The Rotor Flow in the Lee of Mountains

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Summary

After a description of the rotor phenomenon an attempt is made to explain it by the hypothesis of the "heated pressure jump". The simple hydraulic jump theory, although very attractive in many respects, fails to account for the fact that the height of the roll cloud frequently exceeds that of the cap cloud (= "Föhnmauer") and that the mountain wave generally reaches its most intense state in the early afternoon hours. Measurements made during the Sierra Wave Project in California show the effects of rapid ground heating near the foot of the mountain, thereby reducing the inversion on top of the ground layer through turbulent mixing. This removes part of the gravitational constraint on the air mass bouncing upwards in the hydraulic jump.

Heating rates of 1° C per 2 miles ground path and reduction of the inversion by 50 % are observed. If the jump is moderately high part of the energy is radiated away in a wave system ("undulatory jump"). If the jump is intense, up to 50 % of the kinetic energy are transformed into turbulence ("breaking jump"). A diagram is given from which the height of the rotor can be estimated. The "breaking jump" seems to be identical with the most hazardous type of rotor. High cap cloud and strong heating tend to create dangerous conditions.

This study supports the opinion of many glider pilots that the rotor flow plays a primary roll in the development of the mountain wave in contrast to many existing theories.

Ever since the mountain wave has been discovered observers and pilots have been aware of the fact that the smooth lee wave has a strange and rough companion, the "rotor flow". The majority of the lee wave theories consider this disagreeable fellow as an insignificant byproduct of the lee wave or ignore him altogether. In contrast pilots caught in the grip of the rotor have acquired great respect for his manners and tend to consider rather the smooth lee wave as a good natured companion of the rude rotor. Like Cerberus at the gate of Hades the rotor guards the gates to the smooth wave and a flying intruder venturing unsuspectingly into his range is first being clubbed by an unbelievable turbulence, then dumped in a severe down draft and eventually will be happy to beat a hasty retreat. The results of this study tend to support the pilot's viewpoint of the primary importance of the rotor flow.

Description of the rotor phenomenon

Figures 1 to 6 give an idea of the nature of the "rotor" which was probably first described by Koschmieder (1920). Parallel to the mountain range but a few miles leewards it is visible as a line of cumulus which seems to rotate around its horizontal axis (Fig. 1). The height of this "roll cloud" (also called "rotor cloud" or simply "rotor") is generally of the same order as that of the cloud layer covering the mountain crests ("cap cloud" or "Föhnmauer"). The lenticularis clouds lie just over the top of the roll cloud and there may be more roll clouds under the consecutive wave crests further downwind (Fig. 2). In spite of its heavy turbulence the rotor cloud sometimes looks quite harmless or may even be invisible, but in the more severe cases it is a rather impressive mass of cumulus clouds paralleling the mountain range with all its bends (Fig. 3).

Real flight hazards are indicated when the roll cloud forms a solid wall of formidable height at a considerable distance downwind of the mountain range. In this case details in the structure of the mountain range are not reflected in the rotor which represents one straight barrier

