# Thermaling flight optimization

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*Abstract:*

A number of authors [1,2] have investigated the optimization of rate of climb in a thermal. In this paper a systematic approach is presented based on an iterative mathematical optimization technique. Solutions are generated by optimizing the rate of climb in a radially symmetric thermal.

The optimal ascent characteristics provide a quantitative basis for comparing different sailplanes and lead to some general observations concerning the properties of the optimal solution.

## *1. Introduction*

One of the most common techniques for gaining altitude in a sailplane is thermal soaring. Altitude is gained by circling in rising air created by thermal atmospheric instability. If the velocity of the rising air exceeds the rate at which the sailplane sinks relative to the rising air mass altitude is gained. The area of rising air is called a *Thermal*. The Thermal is often a local effect and the sailplane pilot attempts to maximize the rate of ascent by circling so as to center the area of rising air relative to a circular flight path. A further improvement may be achieved by adjusting path radius by varying the roll angle and airspeed. It is the latter optimization which is the subject of this paper.

## *2. Basic Relationships*[3]

The vertical velocity, $V_z$, of a sailplane is the sum of two components

$$V_z = V_s + V_t \tag{1}$$

where $V_s$ is the rate of sink in calm air and $V_t$ is the velocity of the rising air. The equation may be expressed in terms of forces and velocities by equating the instantaneous rate of energy gain to the rate of energy loss. Thus

$$(V_z - V_t)\, W = F_d \cdot V_p \tag{2}$$

where W is the vehicle weight, $V_p$ is the path velocity and $F_d$ is the drag force. Then solving for $V_z$

$$V_z \doteq \frac{F_d \cdot V_p}{W} + V_t \tag{3}$$

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Assuming that the aircraft is in a coordinated turn the drag force is given by

$$F_d = \frac{C_d}{C_l} \cdot \frac{W}{\cos\phi} \tag{4}$$

where $C_d$ and $C_l$ are overall drag and lift coefficients of the glider and $\Phi$ is the bank angle. The sink speed then becomes

$$V_z = \frac{C_d V_p}{C_l \cos\phi} + V_t \tag{5}$$

### 3. *Thermal velocity structure*

The contour of the vertical component of thermal velocity is quite complicated as a result of their basic complex structure [4,5,6,7] and the turbulent nature of the flow. For the purposes of this analysis the velocity, $V_t$, is assumed to be radially symmetric about some central core.

$$V_t = V_t(R) \tag{6}$$

where R is the radius of a flight path centered on the thermal as shown in Fig. 1.

### 4. *Optimization Using a Hill Climbing Approach*

The effect of small changes in $\Phi$ and $V_p$ on $V_z$ may be investigated by forming the differential or change in $V_z$.

$$\delta V_z = \frac{\partial V_z}{\partial\phi}\,\delta\phi + \frac{\partial V_z}{\partial V_p}\,\delta V_p + \text{higher order terms} \tag{7}$$

where $\delta V_z$, $\delta\Phi$ and $\delta V_p$ are changes in $V_z$, $\Phi$ and $V_p$ respectively.
The partial derivatives

$$\frac{\partial V_z}{\partial\phi} = \frac{V_p}{\cos\phi}\,\frac{\partial}{\delta\phi}\left(\frac{C_d}{C_l}\right) + \frac{V_p C_d}{C_l}\,\frac{\sin\phi}{\cos^2\phi} + \frac{\partial V_t}{\partial R}\,\frac{\partial R}{\partial\phi} \tag{8}$$

$$\frac{\partial V_z}{\partial V_p} = \frac{V_p}{\cos\phi}\,\frac{\partial}{\partial V_p}\left(\frac{C_d}{C_l}\right) + \frac{C_d}{\cos\phi\, C_l} + \frac{\partial V_t}{\partial R}\,\frac{\partial R}{\partial V_p} \tag{9}$$

are computed using the physical parameters, the glide polar of the sailplane and the velocity distribution of the thermal.

