Airflow over and in the Lee of an Isolated Hill

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The following is a summary of the paper presented by Dr. R. S. Scorer at the 6[th] OSTIV Congress, St-Yan. The figures have been made available from the Quarterly Journal of the Royal Meteorological Society, in which the full manuscript has been published.

The methods of perturbation theory are used for this study: this has to be done because no methods exist at present for solving the problem except on the assumption that the disturbance is small and that any number of small disturbances can be added together linearly.

One of the simplest ways to synthesise an isolated hill is to add together a lot of ridges whose crests all pass through the central point of the isolated hill but which are inclined at different angles to the airstream. The disturbance produced in an airstream by each of the component ridges can be calculated by methods now well known, and then these disturbances can be added together by a further process of integration. Thus if the height of the component ridges is proportional to the function Φ which varies as $\varnothing$, the angle between a line at right angles to the ridge and the wind direction varies, the disturbance produced by the hill which is compounded by adding the ridges together is equal to

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Phi \zeta (\varnothing)\, d\, \varnothing \tag{1}$$

where ζ is the disturbance (vertical displacement) produced by a ridge of unit height at angle $\varnothing$ to the airstream.

If the integral in (1) is a function $\varnothing$ which has many oscillations in the range $-\frac{\pi}{2} \leq \varnothing \leq \frac{\pi}{2}$ then the integral may

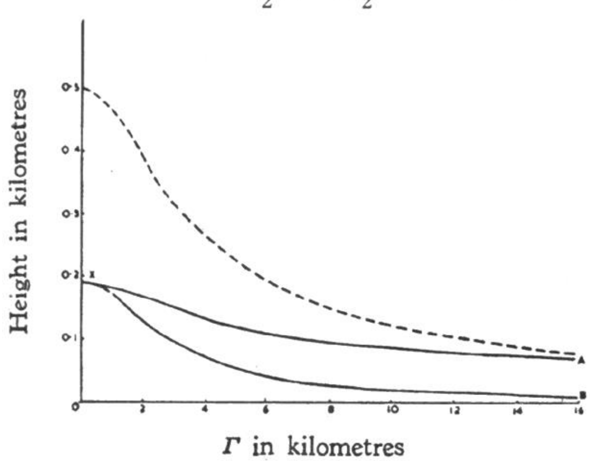

Fig. 1. Sections through centre of isolated hill. Circular hill—dashed line. Oval hill—full lines (sections along major and minor axes)

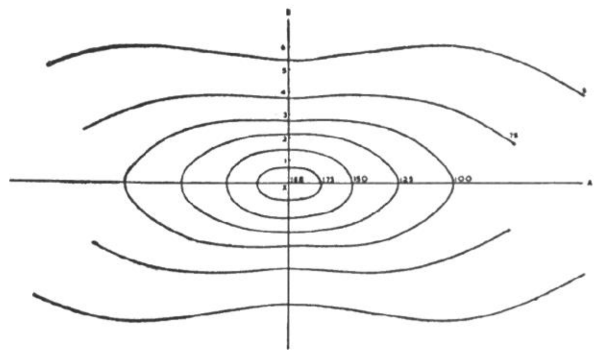

Fig. 2. Contours of oval hill chosen

be approximately evaluated by the principle of stationary phase. Physically this means that each component ridge has its own wave system, and the systems of all the ridges interfere with each other to produce a negligible total affect except in those places where a change in the value of $\varnothing$ produces almost no change in the wave pattern produced by the component ridge. As a consequence the disturbance at each point is due almost entirely to the component ridges within a small angle, or conversely the ridges in a small range of the angle $\varnothing$ contribute appreciably to the disturbance only along one line. This converse approach is the most convenient for calculation and the procedure is as follows: A value of $\varnothing$ is chosen. Corresponding to this value a line is found along which the disturbance is produced mainly by the component ridges whose angle is close to this angle $\varnothing$. The magnitude of the disturbance is then calculated by an approximate formula. In this way for each different value of $\varnothing$ the disturbance along a curved line is evaluated, and by varying $\varnothing$ from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ the whole field is covered as the line corresponding to the value of $\varnothing$ sweeps across it.

In certain parts of the field the approximation used is not good. This part is omitted from the calculation and the contours of the disturbance are drawn across it smoothly from one side to the other. In the diagrams this part is the neighbourhood of the dotted line. Inside this dotted line there are three values of $\varnothing$ corresponding to each point, and the contributions corresponding to the three values have to be added together. On the dotted line two of the values of $\varnothing$ coincide and it is because of this that the approximation breaks down.

