The Air Flow over an Extended Ridge*

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Introduction

THE natural air flow over terrestrial obstructions is distinguished from aerodynamic flows over various, relatively small artificial shapes by the fact that the potential flow is rarely found in natural flows. Theoretical solutions for the two-dimensional potential flow over obstacles have been published by Pockels (1901) for a ridge defined by a harmonic function, Defant (1921) for a half-elliptic cylinder, Ackeret (1922) for a shape defined by a source in the uniform stream, and others. In Figure 1 is shown the solution of Pockels for an obstacle 50 meters high. Lines of constant vertical velocity are shown. The influence of the ridge is evident only in the immediate vicinity of the ridge. However, experimental explorations of natural flows (Wagner, 1926, Koch, 1927-28, Landsberg and Riley, 1943) have failed to to yield the type of flows or the vertical velocities expected from the theoretical studies. Seeking a more complete description of natural flows, Raethjen (1926) included the effect of the lapse rate on the flow and arrived at the general differential equation for atmospheric flows over small regions assuming frictionless, vortex free, two-dimensional flow,

$$\nabla^2 \psi + C.\psi = O \qquad \text{Equation (I)}$$

where ψ is the stream function,

$$\nabla^2 \psi = \frac{\delta^2 \psi}{\delta^2 x^2} + \frac{\delta^2 \psi}{\delta y^2}$$

$$C = \frac{g}{U^2} \cdot \frac{1}{T} \cdot \frac{\delta \Theta}{\delta z}.$$

U is the horizontal wind at a great distance from the obstacle, Θ is the potential temperature, T the absolute temperature, z the altitude above sea level and g the acceleration of gravity. It is evident that equation 1 reduces to Laplace's equation, the case for potential flow when $C = O$ which is true only when $\frac{\delta \Theta}{\delta z} = O$, i.e., when an adiabatic lapse rate exists.

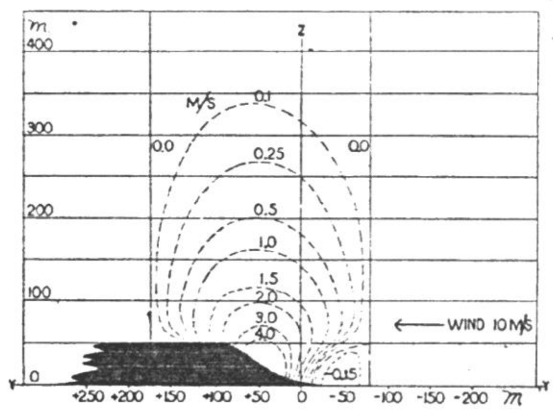

Fig. 1. Pockel's solution for the vertical component of flow over a ridge.

Examination of Raethjen's equation (Eq. 1) also explains why potential flow is rarely found in nature.

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When $C = O$ the convective stability of the air is neutral and consequently any local heating by the sun will have a marked effect on the flow. A steady flow under this condition is hardly possible.
If, on the other hand, the lapse rate is stable, $C < O$, Raethjen (1926) found sinusoidal solutions of the type

$$\psi = \psi_o \sin \sqrt{C}\ \chi .$$

where the period of the wave is

$$P = 2\pi \sqrt{\frac{T}{g} \cdot \frac{1}{\frac{\delta \Theta}{\delta z}}} . \qquad \text{Equation (II)}$$

If the propogation velocity of the waves is equal to the wind speed the waves will be stationary. Lyra (1943) has published a solution for the flow of a stratified fluid over an obstacle showing a system of stationary waves to the lee of the obstacle as well as a general region of lifting over the windward portion of the obstruction. Recently Queney (1947 and 1948) has offered a linearized solution based on the theory of small perturbations not only for the gravity waves studied by Lyra but, also, for the inertial and long atmospheric waves. Queney used a factor which he called the coefficient of vertical stability "S" to define the stratification,

$$S = \sqrt{\frac{g}{\Theta} \cdot \frac{\delta \Theta}{\delta z}} . \qquad \text{Equation (III)}$$

It is evident from a comparison of equations (II) and (III) that $\frac{1}{P} \cong \frac{S}{2\pi}$. The wavelength of the stationary waves computed either from the period or the coefficient of vertical stability is, therefore, identical for either Lyra's or Queney's solution for gravity waves.