If $\delta\Phi$ and $\delta V_p$ are sufficiently small the higher order terms may be neglected and equation (7) written:

$$\delta V_z = \frac{\partial V_t}{\partial\phi}\,\delta\phi + \frac{\partial V_z}{\partial V_p}\,\delta V_p \tag{10}$$

The rate of climb will be improved if $\delta V_z$ is always positive. This may be ensured by choosing

$$\delta\phi = \epsilon_\phi\,\frac{\partial V_z}{\partial\phi} \qquad \epsilon_\phi > 0 \tag{11}$$

$$\delta V_p = \epsilon_{V_p}\,\frac{\partial V_z}{\partial V_p} \qquad \epsilon_{V_p} > 0 \tag{12}$$

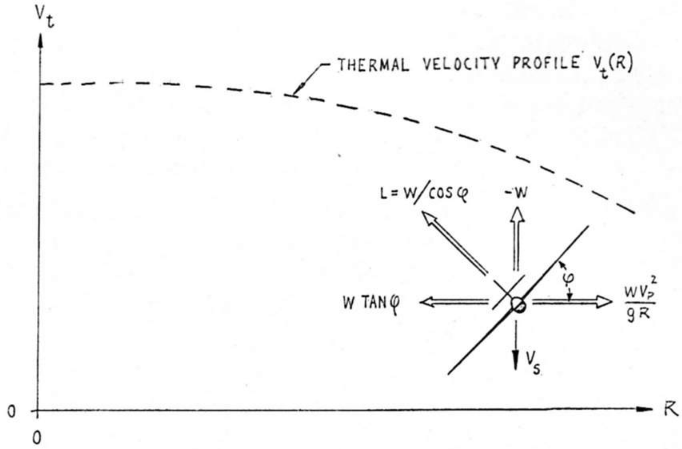


Fig. 1 – A Sailplane n Circling Flight Centered on a Radially Symmetric Thermal

$\varepsilon\Phi$ and $\varepsilon V_p$ are introduced to ensure that $\delta\Phi$ and $\delta V_p$ are small and thus the validity of equation (10) which is then written.

$$\delta V_z = \epsilon_\phi\left(\frac{\partial V_z}{\partial\phi}\right)^2 + \epsilon_{V_p}\left(\frac{\partial V_t}{\partial V_p}\right)^2 \tag{13}$$

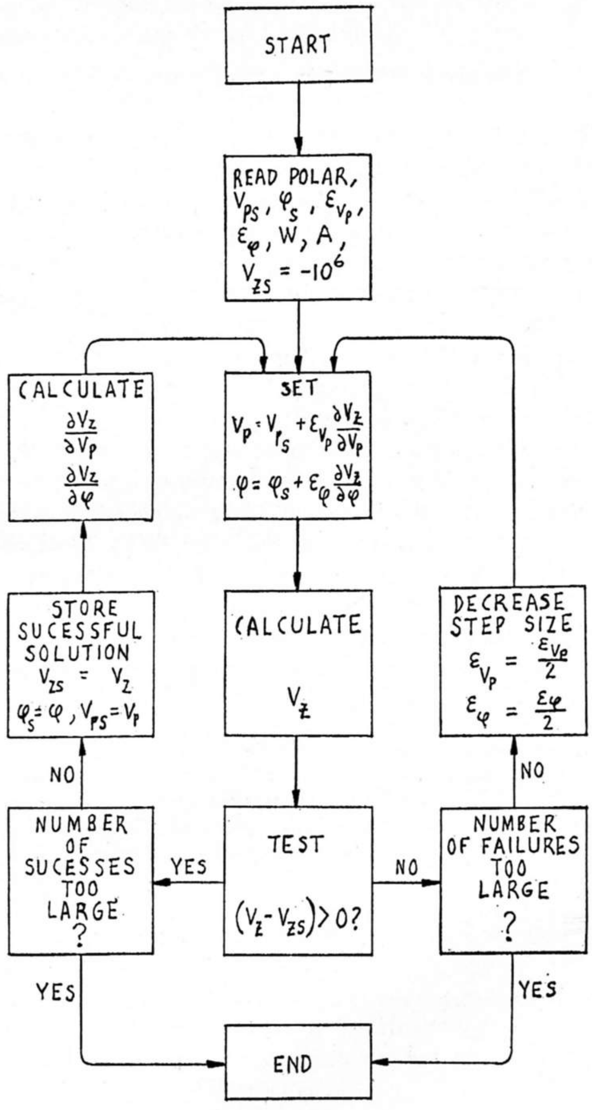


Fig. 2 – A Flow Diagram for Optimizing Rate of Ascent in a Radially Symmetric Thermal

