An introduction to lee waves in the atmosphere

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Have you ever watched the ripples in a shallow brook as the water flows over a submerged rock? The water rises over the rock, dips sharply on the downstream side and, if the rock is in the form of a ridge placed across the stream, the water surface will rise and fall a second, third or several more times downstream. The crests of the ripples form a series of bars parallel to the rock and, with water flowing through them, these bars remain in almost stationary positions in the stream.

Substitute an airstream for the brook, a mountain for the rock and we begin to visualise the form of lee waves in the atmosphere. But before trying to visualise too much let us get a few basic features of the flow fixed in our minds. These features are illustrated in Fig. 1 by the airflow across a very long isolated mountain ridge lying at right angles to the airstream.

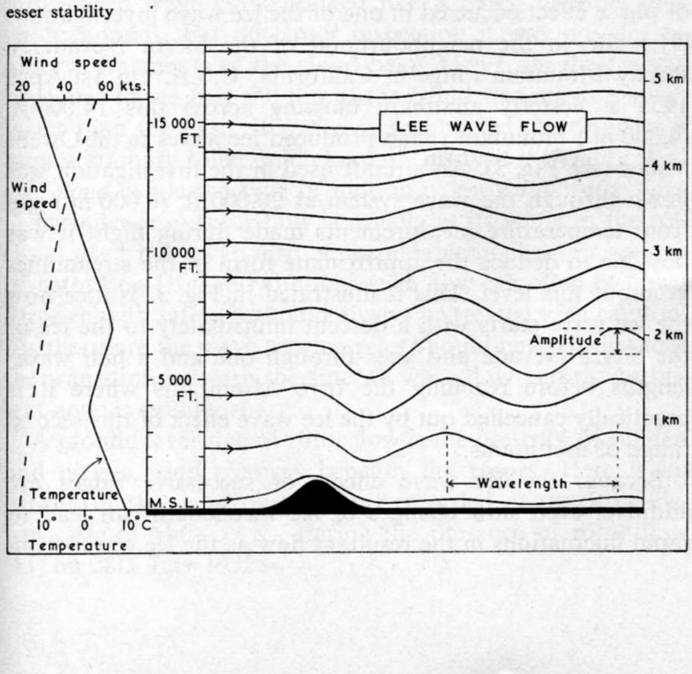
The flow pattern may be dissected into three zones. The first zone contains the undisturbed flow—too far upstream to be diverted from its steady horizontal course. Then comes the mountain sector wherein the streamlines at low levels tend to follow the high ground profile, and finally we have the lee wave flow with its regular undulating stream bearing little apparent relationship to the flat terrain below. It is this lee wave flow that merits detailed discussion and the two dimensions most appropriate to such discussion are the wavelength and amplitude.

The lee wavelength is a measure of the distance from one wave crest to the next—or from trough to trough. Usually between 5 and 50 km., this lee wavelength is determined almost entirely by winds and temperatures at various levels in the undisturbed flow.

The lee wave amplitude is half the vertical distance from wave trough to crest. Notice that the amplitude varies with height. Negligible close to the ground and at very high levels, it attains its maximum at about 6,000 ft. (2,000 m.) in the illustration of Fig. 1.

Either or both of these dimensions are bound up with almost every feature of the wave flow to be discussed.

Fig. 1 Lee waves are often associated with a stable layer sandwiched between air of esser stability



Vertical currents in the wave flow depend upon the amplitude, the wavelength and the wind speed, strong up and down draughts being favoured by large amplitudes, short wavelengths and strong winds. The distribution of vertical speed throughout a wave flow is shown in Fig. 2. The lee wave flow illustrated was evidenced by variations in the rate of ascent of a radio-sonde balloon released from Leuchars,