Experimental Research

With the foregoing theoretical background as a guide, the present study was begun in order to determine if the available theory could predict the type of flow which would take place under the ambient temperature and wind fields. As an approach to an idealized natural obstruction, a terminal morain facing the northwest located on the north shore of eastern Long Island was chosen. The base of the terminal morain is on the Long Island Sound with the plateau about 50 meters above sea level. The slope of the dune is the natural angle of recline of sand, about 30°. Since the ridge is comparatively long, a two-dimensional flow can be expected. The passage of the wind over the smooth and nearly isothermal surface of Long Island Sound yielded a homogeneously stratified atmosphere in most cases.
The apparatus used for exploring the natural flows consisted of a sailplane (Raspet, 1948) equipped with a cine' camera for photographically recording the barometric altimeter, a variometer measuring rate of climb, a low-lag thermometer and a clock. On the ground a mapping of the position of the sailplane synchronized with the recorded data from the sailplane permitted the vertical and horizontal wind fields as well as temperature fields to be plotted along the traverses made by the sailplane. By restricting the sailplane to a plane normal to the ridge and in the surface wind direction, it was possible to map accurately the two-dimensional flow field as well as the ambient wind and temperature fields. In Figures 2, 3, 4, and 5 are shown the various fields made by using the sailplane as a meteorological probe.

Description of the Various Flow Fields Over a Ridge

In Figure 2 is shown the vertical velocity of the air flow over the ridge taken on November 10, 1947. It should be mentioned that the lower two traverses are displaced vertically in order to eliminate confusion. However, the true vertical coordinates are shown for each traverse. The flight path of the sailplane is shown as the base line on which are plotted the values of true vertical velocity of the wind. Time intervals of one second are displayed along the flight path. Study of this flow field shows a long wave on the highest traverse and directly below it a wave of nearly equal wavelength but reversed in phase with respect to the upper wave. Superimposed on the waves and predominant on traverses below them are turbulence waves of much shorter wavelength. That the flow is not steady is evident from a comparison of the lower three traverses where within five minutes the flow is seen to change markedly. However, the ridge lift is apparent in all three traverses.

Returning to a consideration of the longer waves in the two uppermost traverses, one may compute the amplitudes of the waves from the simple relation

$$A = \frac{w}{U} \frac{\lambda}{2\pi} \qquad \text{Equation (IV)}$$

where w is the vertical velocity of the air motion and λ is the wavelength.

The amplitudes are found to be 50 feet and 33 feet reading from the uppermost traverse down. Evidently from this computation the waves are spaced sufficiently that they do not interfere with each other. It is entirely possible then for them to be reversed in phase.

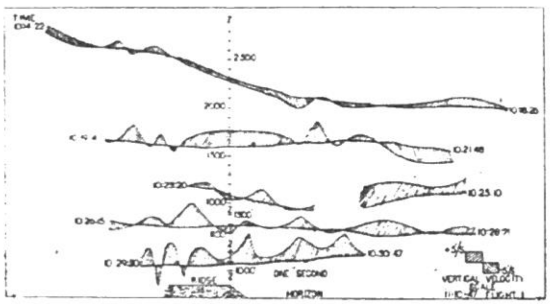

Fig. 2 Vertical velocity of wind over a ridge.

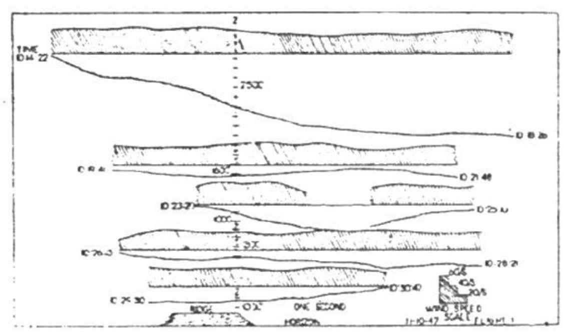

Fig. 3 Horizontal wind component on the ridge

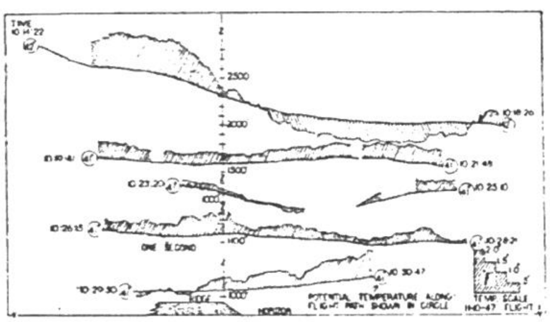

Fig. 4 Potential temperature along flight path.

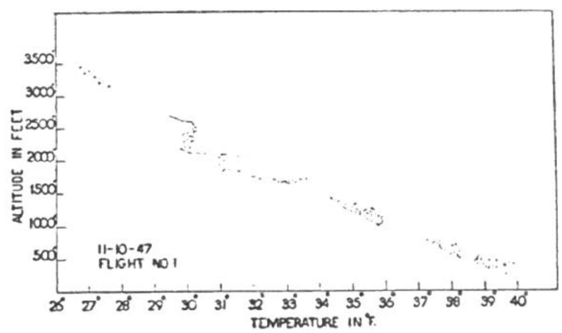

Fig. 5 Lapse rates over ridge.