Fig. 1: Rollcloud over Bishop, Cal. Photo: Ovgard

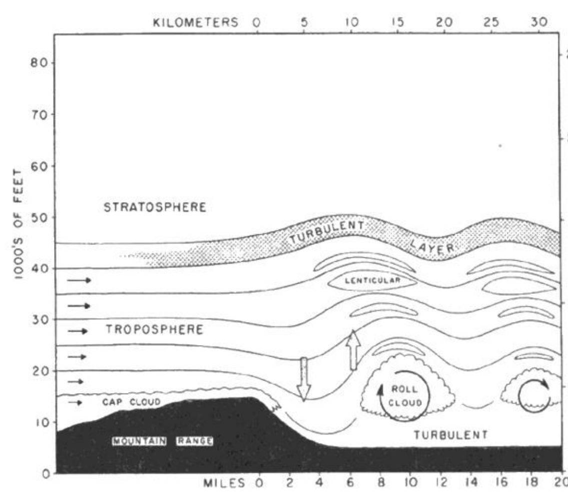

Fig. 2: Schematic diagram of montain wave flow

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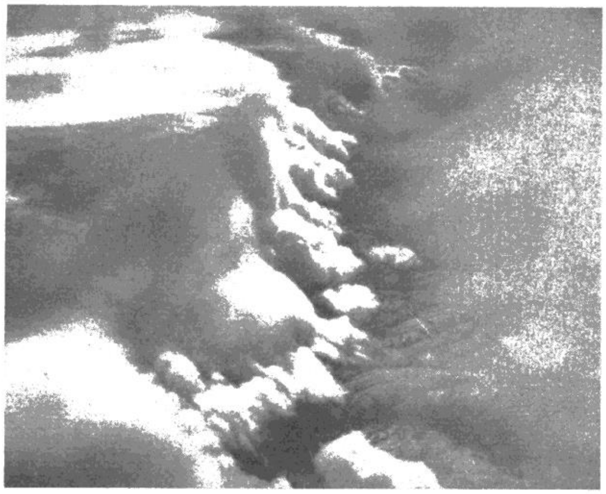

Fig. 3: Airview from 30 000 ft along lee slope of Sierra Nevada showing frontside of rotor. Photo: Henderson, Bishop; California Electric Power Co.

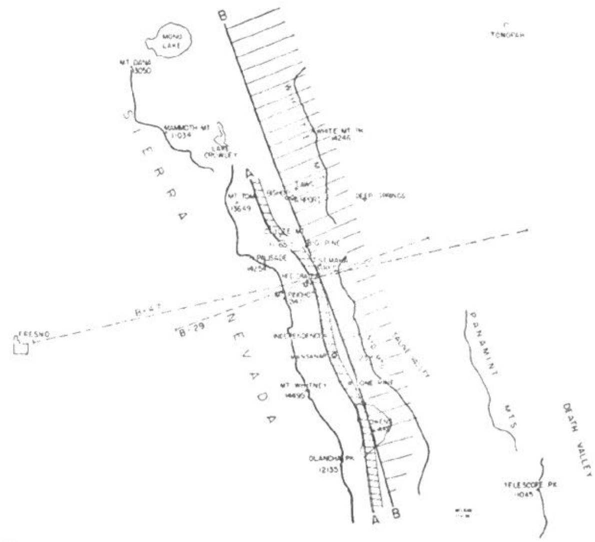

Fig. 4: Normal rotor cloud development in the lee of the Sierra Nevada (line A-A) and severe rotor development (line B-B)

extending sometimes hundreds of kilometers laterally and more than 8 km vertically (Fig. 4). In the afternoon hours its menacing appearance often resembles a thunderstorm-squall line (without precipitation). Fortunately this type is rare. Examples in the Sierra Nevada were the days at which R. Symons soared his P-38 fighter over Bishop to 30,000 feet with feathered props (Fig. 5) or the one when Larry Edgar's Pratt-Read-glider was broken apart by excessive turbulence (Fig. 6). Comparison of these two cases indicates that the severity of the rotor is not simply a matter of humidity, but a more complicated dynamical affair.

Existing Explanations

Numerous theories of the rotor phenomenon have been advanced, some of which may be mentioned here in short terms:

1. Instability of shear-gravity waves at the lower inversion (Kuettner, 1939).
2. Flow separation in the pressure field of the leewave (Lyra, 1943).
3. Hydraulic jump (Knox, 1952; Schweitzer, 1953; Long, 1954; Ball, 1956).
4. Overdeveloped wave in a flow with continuous density gradient (Long, 1955).
5. Karman-Vortex street (Förchtgott, 1955).
6. "Cat's eye effect" at the zero-wind level (Queney, 1955).

We will follow here the general idea of the "hydraulic jump" but with an essential modification which permits us to overcome a basic difficulty of this hypothesis: As mentioned already the rotor frequently reaches heights exceeding those of the cap cloud over the mountain crests. Energy limitations do not permit a simple hydraulic jump to accomplish this feat.

