MEASUREMENTS OF LATERAL CONTROL CHARACTERISTICS.

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In the terminology of sailplane testpilots terms like: the plane handles well on .... or responds quickly to .... are often used. Pilots feel the controls of one sailplane as sufficient and the controls of another as lacking in quick response, but the author also encountered an absurdum, namely that the feeling on one and the same type was very different and influenced through the general opinion. Only a method, giving numerical results could show the rightness or error of pilot's feelings.

The control power of lateral controls can be measured easily only in steady flightconditions like steady turns, as acceleration-measurements are difficult and costly and the moments of inertia of the sailplane usually not known. Lateral control characteristics are $C_{l_{\delta a}}$, $C_{n_{\delta a}}$, $C_{l_{\delta r}}$, $C_{n_{\delta r}}$ determining rolling and yawing due to aileron and rudder deflections. Rolling due to rudder deflection is very small and can be neglected.

Equations, determining steady motions, are:

$$C_{y\beta}\cdot\beta - 2\,d\psi + C_L\cdot\Phi = \theta \quad \text{(1)}$$

$$\mu\cdot C_{l\beta}\cdot\beta + \frac{C_{lr}}{2}\,d\psi + \mu\cdot C_{l_{\delta a}}\cdot\delta a = \theta \quad \text{(2)}$$

$$\mu\cdot C_{n\beta}\cdot\beta + \frac{C_{nr}}{2}\cdot d\psi + \mu(C_{n_{\delta a}}\cdot\delta a + C_{n_{\delta r}}\cdot\delta r) = \theta \quad \text{(3)}$$

$$\frac{C_{lr}}{2}\cdot d\psi + \frac{C_{lp}}{2}\cdot d\Phi - \mu\cdot C_{l_{\delta a}}\cdot\delta a = \theta \quad \text{(4)}$$

The first three equations determine steady circling, the last one the steady rolling motion. Equation 1 is of no interest, as it gives only the connection of the angle of bank versus turning and flight speed. The equations are a system of homogeneous equations and an infinite number of results can be obtained, or exactly, results can be given only as ratios.

The control characteristics alone do not govern the movements about their axes as qualities of the sailplane do interfere, so the results of pilot's feelings are their ratios with respective damping-moment coefficients. These ratios are:

$$\frac{C_{l_{\delta a}}}{C_{lp}} = L_a \quad \text{(5)}$$

$$\frac{C_{n_{\delta r}}}{C_{nr}} = N_r \quad \text{(6)}$$

$$\frac{C_{n_{\delta a}}}{C_{nr}} = N_a \quad \text{(7)}$$

The equations 1 – 4 are in no direct connection among themselves and are determining the motions around the longitudinal and vertical axes. For every axis we must find a sufficient number of equations and these are simply variations of the fundamental equation under special flight conditions. These flight conditions are then exactly determined in flight and measurements taken.

These flight conditions are:

(a) steady coordinated turns .... $\beta = \theta$

(b) steady rolling .... $\beta = \theta$

(c) steady turns with rudder neutral .... $\delta r = \theta$

(d) steady turns with ailerons neutral .... $\delta a = \theta$

(e) steady sideslips .... $d\psi = \theta$

Results for the movements about the longitudinal axis are obtained in steady coordinated turns and in steady rolling. We use equations 2 and 4 in cases (a) and (b) and solve:

$$\frac{C_{1r}}{2} \cdot d\psi_{(a)} + \mu \cdot C_{1_{\delta a}} \cdot \delta a_{(a)} = \theta \qquad (8)$$

$$\frac{C_{1r}}{2} \cdot d\psi_{(b)} + \frac{C_{1p}}{2} \cdot d\Phi_{(b)} - \mu \cdot C_{1_{\delta a}} \cdot \delta a_{(b)} = \theta \qquad (9)$$

giving:

$$L_a = \frac{1}{2\mu} \cdot \frac{d\Phi_{(b)}}{(\delta a_{(a)} + \delta a_{(b)})} \qquad (10)$$