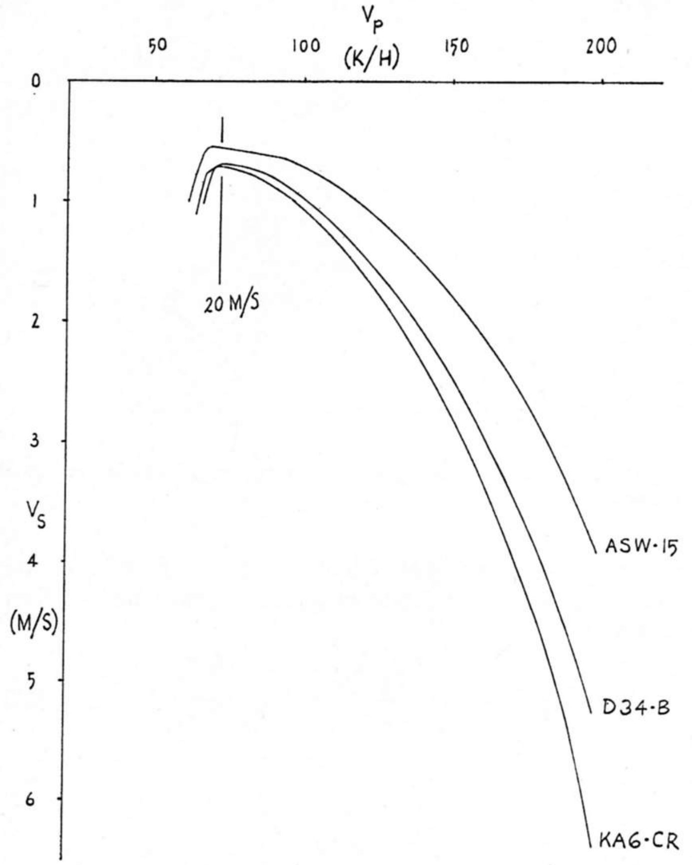


Fig. 3 – Sink Velocity in Still Air Versus Path Velocity

The increment in vertical velocity, $\delta V_z$, is always positive providing equation (10) is a good approximation of equation (7). If $\delta V_z$ is negative a reduction in the size of ${}^{\varepsilon}\Phi$ and ${}^{\varepsilon}V_p$ will lead to a positive vertical velocity increment. A computational scheme which generates a sequence of improving rates of ascent using relationships (11), (12) and (13) is illustrated in Fig. (2). The computation is initiated with $d V_t/d V_p = d V_z/d\Phi = 0$. The mathematical procedure refines the best initial guess, $\Phi_s$, $V_{ps}$, in an iterative fashion. If the procedure fails to improve the vertical velocity the step size, ${}^{\varepsilon}\Phi$, ${}^{\varepsilon}V_p$, is halved. The computation is terminated when a specified number of successful iterations ($\delta V_z > 0$) or unsuccessful iterations ($\delta V_z < 0$) is exceeded. A satisfactory solution is normally reached after 10 to 20 successful iterations. Convergence to a relative maximum ($d V_z/d\Phi = 0$, $d V_z/d V_p = 0$) is guaranteed, however, this maximum may not correspond to the best rate of ascent as a result of the nonlinear character of the vehicle equations.

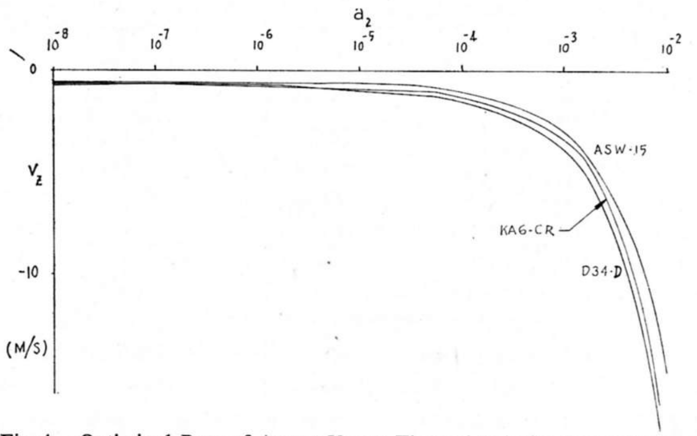


Fig. 4 – Optimized Rate of Ascent Versus Thermal Velocity Parameter $a_2$

Thus some experimentation must be applied in the choice of the starting point $\Phi_s$, $V_{ps}$ to ensure convergence to the optimum solution.