This was one of the reasons why the scientists of the University of California pursuing this idea in the Sierra Wave Project (Knox 1952, 1954; Holmboe and Klieforth, 1957) did not come up with the hydraulic jump as a final answer to the rotor problem. Special measurements conducted on the urging of Prof. J. Bjerknes indicate what kind of mechanism is active. We may call it the "Heated Pressure Jump" ("pressure jump" being the accepted terminology for a "hydraulic jump" in the atmosphere).

In order to explain the heated jump we must first discuss the normal hydraulic jump and its application to the rotor problem. In this connection it may be mentioned that a remarkably clear treatment of the "foehn" as a hydraulic flow has been given by H. Schweitzer (1953) and a very excellent theory of the pressure jump in the lee of mountains by the Australian F. K. Ball (1956).

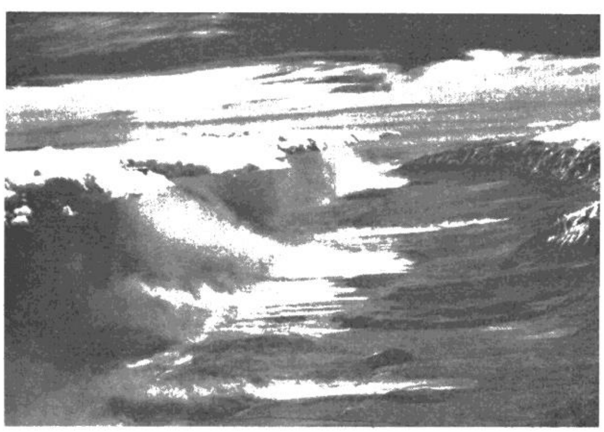

Fig. 5: Severe pressure jump in the lee of the Sierra Nevada
Air photo: R. Symons, Bishop, from 28 000 ft

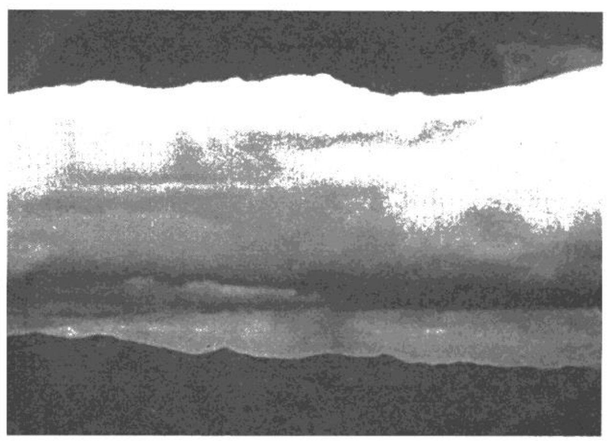

Fig. 6: Powerful rollcloud development near Bishop on 25th April 1955, when severe turbulence destroyed L. Edgar's glider (Air view looking downwind from Bishop)

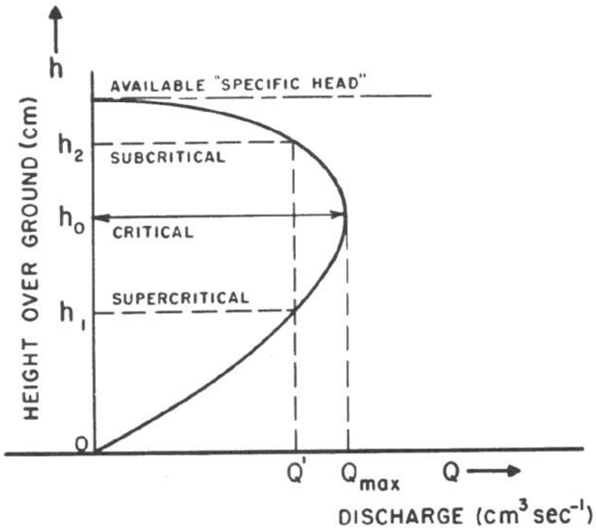

Fig. 7: Discharge diagram. (Explanation in text)

The Hydraulic Jump

Under wave conditions the cap cloud (or "Föhnmauer") makes visible the cold air mass which spills over the mountain range like water over a weir. In a water channel the hydraulic jump forms at the foot of the weir where the water arrives with a high velocity in a "shooting flow". Jumping back to some higher surface level it not only slows down but looses a considerable portion of its energy in turbulence. Down stream of the jump the flow is "tranquil" again as it was upstream of the weir.