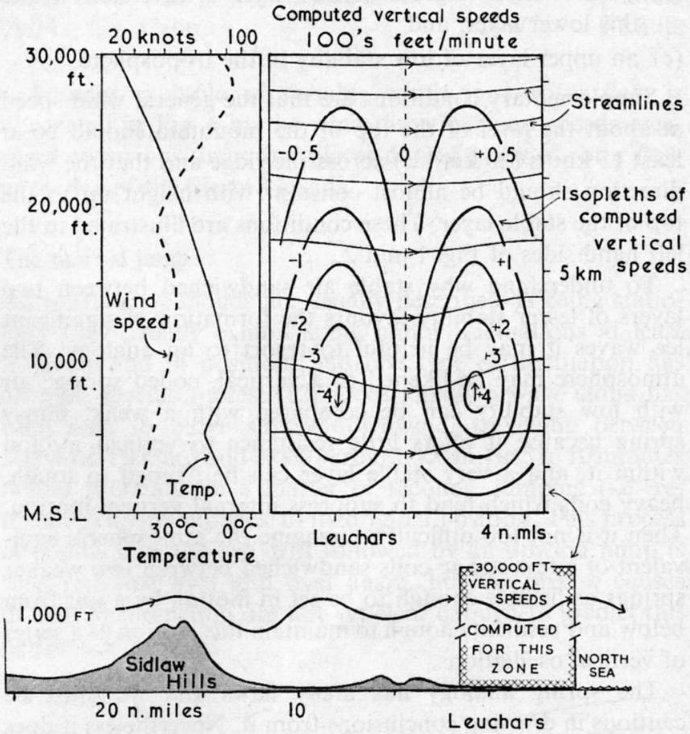


Fig. 2 On 21st December 1953 waves were evident in lee of the Sidlaw Hills, Scotland. Vertical speeds in the neighbourhood of Leuchars were calculated approximately on an experimental but justifiable basis

Scotland, on 21st December 1953 [1], and the detailed structure of this flow was calculated approximately on an experimental but justifiable theoretical basis [2]. In this particular flow it is apparent that a glider with a minimum sinking speed of, say 180 ft. (about 60 m.) per minute could have maintained or gained height in the updraught of the wave between 3,000 ft. (1,000 m.) and 17,000 ft. (about 5,000 m.). Above and below this height interval the vertical currents were too small for soaring.

Wave cloud is another phenomenon linked closely to lee wave amplitude. Air rising in the updraught of a wave cools adiabatically, and if this cooling is sufficient to cause condensation then cloud will form. Subsequent warming in the downdraught causes the condensed water to evaporate, and so by a continuous process of condensation at its leading edge and evaporation at the trailing edge the cloud as a whole appears to be stationary in the sky. At medium levels—between 6,000 ft. (2,000 m.) and 20,000 ft. (6,000 m.) in temperate regions—wave cloud is usually of a lenticular form but at lower levels the wave cloud is often torn into ragged patches by low-level turbulence. The patches as a whole remain more or less stationary but their detailed outline changes quickly and erratically. At high levels cloud is usually composed of ice crystals and, because these crystals form quickly but evaporate slowly, high level cloud forms readily in the updraught of a wave but does not always disappear on the descent; it tends to stretch out some way downstream [3].

Wave conditions

Experience [4] suggests that the conditions commonly associated with waves with appreciable vertical currents comprise:—

(*a*) a layer of low stability (high temperature lapse rate) at low levels,

(*b*) a stable layer (e.g. isothermal layer or inversion) above this lower layer, and

(*c*) an upper layer of low stability in the troposphere.

Supplementary conditions are that the general wind speed at about the level of the top of the mountain should be at least 15 knots (30 km/hr.) across the ridge and that the wind direction should be almost constant with height up to the top of the stable layer. These conditions are illustrated in the left hand sides of Fig. 1 and 2.

To understand why stable air sandwiched between two layers of lesser stability favours the formation of significant lee waves it may be helpful to resort to an analogy. The atmosphere may be likened to a vertical, coiled spring; air with low stability can be compared with a weak, flimsy spring because it offers little resistance to vertical motion within it, and a very stable layer can be likened to tough, heavy coils which tend to suppress internal vertical motion. Then it is not too difficult to imagine the atmospheric equivalent of a few strong coils sandwiched between two weaker springs as flexible enough to be set in motion by a jolt from below and resilient enough to maintain this motion as a series of vertical oscillations.