Having established the method there are two problems to be tackled—what happens immediately over the hill, and what happens far downstream.

It is now well known that trains of lee waves are produced in airstreams in which the stability and velocity vary with height in such a way that the quantity l^2, given by

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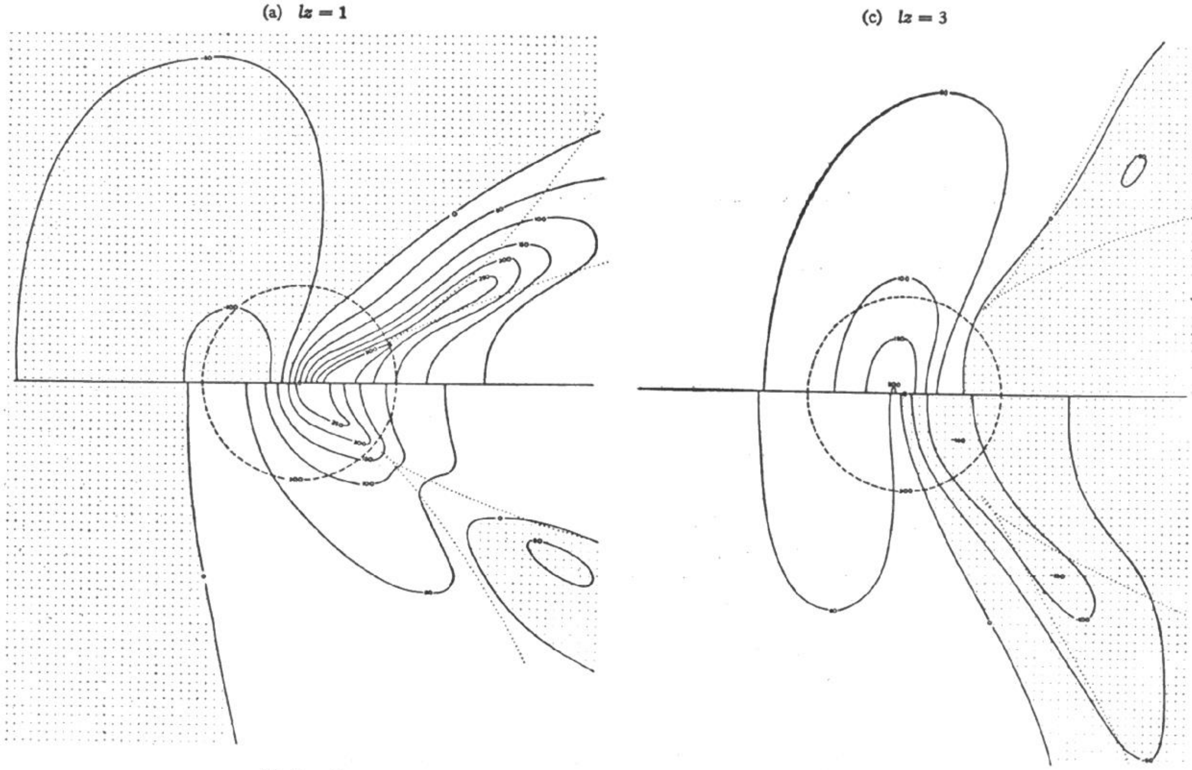
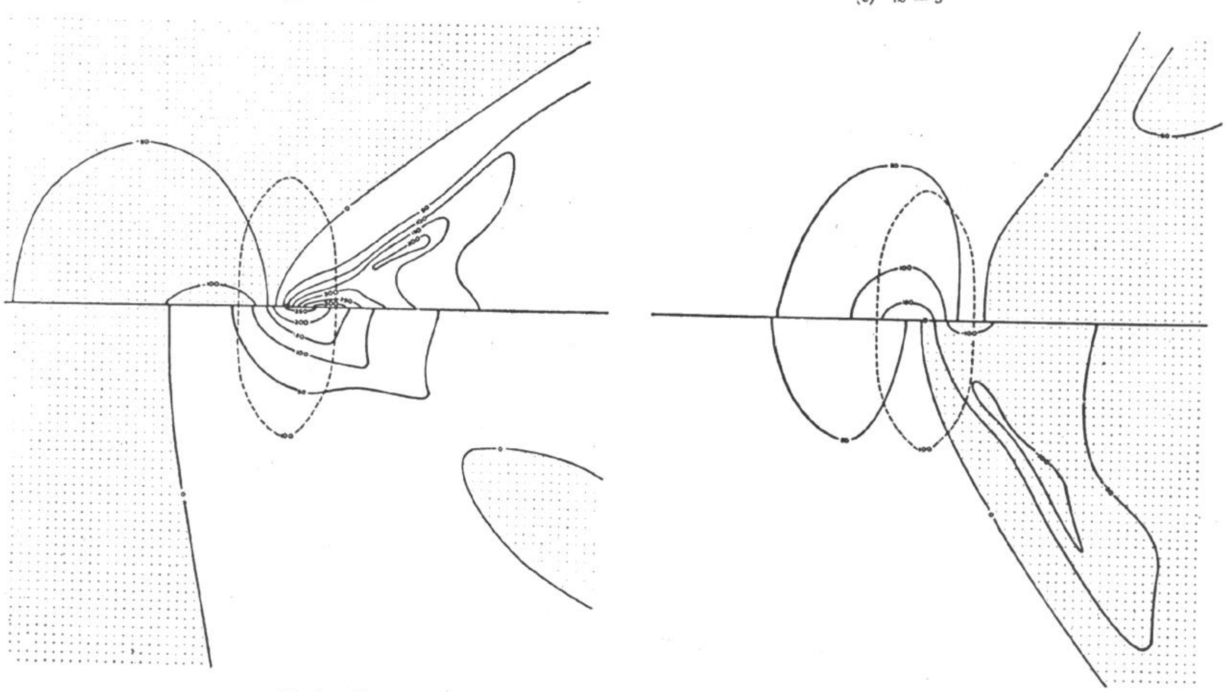
(a) $lz = 1$ (c) $lz = 3$