Figure 3 shows the horizontal winds during the same exploration. Some small variations in wind are present. However, these variations cannot readily be ascribed to the wave motion. The strong anomalies present in the potential temperature field are displayed in Figure 4. The wavelike variation of potential temperature in the upper traverse agrees in wavelength with the vertical flow field. However, no valid conclusions in this respect can be drawn without a better knowledge of the true nature of atmospheric flows. Whether the temperature variations are the cause or result of the wave motion must yet be determined by more refined theoretical and experimental analysis.

In Figure 5 the temperature of the air is displayed as a vertical sounding. The anomalies apparent in Figure 4 as due to horizontal gradients appear as projections from the mean lapse rate of the air. These projections are most prominent at the altitude where the waves occurred. The mean lapse rate obtained from each flight was used as the basis for computing the period of the gravity waves.

Another flow field showing waves is that illustrated in Figure 6. On the two traverses appear two prominent lift regions which might be attributed to convection. However, a computation of the drift of this flow from the lower traverse to the upper proves that the lift is not due to convection. The disturbance is attributed to the ridge and is evidently stationary. In this analysis waves are divided in two classes,

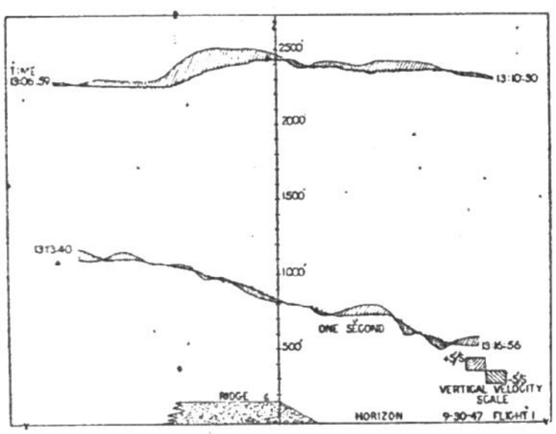

Fig. 6 Vertical velocity of wind over a ridge.

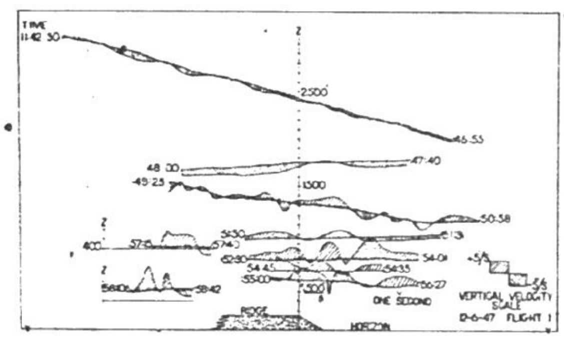

Fig. 7 Vertical velocity of wind over a ridge.

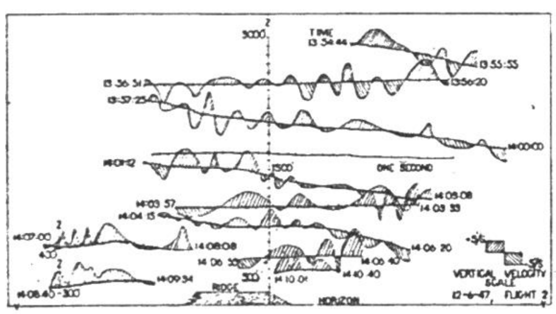

Fig. 8. Vertical velocity of wind over a ridge.

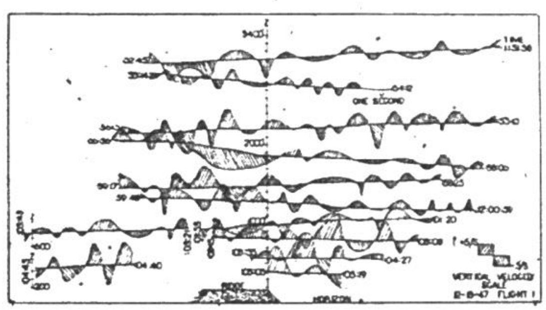

Fig. 9 Vertical velocity of wind over a ridge.

the stationary and those moving with the wind. To the latter class belong the short turbulence waves. With two sailplanes probing the flow, following each other by a fixed time interval, it would be possible to filter out all of the nonstationary disturbances. In the flow of Figure 6 the waves on the two traverses were found to be 0.36 and 1.8 km in wavelength. In Figure 7 a long wave occurs in a layer of very stable air having a potential temperature gradient of 6.02 degrees C/km.