The spring analogy has many flaws and we must be cautious in drawing conclusions from it. Nevertheless it does illustrate several features of lee wave flow. It seems reasonable to suppose that the heavy coils dominate the oscillations, and indeed lee waves in the atmosphere usually do have their maximum amplitude in the stable layer which contributes so much to their existence (see Fig. 1 and 2).

The amplitude and frequency with which the coils of the spring would bounce up and down is related in some close but complicated way to the precise depth and resilience of each part of the spring. It should be no surprise therefore, to learn that (since lee waves are vertical oscillations propelled downstream) lee wavelength and amplitude in the atmosphere are closely related to the vertical distribution of winds and temperatures throughout a deep layer of whatever airstream is being considered.

The relationship between winds, temperatures, lee wavelength and amplitude is intricate and its study usually involves complex mathematical analysis [5]. In such analysis it is often expedient to describe the wind and stability characteristics of an airstream in terms of a particularly relevant parameter. A function of wind speed, U, temperature, T, the temperature lapse rate, $\delta T/\delta z$, and the acceleration due to gravity, g, this wave flow parameter, denoted by λ, is given approximately by the equation

$$\lambda = 2\pi U \sqrt{\frac{T}{g\left(\frac{\delta T}{\delta z} + \Gamma\right)}}$$

where Γ denotes the appropriate adiabatic lapse rate. This λ (which is another version of a parameter denoted by l in much of the literature on lee waves) may be termed the "natural wavelength" of the layer of air in which it is measured.

Theoretical study supported by observational evidence has shown that lee waves in the troposphere are possible when λ increases over some (but not normally the whole) depth of the troposphere and that the significant lee wavelength is likely to have a value somewhere between the maximum and upper level minimum values of λ. Unfortunately although an increase of λ with height denotes the possibility of lee waves forming it gives no indication of the magnitude of the waves [6], and it seems unlikely that simple criteria for large amplitude waves can be formulated [7]. Nevertheless, there is evidence that useful though not infallible supplements to the wave flow conditions already listed are that:—

(*i*) the stable layer associated with the wave flow produces larger amplitudes when it comprises a shallow layer of great stability than when only moderate stability extends over some considerable depth of the troposphere,

(*ii*) long waves with appreciable amplitudes are associated more with strong upper winds than with light winds at high levels.

Hill shape and size

Two requirements are needed for waves to form: the wind and temperature conditions of the airstream must be suitable for wave flow and there must be a mountain ridge to trigger off the actual waves. The effectiveness of this trigger ridge depends not only on the mountain height but also on the width of the ridge in relation to the lee wavelength [7]; there is in fact a sort of resonance effect between lee wavelength and hill, and without attempting the formidable task of formulating a general relationship [8], it is easy to appreciate that short waves tend to be associated with narrow mountain ridges while long waves are at their best over broader mountains. A corollary from this is that large mountains are not necessarily better than small hills for setting off lee waves.

When an airstream suitable for lee waves flows past two ridges in succession the eventual amplitude of the waves depends on whether or not the ridges are in or out of phase with each other for the prevailing lee wavelength. An out of phase effect occurred in one of the lee wave investigations [9] made in the neighbourhood of the Sierra Nevada, a Rocky Mountain range in California, U.S.A. On 1st April 1955 a westerly airstream blowing across this 14,000 ft. (4,300 m.) mountain range produced lee waves in the Owens Valley (see Fig. 3). An aircraft used in the investigation was flown through the wave system at 20,000 ft. (6,000 m.) and from temperature measurements made during flight it was possible to deduce the approximate form of the streamlines at about this level. This is illustrated in Fig. 3. Notice how the lee wave starts with a descent immediately to the lee of the Sierra Nevada and goes through one and a half wavelengths before reaching the Inyo Mountains where it is practically cancelled out by the lee wave effect of this second range of mountains.