## 5. *Comparing the Thermalling Behavior of three Sailplanes*

The preceding results may be applied to compare the performance of the KA 6-CR[8], D 34-D[8] and the ASW-15[9]. The glide polars of these aircraft are illustrated in Fig. 3. The physical parameters of the aircraft are summarized in table 1.

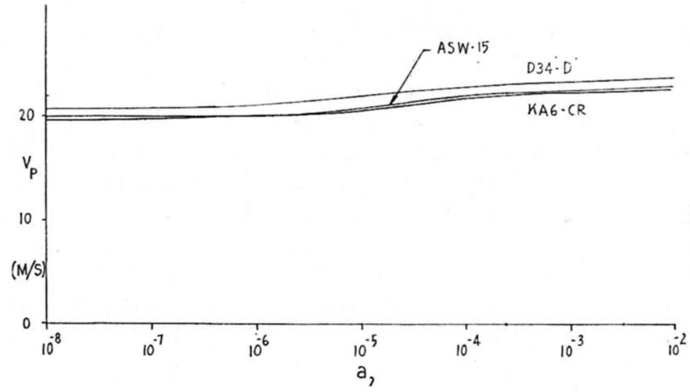


Fig. 5 – Optimum Path Velocity Versus Thermal Velocity Parameter $a_2$

The polars indicate that the low speed characteristics of the D34-D and the KA 6-CR should be comparable. The ASW-15 displays the best low speed characteristics. The improved high speed penetration of the D34-D and the ASW-15 compared to the KA 6-CR is a result of airfoil and fuselage design improvements.

The thermalling behavior was compared by optimizing the rate of ascent in a radially symmetric thermal of the form

$$V_t = a_0 + a_2 R^2 \tag{14}$$

Table 1 – Physical Parameters of Three Sailplanes

| Aircraft | Weight | Wing Area | Wing Loading | Wing Span |
|---|---|---|---|---|
| KA 6-CR | 305 kg | 12.4 $m^2$ | 24.59 $kg/m^2$ | 15.0 m |
| D 34-D | 255.2 kg | 9.18 $m^2$ | 27.79 $kg/m^2$ | 12.65 m |
| ASW 15 | 300 kg | 10.92 $m^2$ | 27.47 $kg/m^2$ | 15.0 m |

The value of $a_0$ was set equal to zero since it does not affect the value of the partial derivatives $d V_z/p\Phi$, and $d V_z/d V_p$.

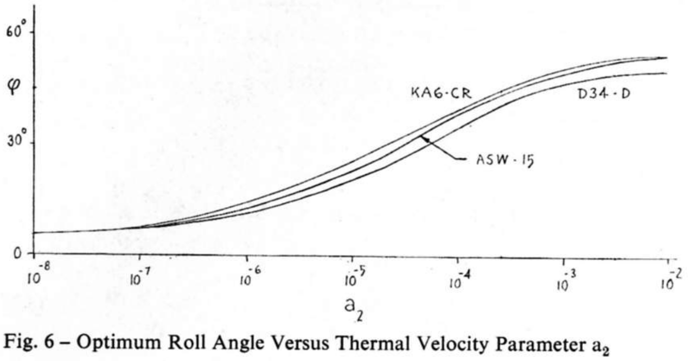


Fig. 6 – Optimum Roll Angle Versus Thermal Velocity Parameter $a_2$

The value of $a_2$ was varied between $10^{-8}$ and $10^{-2}$ meters/sec/meters². The corresponding maximized rate of ascent is shown in Fig. (4). The best circling path geometry is determined by the optimized roll angle and path velocity in Figures 5 and 6. The corresponding circling radius is shown in Fig. (7).

