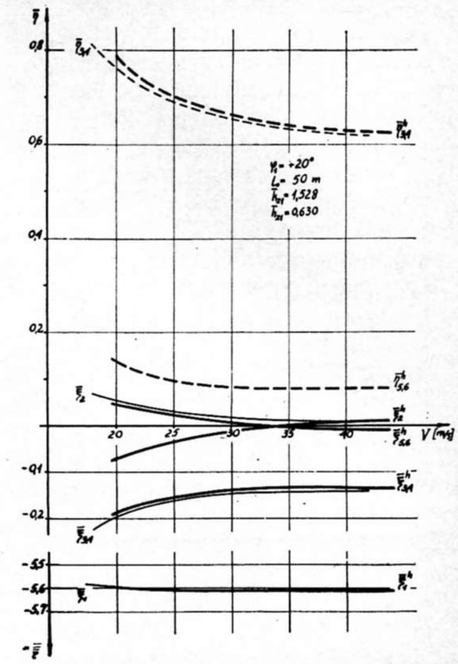

Fig. 4. Changes of damping and oscillation frequency dimensionless coefficients vs. towing velocity.

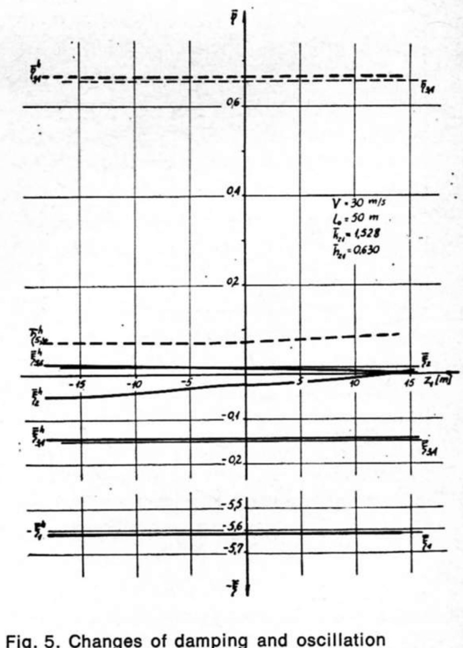

Fig. 5. Changes of damping and oscillation frequency dimensionless coefficients vs. glider position relatively to towing airplane.

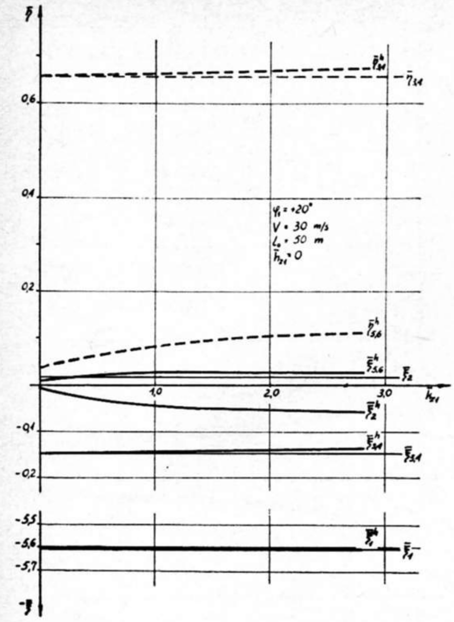

Fig. 6. Changes of damping and oscillation frequency dimensionless coefficients vs. horizontal displacement of towing hook relatively to the glider center of gravity.

The pair of complex roots $\bar{\lambda}^h_{5,6}$ existing only in the towed flight case, corresponds to periodic yawing motion, with a very long period around the glider vertical axis z.
In the figures the thick lines correspond to the towed flight data and the thin lines to the free flight data.