The movement about the vertical axis, defined in equation 3 is measured in cases (a), (c) and (d):

$$\frac{C_{nr}}{2} \cdot d\psi_{(a)} + \mu \cdot (- C_{n_{\delta a}} \cdot \delta a_{(a)} + C_{n_{\delta r}} \cdot \delta r_{(a)}) = \theta \qquad (11)$$

$$\mu \cdot C_{n\beta} \cdot \beta_{(c)} + \frac{C_{nr}}{2} \cdot d\psi_{(c)} + \mu \cdot (- C_{n_{\delta a}} \cdot \delta a_{(c)}) = \theta \qquad (12)$$

$$\mu \cdot C_{n\beta} \cdot \beta_{(d)} + \frac{C_{nr}}{2} \cdot d\psi_{(d)} + \mu \cdot (C_{n_{\delta r}} \cdot \delta r_{(d)}) = \theta \qquad (13)$$

resulting in:

$$N_r = \frac{1}{2\mu} \cdot \frac{d\psi_{(c)} \cdot \beta_{(d)} \cdot \delta a_{(a)} - d\psi_{(d)} \cdot \beta_{(c)} \cdot \delta a_{(a)}}{\delta r_{(a)} \cdot \delta a_{(c)} \cdot \beta_{(d)} + \delta r_{(d)} \cdot \delta a_{(a)} \cdot \beta_{(c)}} \qquad (14)$$

$$N_a = \frac{1}{2\mu} \cdot \frac{\beta_{(d)} \cdot d\psi_{(c)} \cdot \delta r_{(a)} + \beta_{(c)} \cdot d\psi_{(a)} \cdot \delta r_{(d)} - \beta_{(c)} \cdot d\psi_{(d)} \cdot \delta r_{(a)}}{\beta_{(c)} \cdot \delta a_{(a)} \cdot \delta r_{(d)} + \beta_{(d)} \cdot \delta a_{(c)} \cdot \delta r_{(a)}} \qquad (15)$$

The sailplane Triglav was equipped for measurements with instruments indicating angle of sideslip and angle of bank and with registering instruments for aileron and rudder deflections. The sideslip indicator is shown on figure 1. It was mounted on the top of the wing and outside of the interference zone of the fuselage. A maximal angle of sideslip of 30° was supposed and this value proved to be more than the attained angles, so this indicator was never disturbed by the flow over the fuselage. Figure 2 shows the schema of the electrical transmission, of the instrument readings. It was necessary to install an electrical transmission as any mechanical friction would result in great errors and make calibration in air necessary. So the calibration was made on the ground before the take-off and an example of the instrument-calibration curve is shown in figure 3. The registering instrument was installed in the cockpit and mechanical connections with the stick and pedals fitted in, as shown in figure 4. This instrument was calibrated always immediately after the test, again on the ground. Figure 5 is a typical example of recordings. The net indicating the values was drawn later, after the calibration data.

Following flight figures were flown:

I. Steady coordinated turns: The pilot flew with the aid of the calibrated airspeed indicator and wool tufts on the canopy, indicating if there was any sideslip. The angle-of-bank line marked on the canopy against the horizon and the stop-watch gave the time of a complete turn. The control deflections were registered. This was case (a).

II. Steady rolling: The pilot banked the sailplane in a coordinated turn of over 60^{o}. The sense of circling was then changed as quickly as possible while not allowing for sideslip with the help of the wool tufts. When the angle of 45^{o} to the left was passed, time measuring began and ended when the 45^{o} right banking angle was passed. The sailplane was then rolled on for about 15^{o} and with a banking angle of 60^{o} the levelling out of the sailplane began. 15^{o} at the beginning and 15^{o} after measurement should give ample room to steadying the movement of the sailplane. This was measurement case (b).

III. Cases (c) and (d) were flown in steady turns but with stick or pedals in neutral position. The measurements were made as in case (a) with the exception that the angle of sideslip was given by the respective instrument.

IV. Case (e) was a measurement in a steady sideslip. This was the easiest test. The control deflections were registered and the pilot had enough time to read the angles of bank and sideslip.