Strong ridge lift seen to be wavelike in character extends some distance windward of the ridge. This same behavior is apparent in a later flight on the same day as that in Figure 7. Since neither Lyra or Queney explain the existence of such waves before an obstacle, further research appears advisable in order to

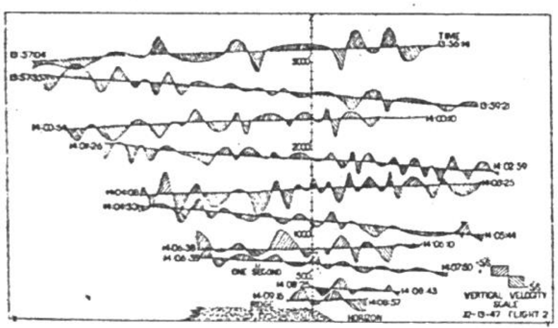

Fig. 10. Vertical velocity of wind over a ridge.

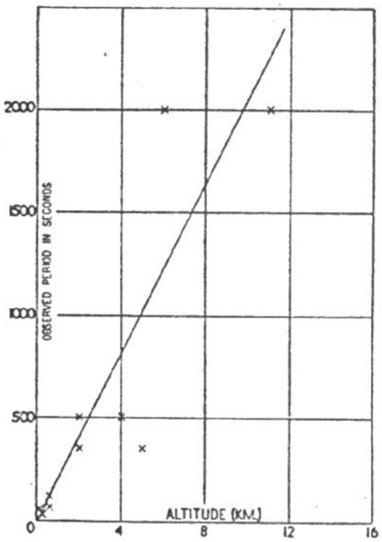

Fig. 11. Period of atmospheric waves vs altitude.

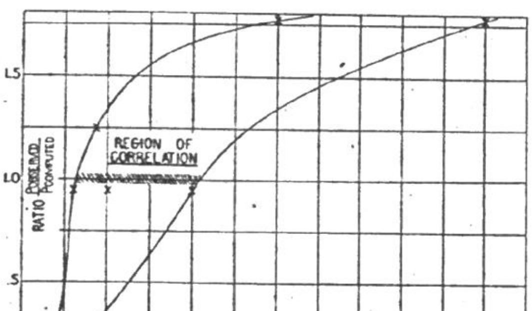

Fig. 12. Ratio of observed and computed periods of atmospheric waves vs altitude.

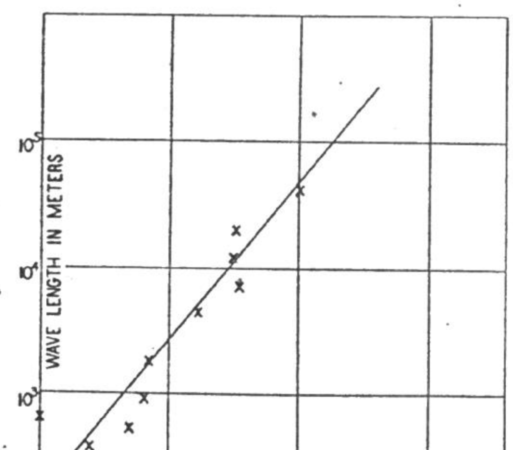

Fig. 13. Wave length of atmospheric waves vs altitude.

Table I. CHARACTERISTICS OF AIR FLOWS

Date	Flow	Wind	$-\frac{\delta\Theta}{\delta z}$	T comp.	T obs.	λ comp.	λ obs.	Z
9-30-47 9-30-47	Wave Wave	17 M/S 17	2.45° C/km 2.45	680 sec. 680	111 sec. 23	11.5 km 11.5	1.8 km 0.36	0.7 km 0.24
11-10-47	Wave	14	2.35	690	60	9.7	0.89	0.65
12-6-47-1	Wave Ridge lift	16 16	6.02 2.16	428 715	57 —	6.8 11.3	0.53 —	0.50 —
12-6-47-2	Ridge lift	15	2.37	680	—	10.4	—	0.1
12-13-47-1	Turbulence Waves	4-8	2.16	690	10.20	2.52	0.06-0.2	0-1
12-13-47-2	Waves	3	1.95	748	10-20	2.28	0.05-0.2	0-1
8-8-38 Ref. 11	Wave	8-10	—	—	40-50	—	0.4	0.1
5-21-37 Ref. 12	Wave	20	—	—	350	—	7.0	2-5
Ref. 13 + 14	Wave	25	4.28	530	500	13.3	10-15	2-4
Ref. 13 + 14	Wave	20	1.0	1130	2000	22.3	40	6-11

determine the nature of these windward waves. It is interesting to remark that F. Ringleb has demonstrated mathematically the symmetry of solutions to Raethjen's equation for a semi-cylindrical obstacle. Ringleb thus finds waves before the obstacle.