To understand what is going on in the hydraulic jump we may visualize a large water reservoir with a sluice gate regulating the water discharge. If the gate is gradually raised, more and more water will discharge, but beyond a certain height of the gate further raising will have no influence upon the discharge. (In fact if it were possible to increase the water depth in the gate further, less water would pass.) Thus, at a "critical height" the outflow reaches a maximum. This height establishes itself automatically if the gate is removed and equals $^2/_3$ of the original height (over the bottom of the gate) inside the reservoir.

This peculiar behaviour is a consequence of two conditions which have to be fulfilled in the discharge. First, the continuity equation requires, once a steady state is reached, that the same amount of water per second flows through every cross section, i. e. that the discharge

$$Q = vh = \text{constant} \tag{1}$$

(v = mean flow velocity, h = height of water surface over channel floor)

Secondly the total available energy E remains conserved along streamlines during the outflow, i. e. the potential energy given by the height h^* (over the channel floor) of the free surface inside the reservoir determines the kinetic

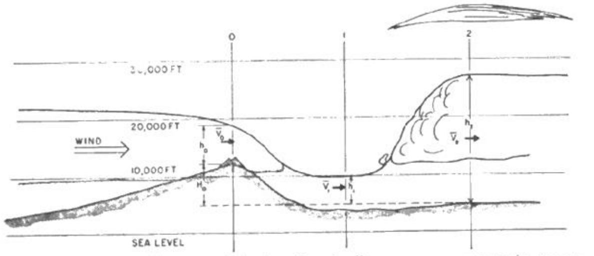

Fig. 8: Schematic diagram of hydraulic air flow over a mountain range. (See text)

Fig. 9: Cloud water fall as seen from Puy-de-Dôme observatory

and potential energy in the discharge. If frictional losses are neglected and streamline flow exists the total energy

$$E = \frac{v^2}{2} + gh = gh^* = \text{constant} \tag{2}$$

(g = gravity. This is Bernoulli's equation along the surface where the pressure vanishes.)

Combining (1) and (2) we find that the discharge has its maximum at a "critical height" h_0, where the velocity at the gate reaches the critical value

$$v_0 = \sqrt{gh_0}. \tag{3}$$

(See discharge diagram, Fig. 7. In hydraulic terminology h^* is called the "specific head".)

The critical velocity has the significance that it is also the well known maximal propagation speed of long surface waves. As a consequence no "swell" of the surface can travel upstream through the gate, once critical velocity has been reached there. This is of importance if the channel floor slopes down outside the sluice gate. Gravity will then accelerate the stream to supercritical "shooting flow", and an eventual return to subcritical "tranquil flow" farther down stream cannot penetrate upstream, but remains fixed at the foot of the slope.

This brings us sufficiently close to the airflow over mountains and we may now look at Fig. 8 which depicts the atmospheric conditions.

In the "reservoir" far to the left the air mass may have the height h^* above the mountain crest. If there were a gate at the crest which could be raised gradually the airflow would reach critical height and velocity at the gate. In practice the air is already streaming towards the reservoir and the mountain barrier under the influence of the large scale pressure field and the critical height h_0 over the crest will be a little more than $^2/_3$ h^*. Since we have no free surface but an interface between a colder and warmer