Because the lee wave effects of successive ridges are additive, even slow changes of lee wavelength can lead to rapid fluctuations in the resultant flow as the lee waves from various ridges are brought in or out of phase with one another, and even when the wave clouds are truly stationary it is sometimes difficult to relate particular clouds with specific ridges in generally rugged terrain.

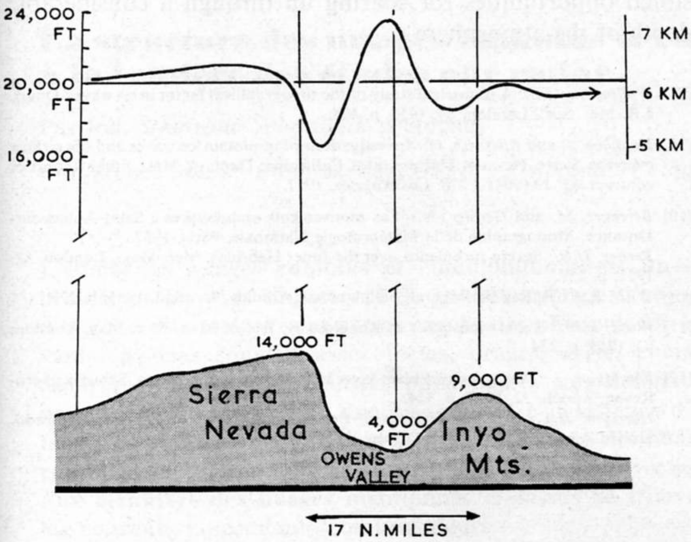


Fig. 3 The streamline depicting the wave flow across the Owens Valley, California, was determined from temperature measurements made during flight through the wave

Rotor flow

Flight conditions in lee waves are often remarkably smooth at medium or high levels, but lee wave conditions can also produce some of the most violent turbulence likely to be encountered in the troposphere.

There is some evidence to suggest that turbulence in wave flow occurs where vertical variation in lee wave amplitude increases a pre-existing wind shear to beyond some critical but not clearly understood limit. At high levels even weak waves may trigger off turbulence in the strong wind shear associated with a jet stream, but it is the lower level variety of turbulence that calls for caution in flight.

The flow in lee of the Sierra Nevada on 18th March 1952 included severe turbulence up to a height of 15,000 ft. (5,000 m.) above the Owens Valley. In the vicinity of the lee wave cloud, shown in Fig. 4, the wave-like flow had broken down into a region of chaotic motion. From a distance this ragged wave cloud looked like a patch of innocuous stratocumulus, but on closer inspection it was possible to observe fragments of the cloud being torn from the trailing edge and to see that the top of the cloud was moving much faster than the base. It was easy to imagine the cloud as a huge stationary roller, and cloud of this type is in fact called *roll cloud,* while the type of motion is known as *rotor flow*.

Turbulence in rotor flow is usually at its worst in the roll cloud itself but it can also be formidable in clear air regions of rotor flow [10], and although such flow can often be flown through with safety it should always be treated with caution. Furthermore the wave soaring pilot should never forget that the transition between the smooth wave flow and the turbulent zone is usually sharp.

A ground level sign of rotor flow is the presence of sudden and erratic wind changes beneath the rotors. Here is an extract of the wind observations recorded at the Ronaldsway, Isle of Man, U.K., during the presence of rotor flow [11] on 28th July 1952:—

Time GMT	Wind at Control Tower		Wind at wind sock 800 yards east of Control Tower	
	Direction	Speed	Direction	Speed
07.00	S	15 knots	—	—
07.16	Calm	—	—	—
07.28	360° Cycle	8 knots	—	—
07.31	SE	14 knots	N	14 knots
07.36	N	10 knots	SW	9 knots
07.41	SW	6 knots	N	14 knots
07.48	ENE	14 knots	N	5 knots
07.52	Calm	—	N	14 knots
07.54	W	10 knots	N	9 knots
08.02	SW	10 knots	N	14 knots
08.08	E	6 knots	NW	9 knots
08.24	Calm	—	NW	9 knots

Another example of variable winds in a rotor flow is illustrated in Fig. 4 by the wind directions and speeds measured at various times and places during a day of rotor flow in the lee of the Sierra Nevada.