(b) $lz = 2$ (d) $lz = 4$

Fig. 3. Vertical displacement of air in a uniform stream (const. 1) over a circular hill whose section is shown in fig. 1, for four different values of l_z. Only half of each diagram is shown. One contour of the hill is shown. In the troposphere $1 = 1\ \mathrm{km}^{-1}$ in a typical case and so the diagrams would in that case represent displacements at heights of 1, 2, 3, 4 km

(a) $lz = 1$ (c) $lz = 3$



(b) $lz = 2$ (d) $lz = 4$

Fig. 4. The same as fig. 3 but for an oval hill across the wind. The displacement is nearly as large although the hill is much smaller. One contour of the hill is shown

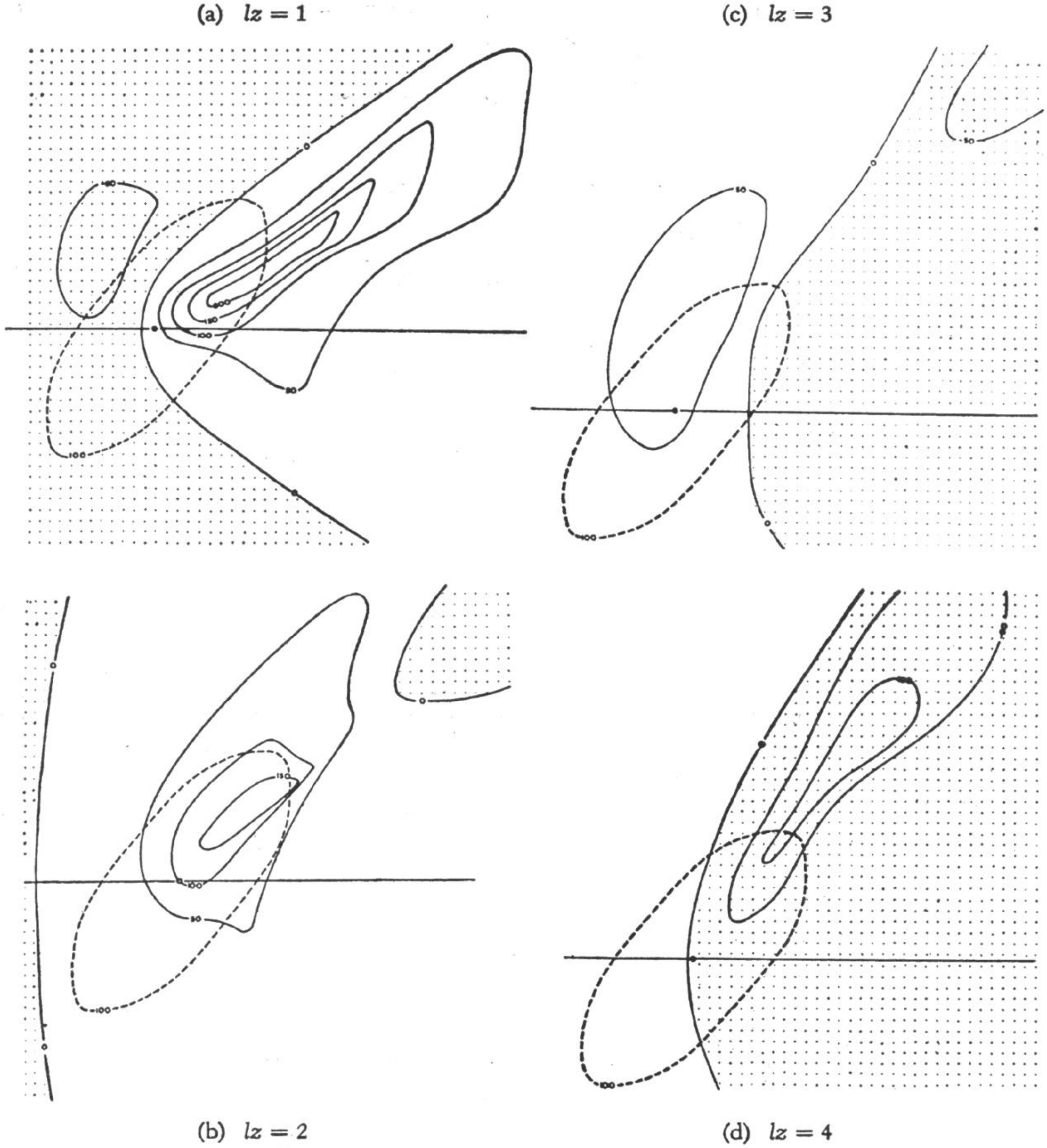


Fig. 5. The same as fig. 4 but with the oval hill turned through 40°. The disturbance is now skew

$$l^2 = \frac{g\,\beta}{U^2} - \frac{U''}{U} \qquad (2)$$

(where $\beta = \dfrac{1}{\Theta}\dfrac{\partial\Theta}{\partial z}$, z = height, Θ = potential temperature, U = undisturbed velocity and $U'' = \partial^2 U/\partial z^2$) has a large value in the lower levels and a smaller value higher up. The possible airstreams that can be chosen are infinite in number.

In order to discover the main features of the results first of all the case of a uniform airstream was taken for the calculation of the disturbance over the hill itself. There are no lee wave trains in this case. The results are shown in fig. 3 for a circular hill, in fig. 4 for the oval hill of fig. 2 lying across the wind (one ground contour shown), in fig. 5 for the oval hill lying at an angle of 40° to the wind and in fig. 6 for oval hill lying along the wind.

The differences according to the inclination of an oval hill to the wind are clearly shown.

Next, in order to study the lee wave pattern produced by an isolated hill, which is analogous to the wave pattern produced by a ship on water, a simple two layer airstream was used. The top layer extends to infinity and has $l = l_s$; the lower layer is of depth h and has $l = l_i$.