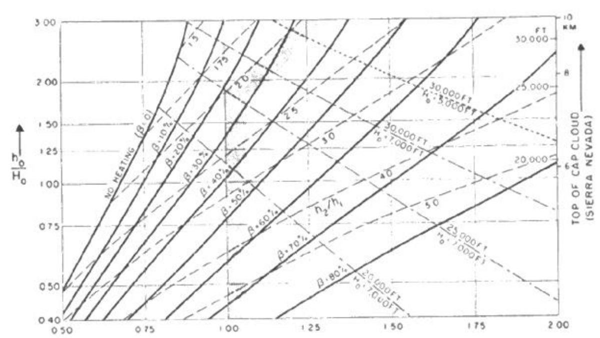

Fig. 10: Nomogram to determine the height of a «heated pressure jump». (Explanation in text)

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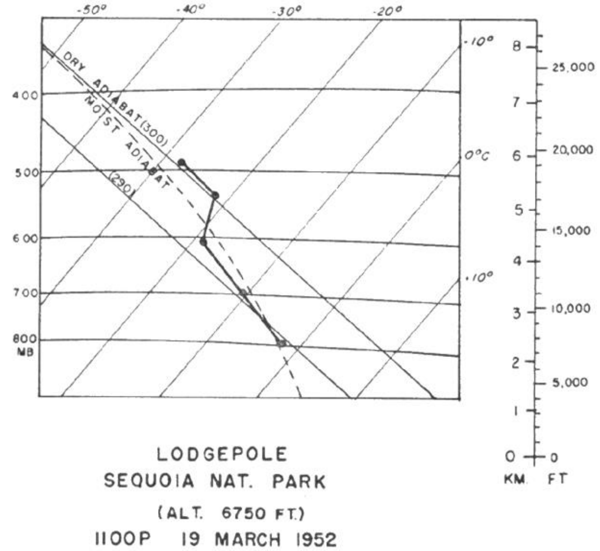

Fig. 11: Temperature sounding over Sierra Nevada of 19th March 1952 showing an inversion layer of 8° potential temperature

air mass the density on top of the flow is not zero but only slightly reduced according to the temperature inversion ΔT of the two air masses. Actually the inversion is rarely a sharp discontinuity, but a finite layer, see Fig. 11, and it is therefore better to take the difference in *potential* temperature $\Delta\Theta$. This reduces the gravitational force acting on the interface to the "*modified gravity*".

$$\gamma = \frac{\Delta\Theta}{\Theta} g \tag{4}$$

which is not a constant any more and may be 50 times smaller than gravity. As a consequence the critical velocity over the mountain crest, as defined by (3) is now reduced to

$$v_0 = \sqrt{\gamma h_0} \tag{5}$$

From now on we replace g by γ and continue to treat the airflow like water[1].

We now want to know what happens to the flow pouring down the mountain slope from Section 0 to Section 1, see Fig. 8. Assuming frictionless streamline flow we may again apply Bernoulli's *energy equation* for the top streamline and compare the two cross sections 0 and 1. If H_0 is

[1] This procedure is precise only if the upper layer were infinite or at rest and if it had a constant potential temperature.

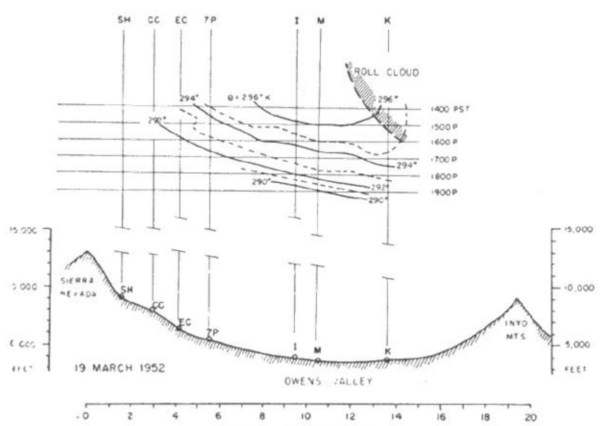

Fig. 12: Potential temperature field and rollcloud position (upper part) measured by mobile weather station over the Owens valley (lower part), after Holmboe and Klieforth, 1957

the height of the mountain over the valley floor we have

$$\gamma (H_0 + h_0 - h_1) = \frac{1}{2} (v_1^2 - v_0^2) \tag{6}$$

In view of the constant discharge Q defined in (1) and the critical velocity v_0 defined in (5) we arrive at a cubic equation for the depths of the flow of the two cross sections:

$$\left(\frac{h_0}{h_1}\right)^3 - \left(2\frac{H}{h_0} + 3\right)\frac{h_0}{h_1} + 2 = 0 \tag{7}$$