The upwind jump

Although wave clouds are usually described as being stationary they usually exhibit some minor fluctuations at least in shape and in position, and one type of fluctuation has aroused special interest [12]. Occasionally a wave cloud has been seen to move slowly downwind, travelling between 500 yards (500 m.) and 2,000 yards (2,000 m.) in 5 to 10 minutes before suddenly, in a matter of seconds, jumping (i.e. reforming) upstream back to its original position. This process of a slow downstream drift followed by an upwind jump is then repeated over and over again, but the precise causes of this phenomenon are not yet clear enough to predict this periodic movement.

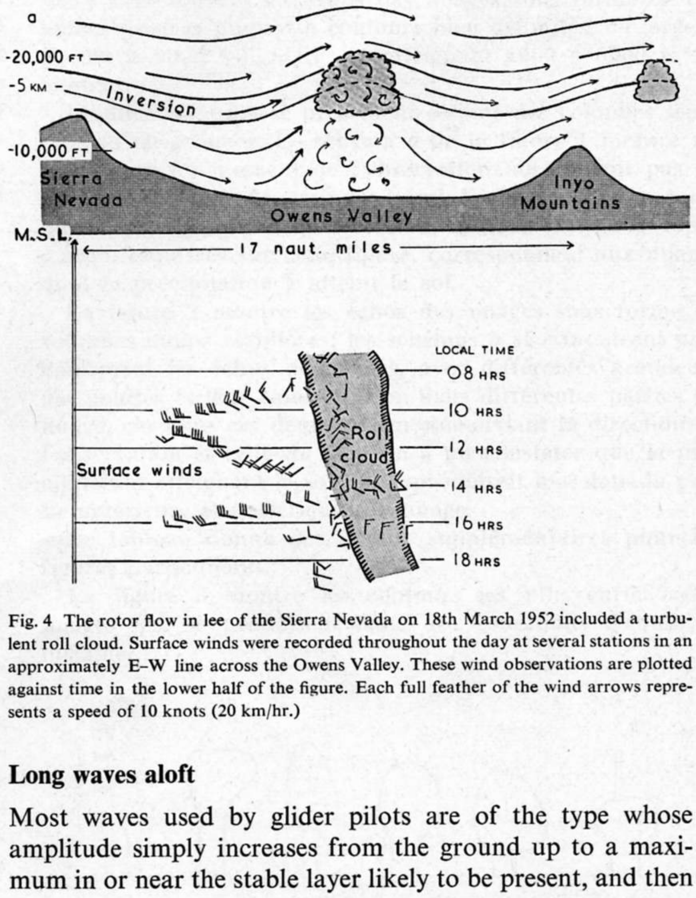


Fig. 4 The rotor flow in lee of the Sierra Nevada on 18th March 1952 included a turbulent roll cloud. Surface winds were recorded throughout the day at several stations in an approximately E–W line across the Owens Valley. These wind observations are plotted against time in the lower half of the figure. Each full feather of the wind arrows represents a speed of 10 knots (20 km/hr.)

Long waves aloft

Most waves used by glider pilots are of the type whose amplitude simply increases from the ground up to a maximum in or near the stable layer likely to be present, and then fades away at higher levels. But some airstreams can undulate on more than one wavelength, and it is not abnormal to find two different wavelengths in operation at once. The shorter wave is most pronounced at some low level while the amplitude of the longer wave is at its maximum at a greater height. Furthermore, the upper part of the long wave pattern is virtually displaced half a wavelength horizontally in relation to its low level section. Thus the combination of the two sets of waves can tilt a stack of wave clouds away from the vertical structure characteristic of the simpler type of lee wave flow. Naturally these double wave systems complicate the flow pattern but they can at least provide occasional opportunities for soaring up through a considerable depth of the atmosphere.

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