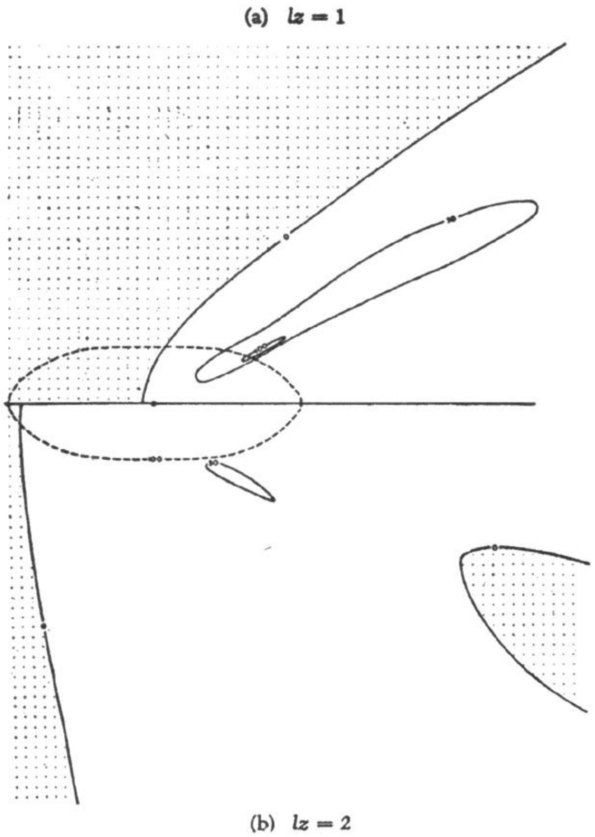


Fig. 6. The same as fig. 3 but with the hill lying along the wind. The disturbance is not shown for $l_z = 3, 4$ because it is very small

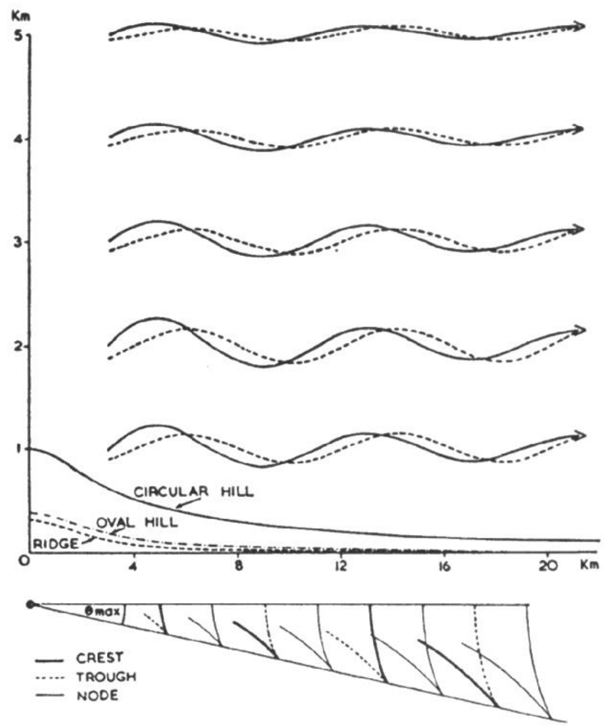


Fig. 8. The streamlines in the central plane $\Theta = 0$, showing the lee-waves of a circular hill (continuous lines) and of a two-dimensional ridge (dashed lines): the latter do not decrease in amplitude downstream. The mountain sections are shown below. The oval hill whose contours are shown in fig. 6 produces the same lee-wave system in this central plane as the circular hill
Below is shown the wave pattern in a horizontal surface for the isolated hills. It lies entirely within the wedge of angle Θ_{max} through the hill centre
The calculations were made for an airstream in which $h = 2$ km, $l_s^2 = 0 \cdot 5 \times 10^{-10}$, $l_i^2 = 1 \cdot 5 \times 10^{-10}$ and $\Theta_{max} \simeq 12 \cdot 3°$
The amplitudes of the mountain and the waves are arbitrary: the height scale applies strictly to the streamlines only

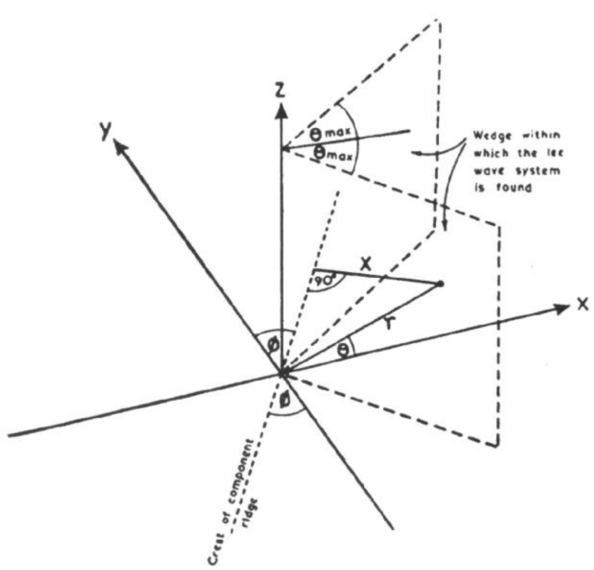


Fig. 7. The system of coordinates used. The origin is under the centre of the hill and the wedge, of half-angle Θ_{max}, within which the lee-wave system lies, is shown.