Remarkably enough gravity γ and discharge rate Q have vanished and the only parameter of importance left is the depth h_0 of the flow over the mountain crest (cap cloud) in comparison to the free slope of the mountain, H_0. This explains why the flow of clouds through mountain passes looks precisely like a true river, Fig. 9 (except for the velocity which cannot be judged from a still photo).

Solution of (7) shows that the depth shrinks by about 50 % and the velocity doubles if $h_0 = 0.4\,H_0$. Equations (1), (5) and (7) indicate that *the most violent winds in the valley must be expected if the "Föhnmauer" (cap cloud) over the mountain is tall and the inversion strong.* This appears to be the basic mechanism of "*foehn*" and "*bora*".

We are now interested in finding out what happens farther downstream, at Section 2 (Fig. 8). As evident from the discharge diagram, Fig. 7, there are always two possibilities to accomplish a given discharge Q, namely a subcritical and a supercritical one. Supercritical or "shooting flow" is never maintained for long in nature, since friction and the undisturbed environment tend towards the subcritical alternative of "tranquil flow". As a consequence we must expect that the flow in the valley raises its depth from h_1 to the corresponding level h_2, see Fig. 7 and 8. Now, if this happens, waves can travel upstream in the subcritical area, near h_2, while in the supercritical area near h_1, they cannot. The wave front will therefore steepen until it breaks between Section 1 and 2. In this way the *hydraulic jump* is formed which becomes stationary over ground.

As much of the linear kinetic energy of the shooting flow is transformed into rotational kinetic energy of turbulence (and eventually into heat) the streamline flow is destroyed by violent mixing and the Bernoulli equation does not hold anymore in the jump area. But the "*momentum equation*" still holds which states that the net pressure force acting on the vertical cross sections to the right and left of the jump equals the rate of momentum change:

$$\frac{\gamma}{2}\ (h_2^2 - h_1^2) = h_1 v_1^2 - h_2 v_2^2 = Q\,(v_1 - v_2) \tag{8}$$

From this relation we can find the height of the jump h_2 in comparison to the depth of the flow upstream of the jump, h_1. It is given by the simple quadratic equation

$$\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2\,F_1^2 = 0. \tag{9}$$

F_1 ist the "Froude number" which expresses the degree to which the flow is supercritical at Section 1:

$$F_1^2 = v_1^2 / \gamma h_1 \tag{10}$$

where γh_1 is the square of the critical velocity at height h_1, as defined in (5). Solution of (9) is the well known hydraulic jump equation

$$\frac{h_2}{h_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2\,F_1^2} \approx F_1\sqrt{2} \tag{11}$$

(for sufficiently large Froude numbers).

To compare the height of the rotor, h_2, with the height of the cap cloud $(H_0 + h_0)$ we have to link equations (11) and (7). This can easily be done through the Froude number F_1 if the "shooting flow" velocity v_1 is expressed in terms of Q by (1) and Q in terms of h_0 by (5). It then turns out that

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$$F_1^2 = (h_0/h_1)^3 \quad (12)$$

Connecting in this way the air flow conditions over the mountains (Section 0) with those in the rotor region (Section 2) we find that the height of the hydraulic jump h_2 can *never* exceed the height of the flow over the barrier $(H_0 + h_0)$. This relation is plotted in Fig. 10 (curve to the left labelled "no heating"). It is quite independent of the slope characteristics, the inversion, the flow velocity etc. The deeper the flow over the mountain, the higher the rotor, but even the rather extreme case where the cap cloud is double as high as the mountain $(h_0/H_0 = 1)$, yields only a jump h_2 of less than $^3/_4$ the cap cloud height $(h_0 + H_0)$. Whe thus arrive at an interesting conclusion: In spite of many similarities (spill over, shooting flow, jump, severe turbulence) the simple pressure jump hypothesis does not explain the remarkable height of the roll cloud which so frequently exceeds the cap cloud. The observations during the Sierra Wave Project suggest that a mechanism is at work which we may call the "heated pressure jump".

