ESTIMATION OF THE ISOBAR PATTERN ON SWEPT WINGS

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I. INTRODUCTION

Both experiments and theory (1, 2) indicate that in a three-dimensional flow near the centre of a swept-back wing, the point of minimum pressure is shifted backwards from the leading edge.

Hence the local lift in the front of the centre section is reduced compared with that of the flat-plate, while the local lift on the rear part has a somewhat higher value.

The maximum local velocity of the flow across the centre section is not necessarily higher compared with that of the sections located at greater distances from the kink.

The available methods for estimation of the velocity distribution in centre section of swept wings are restricted to the case of zero lift.

This note represents an attempt to illustrate a way for determination of an approximate velocity distribution and the isobar pattern in the region of the plane of symmetry including the cases of non zero lift.

Simple expressions are given for calculating the chordwise velocity distributions both for the centre section and the sections at greater spanwise distances from the kink.

Assuming a parabolic spanwise variation of isobars in the region adjacent to the plane of symmetry, a method that can be used to predict the isobar pattern is briefly demonstrated for two particular swept-back and swept-forward constant chord wings with symmetrical airfoils for two values of the local lift.

Since the resultant isobar pattern obtained in this way is only approximate and of empirical character, a firmer theoretical basis is needed.

II. VELOCITY DISTRIBUTION ON A SECTION OF SWEPT WING AT A GREATER DISTANCE FROM THE KINK

It is known (3) that the velocity distribution about a symmetrical airfoil can be considered as composed of two separate and independent components:

$$\frac{V}{V_o} = \frac{v}{V_o} \pm c_1 \frac{\Delta v_a}{V_o} \tag{1}$$

where:

- V - is the local velocity at the wing surface;
- V_o - is the free stream velocity;
- v/V_o - is the velocity - increment ratio corresponding to the velocity distribution over the basic thickness form at zero angle of attack;
- $\Delta v_a/V_o$ - is the velocity - increment ratio corresponding to the additional load distribution associated with angle of attack;
- c_1 - is the lift coefficient for which the velocity or pressure distribution is to be determined.

In the expression (1), the sign (+) corresponds to the upper surface, and the sign (-) to the lower surface of the wing.

For NACA airfoils, the values of ratios (v/V_o) and $(\Delta v_a/V_o)$ are given in reference (3). For other airfoils, these values may be determined using the method exposed in reference (4).

Let us now consider an infinite sheared wing, i.e., a wing of constant chord set into an oblique position with respect to the direction of flight, so that its both leading and trailing edges make an angle (φ) with the normal to the flight direction.

The flow around a section of sheared wing parallel to the flight direction will be similar to that around a section of a swept wing of infinite aspect ratio and constant chord, but at a greater distance from the plane of symmetry.

Though the velocity distribution at such a section is in reality resulting from a three-dimensional flow, it can be determined by means of a two-dimensional method.

The chordwise velocity distribution can be represented by the relation:

$$\frac{V\ (x\ \infty)}{V_o} = 1 + \cos\varphi\ \left(\frac{v}{V_o} \pm c_z \cdot \frac{\Delta\ v_a}{V_o} - 1\right) \quad \ldots\ldots (2)$$

where:

φ - is the angle of sweep;

x - is the chordwise coordinate and

$y = \infty$ - is the spanwise coordinate.

In this formula, the velocity-increment ratios (v/V_o) and ($\Delta\ v_a/V_o$) correspond to that of an infinite non sheared wing, i.e., straight wing, and have thus the same values as in expression (1).

III. VELOCITY DISTRIBUTION AROUND THE CENTRE SECTION OF A SWEPT WING

According to the results given in reference (1), the chordwise velocity distribution around the centre section ($y = 0$) of a swept wing of infinite aspect ratio and at zero incidence can be calculated from the following expression:

$$\frac{v\ (x\ o)}{V_o} = 1 + \cos\varphi\ \left(\frac{v}{V_o} - 1\right) - f\ (\varphi)\ .\ F'\ (x) \quad \ldots\ldots (3)$$

where:

F' (x) - denotes the first derivative of the function

$$z = F\ (x) \quad \ldots\ldots (4)$$

which determines the upper boundary of the wing profile, and

$$f\ (\varphi) = \frac{\cos\varphi}{\pi}\ .\ \ln\ \frac{1 + \sin\varphi}{1 - \sin\varphi} \quad \ldots\ldots (5)$$

is a function of the angle of sweep (Table I, see the next page).

