CRITICAL GUST.

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I. INTRODUCTION.

The purpose of this analysis is to present a new, simple, and approximate method of calculating the characteristics of the critical graded gust, as a function of physical characteristics of the aeroplane under study. The effect of this gust on the normal load factor, and design requirements for wing flexibility are also presented.

II. NOTATION.

a_1 slope of the wing lift/incidence curve for finite aspect ratio, per radian;

ϑ flight path angle, measured between the horizon and a tangent to the flight path, in radians;

g standard acceleration of gravity, meters per sec^2;

ρ standard density, kg sec^2 per meter^4;

V aeroplane speed, meters per second;

U gust velocity, meters per second;

H gradient distance of the gust, in meters;

G weight of the aeroplane in kilograms;

S wing area in square meters;

c wing chord in meters;

n normal acceleration coefficient;

$a = \dfrac{\rho\, g\, S\, a_1}{2\, G}$ coefficient describing type of aeroplane;

F natural frequency of the wing in flexibility (bending) in cycles per second;

$s_g = 2H/c$ double value of non-dimensional gradient distance;

$\lambda = \dfrac{1}{ac}$ non-dimensional coefficient depending on type and size of aeroplane;

η corrective factor for Wagner-effect.

III. AIMS OF THE PROBLEM.

It is assumed that an aeroplane, moving in a straight line with a uniform velocity V, encounters a gust which acts normal to the flight path. The gust has a gradient distance H and maximum velocity U. Reference 1 presents a statistically determined relationship between gradient distance and gust velocity.

$$\frac{U}{15} = \sqrt{\frac{H}{30}} \quad \text{.......... (1)}$$

Under the influence of the gust, the flight path becomes curved resulting from an increment of normal acceleration:

$$\Delta n = \frac{1}{g} \cdot \frac{d\gamma}{dt} \cdot V \qquad (2)$$

The value of $\frac{d\gamma}{dt}$ increases as U increases, and decreases as H increases. Since U and H are dependent upon one another, $\frac{d\gamma}{dt}$ will have a maximum value with a corresponding maximum possible Δn. The gust which produces the maximum Δn is defined as the „Critical Gust". Gusts of greater or lower intensity than the critical gust do not furnish valid structural criteria. The aim of this analysis is to find the „Critical Gust" and corresponding maximum Δn for various types of aeroplanes.

IV. MODE OF CALCULATION.

In the analysis it is assumed that:

(a) the speed of the aeroplane upon entering the gust remains unaltered;
(b) the position of the aeroplane with respect to the horizon remains unchanged;
(c) the rate of load increase is of no importance;
(d) the aeroplane is infinitely rigid.

The initial equation of the problem results from equating the increment of lift force to the inertia force.

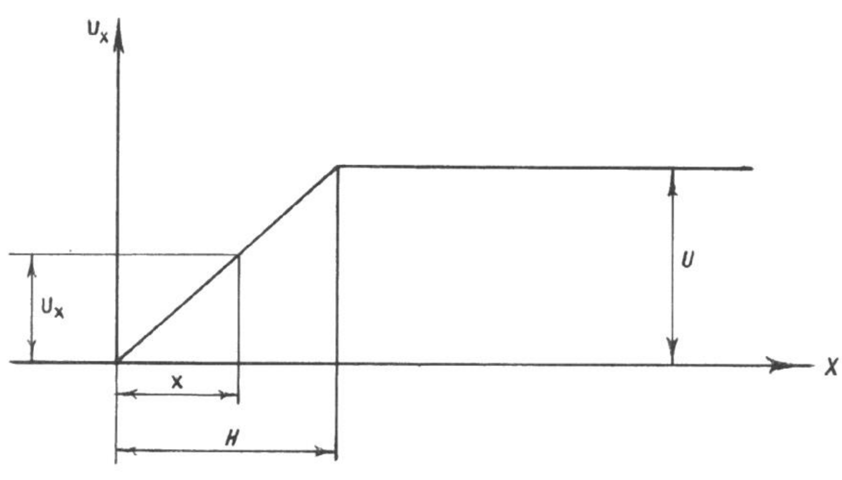
$$a_1 \left(\frac{u_x}{V} - \gamma \right) S \frac{\rho}{2} V^2 = \frac{G}{g} \frac{d\gamma}{dt} V \qquad (3)$$