The Heated Pressure Jump

Full development of lee wave activity is generally not reached before the early afternoon, much to the dismay of glider pilots who want to fly cross country during the short winterly daylight hours. The "4 o'clock wave" has become notorious among the Sierra-Wave-Project-pilots. Their usual conclusion that thermal activity causes the swelling of the roll cloud is quite close to the point although the mechanism involved is a bit more complicated.

Continuous cruises with a mobile weather station across the Owens Valley and the Sierra slopes have revealed that during the early afternoon hours the air coming down the mountains is heated in the valley floor by about 1° potential temperature every 2 miles of its path (Holmboe and Klieforth, 1957). Fig. 12 indicates a total heating of 4° on an 8 mile stretch from the foot of the mountain to the front side of the rotor during the hours between 14 and 16 o'clock. In this case (19 March 1952) the mountain wave was very powerful with a rotor top of almost 25,000 ft. (Glider flights of over 13 km height and 600 km distance were accomplished.) Since mixing is very violent in the rotor zone (making it almost impossible to trace clear streamlines along the potential isotherms, Knox, 1952), the heat will be distributed throughout the layer so as to reduce the inversion. Fig. 11 shows a radiosounding from the upwind side of the Sierras at 11 hours of the same day. The inversion amounted there to about 8° in potential temperature (generally it is even stronger). We must therefore expect a 50 % reduction of the inversion on the leeside of the mountain. This will allow the hydraulic jump to bounce back to far greater heights, as the gravitational constraint on the inversion surface is eased.

Comparing cross sections 1 and 2 (Fig. 8) the modified gravity changes from

$$\gamma_1 = \frac{\Delta\,\Theta_1}{\Theta_1}\,g \text{ to } \gamma_2 = \frac{\Delta\,\Theta_2}{\Theta_2}\,g_1 \quad (13)$$

which, if introduced into (8) leads to the cubic equation

$$\left(\frac{h_2}{h_1}\right)^3 - \frac{\gamma_1}{\gamma_2}\left(1 + 2\,F_1^2\right)\frac{h_2}{h_1} + 2\,\frac{\gamma_1}{\gamma_2}\,F_1^2 = 0 \quad (14)$$

instead of (9). Connection to the upstream conditions is again given by the Froude number, equation (12) and (7). The result is startling. Solutions for different heating rates $\beta = \frac{\gamma_1 - \gamma_2}{\gamma_1}(\%)$ are plotted in Fig. 10. If 50 % of the inversion are destroyed (as on the 19 March 1952) the rotor is lifted by over 50 % and exceeds the height of the cap cloud provided the flow over the mountain is deep enough $\left(h_0 > \frac{3}{4}H_0\right)$

The use of Fig. 10 may be illustrated by the following example which fits the case of 19 March 1952. The height h_0 of the cap cloud over the mountain crest is the same as that of the mountain crest H_0 over the valley, i. e. $h_0/H_0 = 1$ (vertical scale on the left). The heating may destroy 50 % of the inversion so that we have to follow the line $\beta = 50\,\%$ until it meets the horizontal line $h_0/H_0 = 1$. We then read vertically down and find, on the horizontal scale $(h_2/H_0 + h_0) = 1.1$. In other words, the height h_2 of the rotor exceeds the total height $(H_0 + h_0)$ of the cap cloud over the valley floor by 10 %. Since the Sierra Nevada, south of Bishop, is about 13,000 feet high and the valley floor 4,000 feet, the cap cloud should be in this case at 22,000 feet and the rotor at 24,000 feet m. s. l. (while without heating, it would not exceed 17,000 feet). This is very close to the conditions observed in the afternoon of the 19 March 1952 (Kuettner, 1953).