The velocity distribution at the kink in the zero lift case may therefore be determinded immediately from the results obtained for a given section in two-dimensional flow.

In reference (2) it is shown that the chordwise lift distribution at the centre of a thin swept-back wing differs, in the first approximation, from the ordinary flat-plate lift distribution only in the exponent which instead of ($\frac{1}{2}$) becomes $[(\frac{1}{2}) - (\varphi/\pi)]$.

It is suggested that this change of the exponent is more appropriate for the centre section of swept wings, and that it may replace the ordinary flat-plate distribution in all those cases when the lifting surface is swept.

Combining this result with the method exposed in reference (4), the following general expression for calculating the chordwise velocity distribution around the centre section of a swept wing may be obtained:

$$\frac{V\ (x\ o)}{V_o} = \frac{v\ (x\ o)}{V_o} \pm c_z\ .\ g\ (\varphi)\ .\ \frac{\Delta\ v_a\ (x\ o)}{V_o} \quad \ldots\ldots (6)$$

where the first term at right side is given by the relation (3), and

c_1 - is local lift coefficient on a section at greater spanwise distance from the plane of symmetry.

The sign (+) corresponds to the upper surface and (-) to the lower surface of the wing.

The function g (φ) represents the kink effect, and is used in the above formula as a factor to obtain the corresponding local lift coefficient in the centre section.

This factor is defined as follows:

$$g(\varphi) = \frac{\int_0^1 \left[\frac{1 - \frac{x}{z}}{\frac{x}{z}}\right]^{\left(\frac{1}{2} - \frac{\varphi}{\pi}\right)} d\left(\frac{x}{z}\right)}{\int_0^1 \left[\frac{1 - \frac{x}{z}}{\frac{x}{z}}\right]^{\frac{1}{2}} d\left(\frac{x}{z}\right)} \quad \ldots\ldots (7)$$

z - being the chord of the airfoil.

The last term in the expression (6) is given by:

$$\frac{\Delta v_a \,(x\,o)}{V_o} = \frac{A}{4\,B} \quad \ldots\ldots (3)$$

where:

$$A = \frac{2}{\pi} \left[\frac{1 - \frac{x}{z}}{\frac{x}{z}}\right]^{\left(\frac{1}{2} - \frac{\varphi}{\pi}\right)} \quad \ldots\ldots (9)$$

and

$$B = \int_0^1 \frac{v\,(x\,o)}{V_o} \,.\, A \,.\, d\left(\frac{x}{z}\right) \quad \ldots\ldots (10)$$

The evaluation of the integral in numerator of the expression (7) requires some special care. In order to avoid the infinite values of the integrand at the leading edge, for $(x = 0)$, an approximate solution is obtained by integrating grafically from:

$$\frac{x}{z} = 0{,}025 \quad \text{to} \quad \frac{x}{z} = 1$$

In table I are summerized the calculated values of the kink effect factors f (φ) and g (φ) for different angles of sweep.

TABLE I

φ	0^o	10^o	20^o	30^o	40^o	50^o	60^o
f (φ)	0	0,1100	0,2132	0,3029	0,3721	0,4136	0,4192
g (φ)	1,0	0,9434	0,3955	0,8561	0,8247	0,8000	0,7823
φ		-10^o	-20^o	-30^o	-40^o	-50^o	-60^o
f (φ)		-0,1100	-0,2132	-0,3029	-0,3721	-0,4136	-0,4192
g (φ)		1,0680	1,1503	1,2403	1,3493	1,4762	1,6238

IV. VELOCITY DISTRIBUTION OVER THE WING SURFACE AND THE ISOBAR PATTERN PLOTTING

By means of the expressions (2) and (6), the chordwise velocity distributions on a section at greater spanwise distance from the kink and in centre section of a swept wing of constant chord may be calculated, for various prea sumed values of the local lift coefficient. In order to obtain approximate velocity distribution (or pressure distribution, which is the same, since the two are strictly connected by BERNOULLI's equation) over the surface of the wing, the following simple method is adapted.

It is known that in the centre section, the isobars or lines of equal pressure must be at right angles to the direction of flight. Assuming now a parabolic spanwise variation of the isobars in the region of the kink, the lines of equal pressure or "pressure contours" may be plotted.