The measured values are:

TABLE I

Case:	(a)	(b)	(c)	(d)	(e)
δ_a =	$0,3^{o}$	22^{o}	$3,3^{o}$	---	$6-6,5^{o}$
δ_r =	$2,6^{o}$	---	---	$2,5^{o}$	19^{o}
β =	----	---	$5,15^{o}$	$5,20^{o}$	21^{o}
ψ =	37^{o}	90^{o}	----	----	18^{o}
t =	18"	5,6"	32"	26"	---

Following corrections were introduced:

I. All angle data are given in radians.

II. The time data are non-dimensionalized by the operator $\frac{d}{d\tau}$

$$\tau = m / \rho . S_W . V = 0,87 \text{ sec}$$

$m = W/g$, the mass of the sailplane;
$W = 238$ kg, flight weight;
$g = 9,81 \text{ m/sec}^2$;
$S_W = 13,25 \text{ m}^2$, wing area;
$V = 20$ m/sec, horizontal speed;
$H = 1600$ m ASL, the average height;
$\rho = 0,105$ air density.

III. The density of the sailplane is:

$$\mu = m / \rho . S_W . b = 1,16$$

IV. The differential gearing of the ailerons was 2:1. Now we are using a symmetrical deflection of the ailerons which corresponds to the unsymmetrical case by a simple relation

$$\delta_a' = \frac{\delta_a^+ + \delta_a^-}{2} ;$$

$$\delta_a' = 0,75 \ . \ \delta_a^-$$

δ_a^- - the upward deflection;

$$\delta_a^+ = 0,5 \ . \ \delta_a^-$$

δ_a' - the downward deflection;

δ_a^+ - the symmetrical deflection.

All these data are collected in:

TABLE II

Case:	(a)	(b)	(c)	(d)	(e)
δ_a =	0,0039	0,2880	0,0433	θ	0,1700
δ_r =	0,0279	θ	θ	0,0436	0,3318
β =	θ	θ	0,090	0,0907	0,3670
Φ =	0,6460	1.5710	θ	θ	0,3140
t =	20,66	6,44	36,77	29,90	θ
$d\psi$ =	0,3040	θ	0,1710	0,200	θ
$d\Phi$ =	θ	0,235	θ	θ	θ

The rolling velocity was:

$$p = \frac{\Phi_{(b)}}{t_{(b)}} = 0,2804$$

And the aileron control power:

$$\frac{p.b}{2\ v} = 0,1051$$

Solving equations 10, 14 and 15 with the values from Table II, we obtain:

L_a = 0,349

N_r = - 0,0476

N_a = 0,38

With the use of mentioned reference, these values can be determined theoretically as:

$$L_a = \frac{C_{1_{\delta a}}}{C_{1p}} = \frac{C_{1_{\delta a}}}{\tau} . \frac{\tau}{C_{1p}} = 0,463$$

τ = 0,6 (from diagrams)

$\frac{C_{1_{\delta a}}}{\tau}$ = 0,41 (from diagrams)

C_{1p} = 0,532 (from diagrams)

$$N_r = \frac{C_{n_{\delta r}}}{C_{nr}} = - 0,115$$

$$C_{n_{\delta r}} = a_v . \ \eta_v . \tau \ . \ S_v . \ 1_v . / \ S_w . \ b = 0,000655$$

$a_V \cdot \eta_V = 0{,}0042$

$\tau = 0{,}65$

$S_V = 1{,}16\ m^2$; $S_W = 13{,}25\ m^2$

$l_V = 4{,}1\ m$; $b = 15\ m$

$C_{nr} = -0{,}25\ C_{Dw} - k_f \cdot S_S \cdot l_S^2 / S_W \cdot b^2 - 2\ a_V \cdot \eta_V \cdot S_V \cdot l_V^2 / S_W \cdot b^2 = -0{,}00566$

$S_S = 3{,}8\ m^2$ ..... fuselage side area

$l_S = 6{,}0\ m$ ..... fuselage length

$C_{Dw} = 0{,}02$ ..... wing drag coefficient

$k_f = 0{,}0024$

No empirical data are available to the author for N_a.