By assuming a linear variation of gust velocity in the transition region (Fig. 1) we find that:

$$U_x = U \frac{x}{H}, \quad V = \frac{dx}{dt}, \quad a = \frac{g}{G} S \frac{\rho}{2} a_1$$

and for $x = H$ according to reference 2 we find:

$$\frac{d\gamma}{dt} = \frac{U}{H} \left(1 - e^{-aH} \right) \qquad (4)$$



Therefore $\Delta n = \frac{U.V}{g.H} (1 - e^{-aH})$ .... (5)

It is obvious from these results that $\frac{d\gamma}{dt}$ is not dependent upon aeroplane speed but depends only on the type of aeroplane and the meteorological elements ($U = f(H)$). If we consider the choice of speed V for each aeroplane as a specific one, we see that Δn is a maximum when the value of $\frac{d\gamma}{dt}$ is a maximum.

Combining equations (1) and (4) we find:

$$\frac{d\gamma}{dt} = \sqrt{7.5} \cdot (1 - e^{-aH}) \; / \; \sqrt{H}$$

This equation, when differentiated with respect to H and set equal to zero, yields: $2aH + 1 = e^{aH}$ from which one obtains: $aH_{critical} = 1.277$ .... (6)

This equation relates the critical meteorological conditions to the characteristics of the aeroplane under study. Refering back to equation (1) we find:

$U_{critical} = 3.095 / \sqrt{a}$ .... (7)

and $\Delta n_{max} = \frac{\sqrt{7.5}}{g} \cdot \frac{1 - e^{-1.277}}{\sqrt{1.277 / a}} \cdot V = 0.178 \sqrt{a} \cdot V = k.V$ .... (8)

The solution of these equations are given in table 1 and diagram 1.

TABLE 1

a	$\sqrt{a}$	$U_{crit.}$	k	$H_{crit.} = 1.277/a$
0.01	0.1000	30.9	0.0178	127.7
0.02	0.1414	21.9	0.0252	63.5
0.03	0.1732	17.9	0.0308	42.3
0.04	0.2000	15.5	0.0356	31.7
0.05	0.2236	13.8	0.0398	25.5
0.06	0.2449	12.6	0.0436	21.3
0.08	0.2828	10.9	0.0504	15.9
0.10	0.3162	9.8	0.0564	12.7
0.12	0.3464	8.9	0.0617	10.6
0.14	0.3742	8.3	0.0666	9.12
0.16	0.4000	7.7	0.0712	8.0
0.18	0.4242	7.3	0.0755	7.1
0.20	0.4472	6.9	0.0797	6.4

Equation (8) gives the simplest relationship between critical load and flight speed. If the aeroplane structure is not strong enough to withstand the critical gust at the higher speeds, equation (8) may be used to define the maximum permissible flight speed in gusty air.

$V = \frac{\Delta n}{k} = \frac{n - 1}{k}$ .... (9)

V. WAGNER EFFECT.

Reference 4 provides a refinement to the theory known as Wagner effect. In the calculation we introduce new variables:

$$\lambda = \frac{1}{ac} \qquad \text{and} \qquad s_g = \frac{2\,H}{c}$$

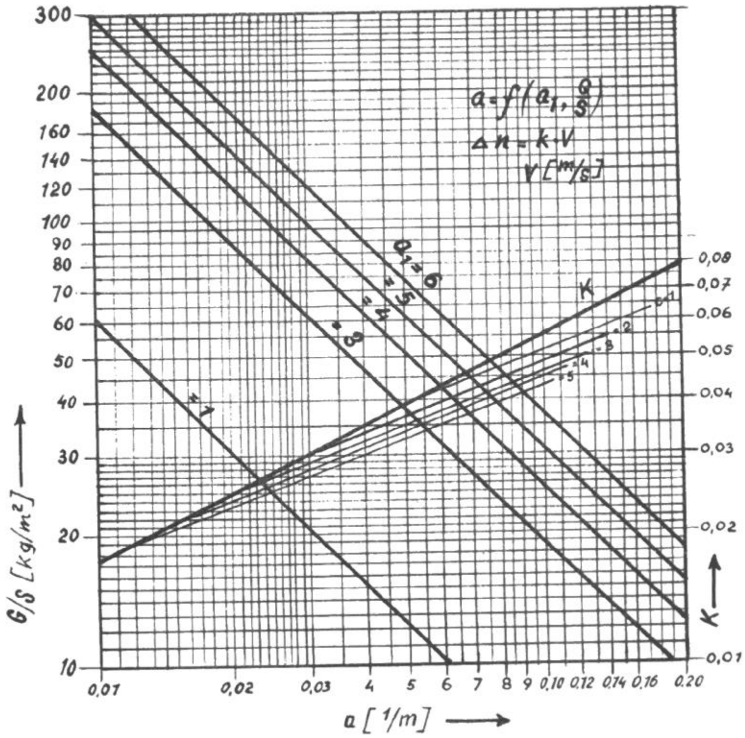
Substituting the critical condition $(aH)_{crit.} = 1.277$ yields;

$$s_g = \frac{2\ (1.277)}{ac} = 2.554\ \lambda$$

The Wagner alleviating factor η follows from the comparison of Δn calculations with and without Wagner effects.

λ	s_g	η
5	12.8	0.86
10	25.5	0.93
15	38.3	0.96

These corrective values are taken into account in diagram 1 by a modification to the factor K as a function of chord length c.



VI. INFLUENCE OF FLEXURAL DEFLECTION OF WINGS IN FLIGHT.

According to the Netherlands investigation (reference 1) the flexibility criterium is given by

$$\frac{H}{V} \geqslant \frac{1}{2\ F} \quad \ldots\ldots\ldots\ldots (10)$$

As determined above, in the case of critical load the product aH is a constant. Thus if a is large, H will be low and the minimum allowable natural frequency in bending will be high. This is a very important factor in the design of high speed, high performance sailplanes where the old criteria of 2 cycles per second is no longer applicable. The new criteria becomes:

$$\frac{1.277}{a\ V} \geqslant \frac{1}{2\ F} \quad \ldots\ldots\ldots\ldots (11)$$

VII. ILLUSTRATIVE EXAMPLE.

From preliminary design data for a high performance sailplane a wing is to have a_1 - 5.335, an aspect ratio of 20.29 and a mean chord of 0.937 meters (36.9 inches). The wing loading G/S equals 22.5 kg/m^2 (4.69 pound per square ft).

a) What will be the critical gust?

b) What will be increment of normal acceleration?

c) What will be the required natural frequency of the wing in bending (cycles per second)?

The required answers are determined with the use of diagram 1 and table 1.

a) In diagram 1 for a wing loading of 22.5 kg/m^2 the intersection with $a_1 = 5.335$ gives a value of $a = 0.147$. From table 1 for $a = 0.147$ the critical gust $U = 8$ m/sec (26.23 ft/sec) appears.

b) In diagram 1 the intersection of the vertical projection of the above mentioned intersection with $c = 0.937$ m gives the factor $K = 0.062$. If the sailplane has a forward speed of 150 km/h = 42 m/sec = 93.2 mph the increment of normal acceleration is given by equation (8).

$$\Delta n = 0.062 \times 42 = 2.6 \qquad n = 3.6$$

c) Refering to equation (14)

$$F \geqslant \frac{1}{2}\,\frac{a\,V}{1.277} = \frac{1}{2}\,\frac{0.147 \times 42}{1.277} = 2.42 \text{ cycles per sec.} = 145.2 \text{ cycles per min.}$$

VIII. SUMMARY.

This study has made it evident that it is illogical to fix the magnitude of the design gust velocity as proposed in some present day airworthiness requirements (e.g. U = 10 m/sec; H = 30 m). Likewise, the standardization of distance H is illogical for the determination of structural flexibility criteria. This paper then, makes the first attempt to establish values of critical U and H dependent on the characteristics of the particular aeroplane under study.

The seriousness of wing flexibility in high performance sailplanes was emphasised in paragraph 6 and covered by equation (11). This equation may not be strictly applicable to heavily loaded aeroplanes. The relationship given in equation (6) is perhaps modified for this type of aeroplane.

IX. ACKNOWLEDGMENT.

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