The series of figures which follow show the results of calculations for two infinite swept-back and swept-forward wings of constant chord, having both NACA 0012-64 airfoil and a 35^{o} angle of sweep.

Figures 1 and 2 (see the next page) show the resulting isobar pattern in the case of zero lift while figures 3 and 4 (see page) represent the same for a definite value of the local lift coefficient ($c_l = 0.25$).

On these figures, the kink effect can be seen clearly and at some distance from the centre the isobars become almost parallel to the edges.

In view of the difficulties of the three-dimensional flow, it is evident that the results of calculations and the method of plotting the isobar pattern are only very approximative.

Further experimental investigations are needed to give a more detailed information applicable in practical cases.

V. REFERENCES

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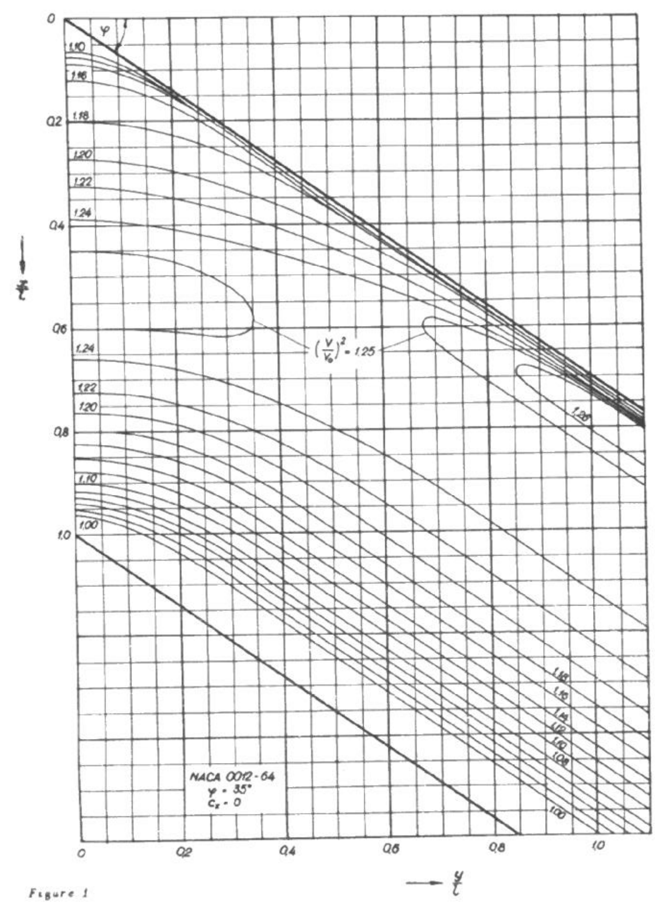