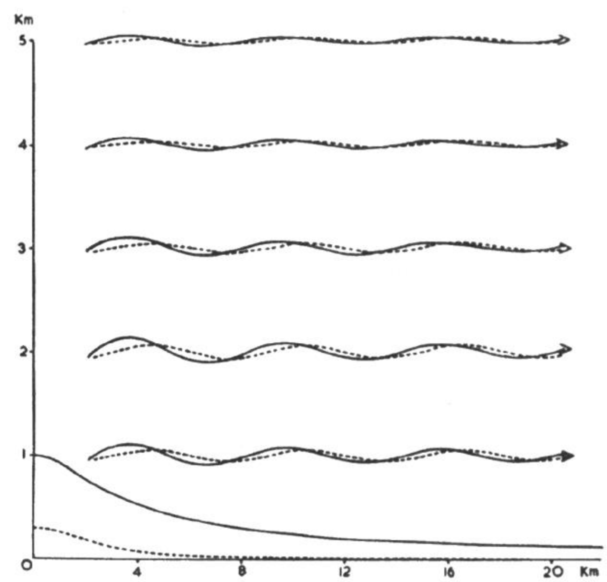


Fig. 9. The lee waves of a circular hill and a two-dimensional ridge for the case $h = 2$ km, $l_s^2 = 10^{-10}$, $l_i^2 = 2 \times 10^{-10}$, for which $\Theta_{max} \simeq 7 \cdot 8°$

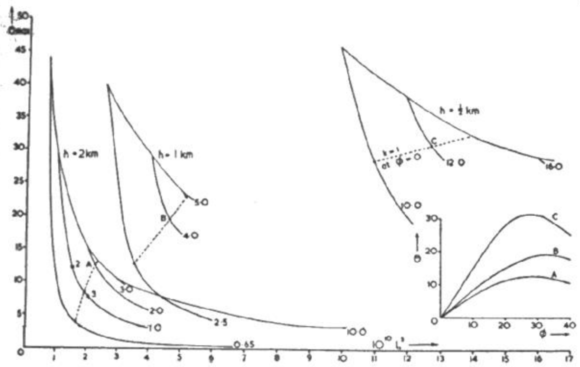


Fig. 10. Values of Θ_{max}, the angle of the wedge containing the lee-wave system, for variations in the airstream. Three groups of curves are shown, for $h = 2$ km, 1 km, $^1/_2$ km. In each of these the curves are drawn for those values of l_i^2 and l_s^2 for which there are lee waves at $\varnothing = 0$, viz. $l_i^2 - l_s^2 > \pi^2/4\,h^2$. The curves are for constant values of $l_i^2 - l_s^2$, and $10^{10}\,(l_i^2 - l_s^2)$ is written against each curve. The abscissa is $10^{10}\,l_i^2$
The dotted lines show the airstreams for which $k = 1$ at $\varnothing = 0$, and the two crosses represent the airstreams of figs. 2 and 3
Inset is a diagram showing how Θ varies as a function of $\varnothing$. It is seen that Θ_{max} corresponds very nearly to $\varnothing = 30°$. The curves in this figure were all calculated for $\varnothing = 30°$, and it was assumed that Θ_{max} was very close to the value of Θ thus obtained. The inset curves correspond to the points A, B, C on the main diagram

In this second case the disturbance over the hill was not computed, but only the interference pattern of the lee wave systems of the component ridges. It is found that outside a wedge of half angle θ_{max} there is no disturbance (see fig. 7). An idea in plan and in vertical section downwind of the hill-top is given in fig. 8 in which sections of the circular and oval hills are shown, and the amplitude of the waves is compared with those in the lee of a two dimensional ridge. In fig. 9 another case is shown.

In all these cases the amplitude of that part of the wave pattern forward from the cusp at the edge of the wave area (where the amplitude is zero) is of negligible amplitude and is unlikely to be ever observed.

In fig. 10 is shown how the angle θ_{max} which the lee wave pattern subtends at the mountain peak varies with different values of l_i, l_s and h. In order to have values of θ_{max} in excess of 15° it is necessary either to have shallow layers of small values of l_i and l_s. 15° seems to be about as wide an angle as is likely to be found in common cases.

The details of the analysis can be found in the papers referred to below.

References

Scorer R. S.: 1956, Airflow over an isolated hill. Quart. J. RR. Met. Soc. *82*, p. 75.

Scorer R. S. and *Wilkinson M.:* 1956, Waves in the lee of an isolated hill. Ibid, *82*, p. 419.