Linear and non-linear periodic waves of large amplitude

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Presented at the 8th OSTIV Congress, Cologne, Germany, June 1960

Summary

This paper is concerned with steady flow of a two-dimensional model of an incompressible, inviscid, stably stratified fluid. Special cases of linear and non-linear periodic waves and rotors are discussed. In contrast to the much studied linear periodic waves, the more realistic non-linear waves show that nodal points do not remain stationary as the amplitude is increased, and that symmetry is no longer preserved about the center-line of the fluid. For example, rotors might appear at the bottom without any appearing at the top and vice versa, etc.

1. Introduction

Disturbances produced by mountains have for some time been studied experimentally and numerically by meteorologists. The wave disturbances set up by a mountain could be categorised into three main types:

The disturbance immediately near the mountain.
Tropopause waves.
Lee waves.

This paper is concerned with some types of lee waves. These extend in theory to infinity downstream behind the disturbance, and they occur when the streams have suitable stability profiles.

2. Short Historical Background

Lee wave problems were studied by Rayleigh (1883) and Kelvin (1886). In both cases, surface waves were investigated in the lee of some obstacle.

From the meteorologist's view-point, Lyra (1943) and Queney (1947) investigated waves in the atmosphere and obtained patterns for disturbances immediately near the mountain. Scorer (1949 etc.) produced more realistic solutions by considering 2-layer models of the atmosphere. This idea was further extended by Wurtele (1953), Palm (1958) and Wallington (1959).

Long (1953, etc.) obtained the general theory for incompressible stratified fluids, and showed that there existed a qualitative analogy with the atmospheric case, but a quantitative comparison has not been attempted, due to dissimilarity at the "upper boundary".

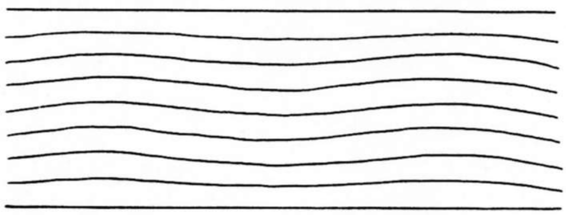
3. Theory

The model under consideration is a two-dimensional one with horizontal boundaries, and the fluid is incompressible, inviscid and stably stratified. The wave motion is in one direction under steady conditions.

The equation used has been a modification of Long's equation (1953). The boundary condition is that top and bottom streamlines are undisturbed.

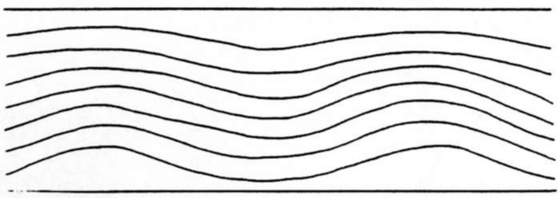
A linear variation of the stability parameter with height has been investigated numerically. The solution was effected by writing a harmonic series solution for the displacement of the streamlines.

Fig. 1



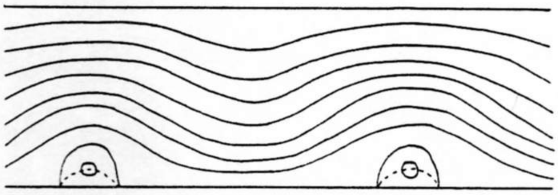
Small Amplitude Wave. Wavelength = 13.38 in

Fig. 2



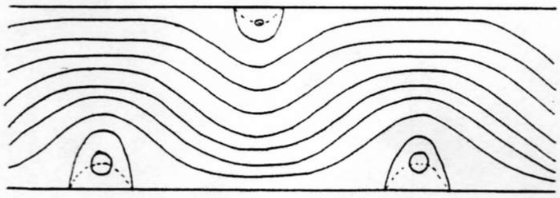
Increased Amplitude Wave. Wavelength = 14.02 in

Fig. 3



Large Amplitude Wave. Rotors appear at the bottom.
Wavelength = 14.15 in

Fig. 4



Very Large Amplitude Wave. Rotors appear at the bottom and top.
Wavelength = 14.27 in

The diagrams show non-linear wave profiles developing with increasing amplitude. The profile of the stability parameter (linear decrease with height) is the same for all 4 figures, but has an important bearing on the nature of the developing waves and rotors, as the amplitude is increased. The waves drawn are in relation to a particular model whose total depth was 8 in.

4. Results

It is well known that linear periodic waves have a wavelength which is independent of the amplitude. Nodal points remain stationary as the amplitude is increased. Symmetry or anti-symmetry is preserved about the center-line of the fluid.

In contrast non-linear periodic waves change their wavelength with the amplitude (Fig. 1 to 4). Nodal points do not remain stationary if the amplitude is altered. Symmetry is no longer preserved. For example, rotors might appear at the bottom without any appearing at the top, and vice versa, etc.

5. References

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Long, R.R., 1953, Tellus 5, 42—58
Scorer, R.S., 1949, Q.J. Roy. Met. Soc. 75, 41—56