Figure 1

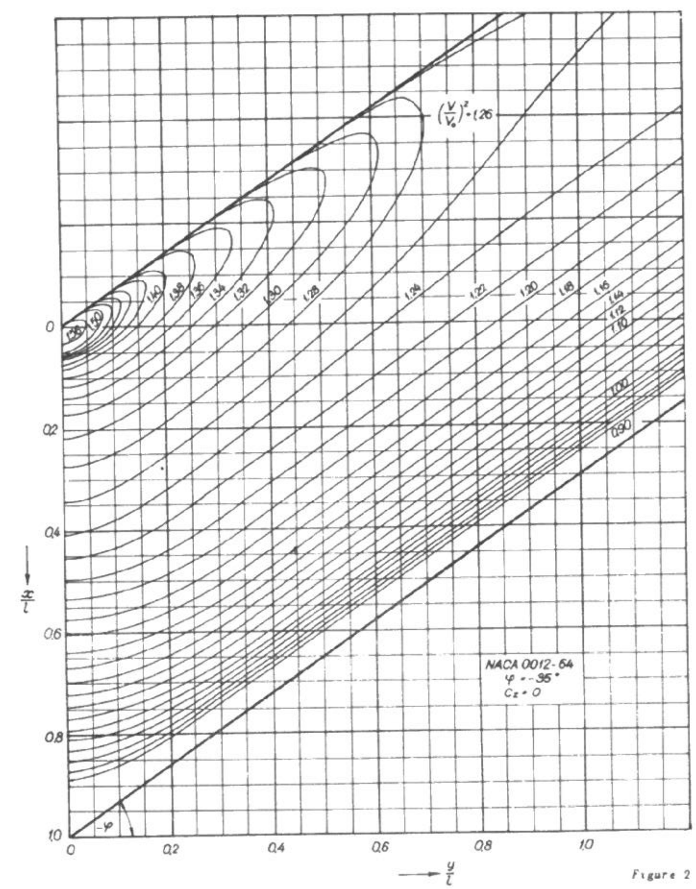

Figure 2

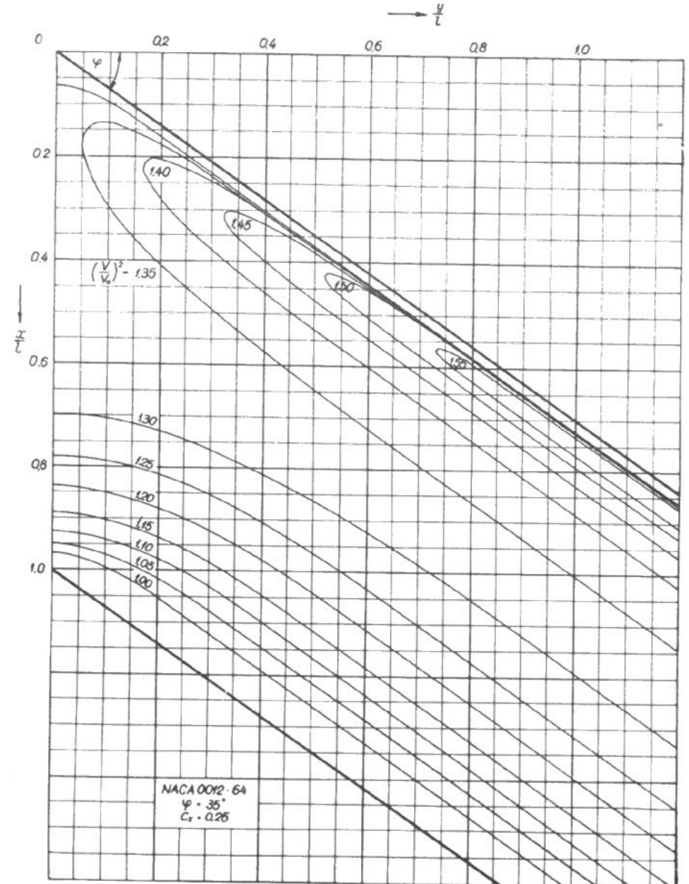

Figure 3

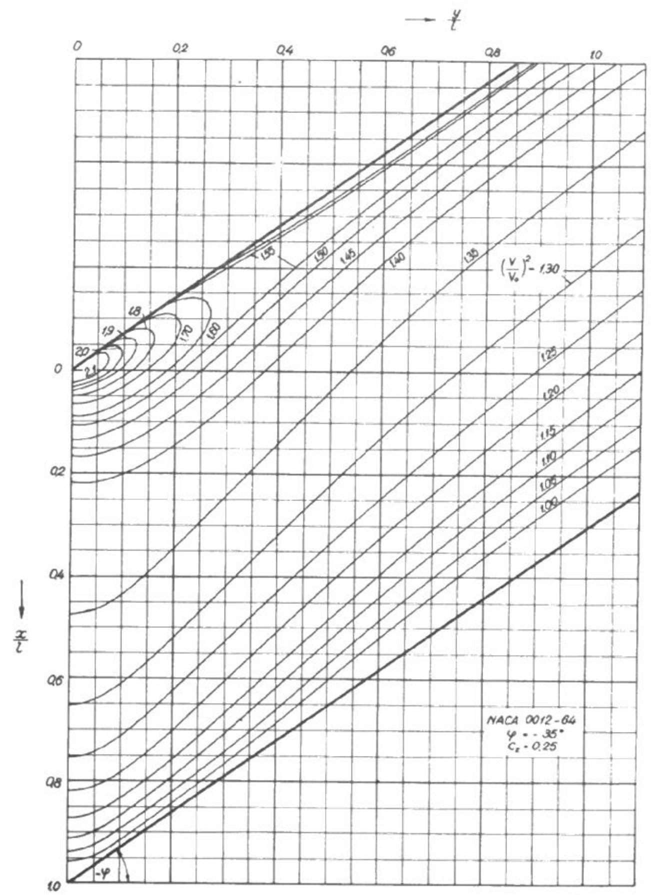

Figure 4