At low values of $a_2$ the thermal approximates a uniformly rising air mass. As a result the best path approaches a straight line and the roll angle approaches zero. The optimum path velocity approaches the velocity for minimum sink in level flight. As $a_2$ increases the roll angle and the path velocity increase in a monotone fashion. The change in path velocity is quite small ($\approx 15\%$) while the change in roll angle is very large ($0 \rightarrow 60°$).

## 6. *Conclusions*

A technique has been presented for mathematically optimizing the rate of ascent of a sailplane in a radially symmetric thermal. The method is demonstrated for three sailplanes in a thermal with a parabolic vertical velocity characteristic. The method may be extended to thermals with a more complex structure.

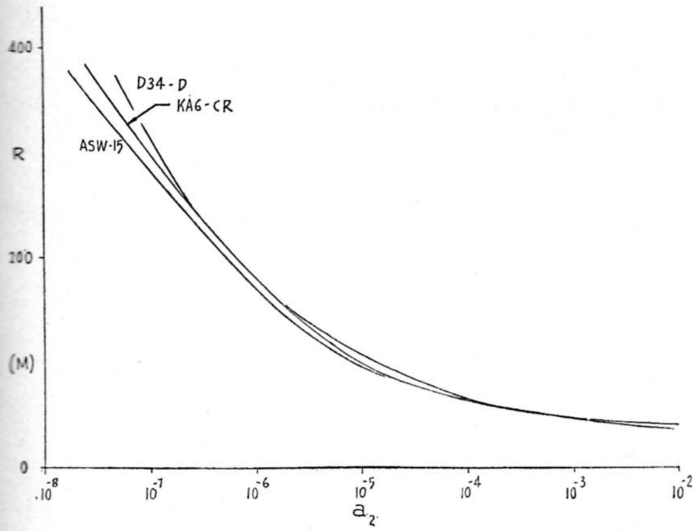


Fig. 7 – Optimum Circling Radius Versus Thermal Velocity Parameter $a_2$

While the results presented are not conclusive the optimal circling velocity appears to lie in a narrow range between the velocity for minimum sink in level flight and a velocity 1.15 times as large. The optimum roll angle lies in the range between zero and 60 degrees. These results suggest a flight procedure consisting of two steps.

1. Maintain an essentially constant path velocity slightly above the minimum sink velocity in level flight.
2. Modify the roll angle to achieve centering and rate of climb maximization.

## 7. *Acknowledgements*

The author would like to thank Mr. Paul Madden for his many helpful comments during the course of the research leading to this paper.

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# Zusammenfassung der Arbeit von MacKinnon

Die am Schluss des Vortrages erwähnten Verfasser haben die bestmöglichste Steiggeschwindigkeit in thermischen Aufwinden untersucht. Die vorliegende Arbeit zeigt einen mathematischen Weg dieses Optimum zu berechnen. Dabei wurde eine radial symmetrische Aufwindstärke angenommen. Die gefundenen mathematischen Lösungen werden im Abschnitt 5 benutz, um die Leistungen der drei Segelflugzeuge Ka 6 CR, D 34 D und ASW 15 zu vergleichen. Die Geschwindigkeitspolaren dieser Flugzeuge sind in Figur 3 dargestellt, die technischen Daten in Tabelle 1 enthalten.

Die Polaren zeigen, dass die Langsamflugeigenschaften der Ka 6 CR und der D 34 D ähnlich sind. Die ASW 15 zeigt die besten Langsamflugeigenschaften. Die Abweichungen der D 34 D und der ASW 15 in den höheren Geschwindigkeiten sind die Folge von Flügelprofil- und Rumpfformverbesserungen.

Obwohl die dargestellten Ergebnisse nicht endgültig sind, kann doch gesagt werden, dass die beste Kreisgeschwindigkeit in einem schmalen Bereich liegen muss, nämlich zwischen der Geschwindigkeit mit minimalem Sinken im Geradeausflug und einer Geschwindigkeit, die 15% über dieser Limite liegt. Die optimale Schräglage liegt zwischen Null und 60°.

Diesen beiden Punkten soll man durch folgende Flugtaktik bei Thermikkreisen Rechnung tragen:

1. Halte eine konstante Geschwindigkeit, die wenig über der Geschwindigkeit mit minimalem Sinken im Geradeausflug liegt.
2. Korrigiere nur die Querlage zum Zentrieren und zur Erreichung der höchsten Steiggeschwindigkeit.

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