Since the height of the mountain range varies, we have also plotted on Fig. 10 the height of the rotor over sea level for $H_0 = 7000$ ft. and $H_0 = 5000$ ft. (dash-dotted lines) assuming a mean-valley floor of 5000 ft.

Also plotted are the relative heights of the hydraulic jump h_2/h_1 (dashed lines) for the following reason: If the jump is small or moderate much of the released energy is radiated away in a wave system downstream of the jump ("undulatory jump"). If the jump is high $(h_2/h_1 > 2)$ most of the released energy goes into turbulence ("breaking jump"). As much as 50 % of the kinetic energy may be transformed into turbulence in this way.

We have therefore shaded the *"dangerous area"* of the rotor flow which is defined by $(h_2/h_1) > 2$ (breaking jump) and $(h_2/H_0 + h_0) > 1$ (rotor height exceeding cap cloud height). The danger is maximized by strong heating rates β. The shallower the flow, the stronger the heating required.

For deep flows rotor heights of 30,000 ft. (9 km) may be reached if the heating rate exceeds 75 %. This may be the reason why the severe rotor formations are found way back in the valley (fig. 4—6) allowing the flow to heat strongly along the valley ground (which in the Sierra Nevada, consists of hot, sunny desert).

Finally an energy consideration may show what a powerful agent this heating process is. Kinetic energy if transformed into heat creates the following temperature increase:

$$\Delta T = \varepsilon v^2/2\,c_p \quad (15)$$

where ε = heat equivalent of mechanical energy

c_p = specific heat of air at constant pressure.

If in a breaking jump, 50 % of the kinetic energy is transformed through turbulence into heat, a foehn storm of 100 km/h velocity would raise the temperature by only $^1/_4$° C. The heat added at the ground in front of the rotor is therefore a multiple of the kinetic energy which is responsible for the normal hydraulic jump. This confirms the opinion of glider pilots that thermal growth of the roll cloud intensifies the mountain wave in the afternoon.

The theory outlined here tends to support the idea first advanced by Schweitzer (1953) that the "foehn" is a hydraulic phenomenon with supercritical flow.

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Résumé

Après avoir décrit le phénomène du rotor, l'auteur essaye de l'expliquer par l'hypothèse d'un «seuil de pression thermique». La seule théorie du seuil hydraulique, quoique très plaisante à maints égards, ne permet pas d'expliquer le fait que l'altitude du nuage de rotor dépasse souvent celle de la cape nuageuse (mur de fœhn) et que l'onde de relief atteint son intensité maximum dans les premières heures de l'après-midi. Des mesures faites pendant le «Sierra Wave Project» en Californie montrent les effets du rapide réchauffement du sol au voisinage du pied de la montagne, qui provoque une réduction de l'inversion à la partie supérieure de la couche au sol grâce à un effet de mélange dû à la turbulence. Cela supprime une part de la contrainte gravitationnelle sur les masses d'air projetées vers le haut sous l'effet du seuil hydraulique.

Des réchauffements atteignant 1° sur deux milles de trajectoire au sol et une réduction de l'inversion de 50 % sont observés. Si le seuil est de hauteur modérée, une partie de l'énergie est utilisée par le système d'onde («seuil ondulatoire»). Si le seuil est très marqué, jusqu'à 50 % de l'énergie cinétique se transforme en turbulence («seuil de rupture»). Un diagramme est reproduit permettant d'estimer la hauteur du rotor. Le «seuil de rupture» semble s'identifier avec le type de rotor le plus dangereux. Un mur de fœhn élevé et un fort échauffement tendent à créer des conditions dangereuses.

Cette étude appuye l'opinion de beaucoup de pilotes de planeurs que la circulation dans le rotor joue un rôle primordial dans le développement de l'onde de relief malgré ce que prétendent beaucoup de théories existantes.

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