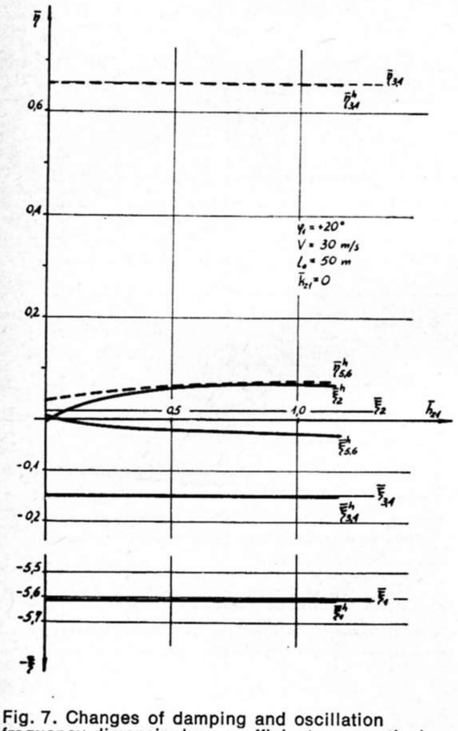

Fig. 7. Changes of damping and oscillation frequency dimensionless coefficients vs. vertical displacement of towing hook relatively to the glider center of gravity.

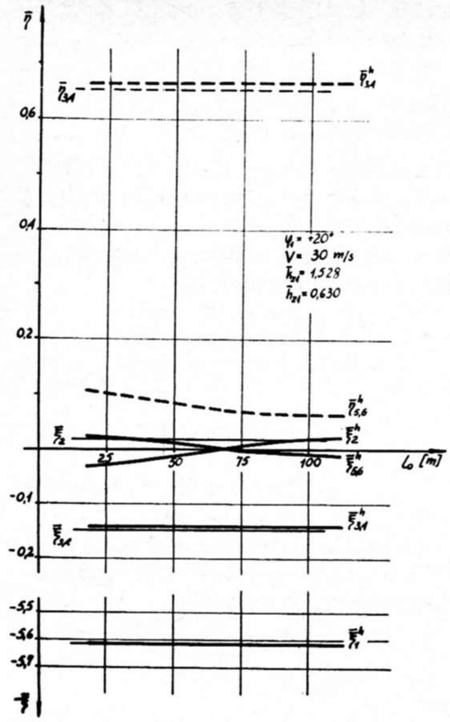

Fig. 8. Changes of damping and oscillation frequency dimensionless coefficients vs. length of towing rope.

The conclusions given below are based on the numerical calculations for one glider type only, and may not be valid for other gliders.
(1) According to the results of the calculations, the towed glider is unstable. For typical flight conditions, the period of the unstable oscillations is T = 50 s, and the time of double amplitude is T_2 = 28 s. As a typical time of pilot's reaction t_R is $0{,}20 < t_R < 0{,}54$ s, the glider instability is not dangerous.
(2) Towing has only a small influence on the damping and frequency of the sailplane's lateral oscillation.
(3) Towing does not affect the glider aperiodic motion. Below the angle of stall, this motion is very strongly damped in free as well as towed flight.
(4) Towing has an influence on the spiral motion and new type of low frequency yawing oscillation appears. Both these motions are mutually dependent; increase in damping of one of them involves decrease in damping of the other.
(5) From the point of view of stability, the position of the towed sailplane below the towing airplane is favorable; it assures better spiral stability and only slightly decreases the damping of the low frequency yawing oscillation.
(6) The increase of towing velocity above 30 m/s is desirable. It improves the spiral and yawing stability.
(7) The towing hook attachment, being in front of the glider center of gravity, improves the spiral stability and slightly decreases the yawing stability.
(8) Positioning the hook below the glider center of gravity has the opposite effect to the one mentioned above.
The methods presented in this paper, together with those of (4), (5) and (6) with respect to longitudinal stability, make possible the quantitative estimates of the parameters, having an influence on the glider equilibrium, stability and characteristics.

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Symbols

b_1	m	– the span of the wing of glider,
C_n, C_t	–	– dimensionless aerodynamic coefficients of forces normal and tangential to the rope and determined in relation to the diameter and the length unity of this rope,
d	m	– diameter of towed rope,
n	kg/m	– aerodynamic forces normal to the rope acting on 1 m length of the rope,
q	kg/m	– weight of 1 m of the rope,
Q_1	kg	– weight of glider,
S_1	m²	– wing area of glider,
t	kg/m	– aerodynamic forces tangential to the rope acting on 1 m length of the rope,
T_1	kg	– forces acting on the hook of the glider and incidence by the towed rope,
φ_1, φ_2	–	– angles of slope of the towing rope in relation to the trajectory of flight and measured on the hook of glider and the hook of towed airplane,
λ	1/kg	– coefficient of elasticity of the towing rope.