The control characteristics are always used together with the stability derivates. In this little research, we can study the conditions of the spiral stability. These are given by the inequation:

$$C_{1\beta} \cdot C_{nr} > C_{n\beta} \cdot C_{1r} \quad \text{(16)}$$

which can be rearranged to obtain ratios:

$$\frac{C_{1\beta}}{C_{1r}} > \frac{C_{n\beta}}{C_{nr}} \quad \text{(17)}$$

These two values can be found through our measurements but a new steady motion is required. Suppose the ration N_a and N_r as already known. Let the new steady motion be determined by the condition $d\psi = \theta$, giving a steady sideslip as case (e). We use equations 2 and 3 again and solve:

$$\mu \cdot C_{1\beta} \cdot \beta_{(d)} + \frac{C_{1r}}{2} \cdot d\psi_{(d)} = \theta \quad \text{(18)}$$

$$\mu \cdot C_{n\beta} \cdot \beta_{(e)} + \mu\,(- C_{n_{\delta a}} \cdot \delta a_{(e)} + C_{n_{\delta r}} \cdot \delta r_{(e)}) = \theta \quad \text{(19)}$$

The inequation becomes the form:

$$\frac{1}{2\mu} \cdot \frac{d\psi_{(d)}}{\beta_{(d)}} > \frac{N_a \cdot \delta a_{(e)} - N_r \cdot \delta r_{(e)}}{\beta_{(e)}} \quad \text{(20)}$$

This is again the condition for spiral stability. Our measurements when values from Table II are introduced, show that:

$$\frac{1}{2\mu} \cdot \frac{d\psi_{(d)}}{\beta_{(d)}} = 1{,}0$$

$$\frac{N_a \cdot \delta a_{(e)} - N_r \cdot \delta r_{(e)}}{\beta_{(e)}} = 1{,}8$$

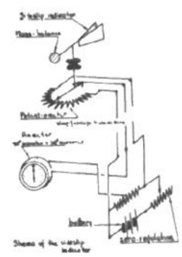
and prove, that this sailplane, when flying with the flown lift coefficient ($c_L = 0{,}75$) is not spirally stable.

REFERENCE:

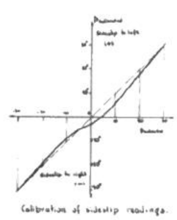
Perkins & Hage: Airplane Performance, Stability and Control, John Wiley & Sons Inc. N. Y., 1950

APPENDIX I

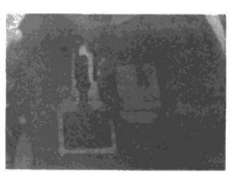
The sideslip indicator



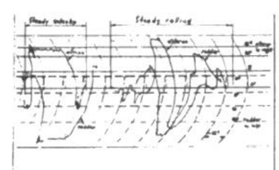
Schema of the electrical installation



Sideslip-indicator calibration



Installation of the registrating instrument in the cockpit



Example of recordings

APPENDIX II

SYMBOLS

C_L .... lift coefficient

C_D .... drag coefficient

$C_{y\beta} = \frac{dC_y}{d\beta}$ .... coefficient of lateral force due to sideslip

$C_{1\beta} = \frac{dC_1}{d\beta}$ .... coefficient of rolling moment due to sideslip

$C_{n\beta} = \frac{dC_n}{d\beta}$ .... coefficient of yawing moment due to sideslip

$C_{1p} = \frac{dC_1}{d\frac{p \cdot b}{2\ v}}$ .... coefficient of rolling moment due to rolling

$C_{np} = \frac{dC_n}{d\frac{p \cdot b}{2\ v}}$ .... coefficient of yawing moment due to rolling

$C_{1r} = \frac{dC_1}{d\frac{r \cdot b}{2\ v}}$ .... coefficient of rolling moment due to yawing

$C_{nr} = \frac{dC_n}{d\frac{r \cdot b}{2\ v}}$ .... coefficient of yawing moment due to yawing

$C_{1_{\delta a}} = \frac{dC_1}{d\ \delta a}$ .... coefficient of rolling moment due to aileron deflection

$C_{n_{\delta a}} = \frac{dC_n}{d\ \delta a}$ .... coefficient of yawing moment due to aileron deflection

$C_{1_{\delta r}} = \frac{dC_1}{d\ \delta r}$ .... coefficient of rolling moment due to rudder deflection

$C_{n_{\delta r}} = \frac{dC_n}{d\ \delta r}$ .... coefficient of yawing moment due to rudder deflection

β .... angle of sideslip

Φ .... angle of bank

ψ .... angle of yaw

δa .... aileron deflection

δr .... rudder deflection