The flow fields of Figures 9 and 10 are shown to illustrate the turbulence waves in a stable atmosphere in a comparatively weak wind. The flight paths are seen to be nearly straight lines sloping downward at the glide angle of the sailplane. The small oscillations of vertical velocity are obtained from a differentiation of the theodolite plot of the track of the sailplane. By this method the short period oscillations can be measured without their being damped by instrument lag.

A summary of the data obtained in this exploration as well as some data from the literature (Klanke 1938, Kuettner 1939, Georgii 1947, and Krug-Pielsticker 1942) is shown in Table 1. It is apparent that the potential temperature gradient is nearly identical in several of the flow fields yet the nature of the flow is extremely different. According to Raethjen's theory the temperature field should be expected to exercise the principal control in determining the flow field. If two examples are selected, those of Figures 2 and 8, which have a nearly equal horizontal wind and nearly identical lapse rate, the flows are found to be quite dissimilar. In other words, this data demonstrates that the Raethjen equation is not sufficient to define the flow in the realm explored in this study, that is, up to about one kilometer above the surface.

Examination of the flow data plotted in Figure 11 shows that the period of the waves increases with the altitude. That gravity wave theories are not sufficient to cover this behavior is shown in Figure 12 where the ratio of the observed period of the waves and that computed from the existing temperature gradient is plotted against altitude. In the level 1 to 4 km, the gravity wave theory holds true, outside this region the theory breaks down.

When the observed wavelengths from Table 1 are plotted logarithmically against altitude, Figure 13, the wavelength is found to be given empirically

$$\lambda = K \, . \, z^{1,7}$$

where K is the constant of proportionality.

This experimental evidence thus shows that the so-called gravity wave theory should include an air density term. The theoretical airflow patterns determined by the linearized flow equations of Raethjen, Lyra, and Queney, are not, therefore, sufficiently rigorous to explain either high altitude waves or those near the ground. Of interest is the comment by Hess and Wagner (1948) that the observed height of nodal planes showed a discrepancy of 100% over that computed at an altitude of 24,000 feet. From Figure 12 the ratio of $\frac{\text{P obs.}}{\text{P comp.}}$ at this altitude is about 1.8 which is in rather close agreement with the result of Hess and Wagner.

As part of this project Ringleb (1948) has attacked the theory of atmospheric flow from a more general viewpoint deriving a following differential equation

$$\nabla^2 \psi + \frac{1}{2} \left[\frac{d}{dz} \left(\frac{\rho'}{\rho} \right) - \frac{1}{2} \left(\frac{\rho'}{\rho} \right)^2 \right] . \psi = 0$$

where ρ = air density and ρ' — density gradient with respect to altitude, so that $\rho' = \frac{d\rho}{dz}$. It is evident that this form does include the air density in the term from which would be computed the period or wavelength. However, considerable more study is required to reduce the equation to a form by which its validity may be tested experimentally.

Sekera (1938) has also studied the problem of atmospheric wave motion in a more general manner by including the effect of a vertically changing wind field. Waves have been reported by Ross (1948) under conditions when the wind increased from 0 at the ground to 96 mph at 25,000 feet. Under such a wind field theory based on a uniform wind cannot be used.

Conclusion

This study has aimed at correlating experimentally determined atmospheric flow fields against available theories for air flow over natural obstacles. As a result of these attempts it may be stated that:

a. From the differential equation of micrometeorological flow of Raethjen, it was not possible to define the type of flow from the parameters included in the theory.

b. For the wave-like airflows, wavelengths computed on the basis of short gravity waves were found to agree with experimental results in the level 1 to 4 km only.

c. The theories of Lyra and Queney predict waves to the lee of the obstacle. In the exploration waves were also found windward of the ridge.

The theories of Ringleb and Sekera appear worthy of experimental test. For the Ringleb theory a special instrument which measures air density directly will be needed. The physical basis for such an instrument is available by utilizing the aerodynamic properties of the sailplane. The extension of the exploration to a larger ridge is indicated by the experiences of this experiment.

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REFERENCES

Ackeret, J., 1922, Ein Beispiel zum statischen Segelflug, Zeit. Flugtechnik u. Motorluftschiffahrt, 14, 86–89.

Defant, A., 1921, Über die Dynamik der Böen, Beiträge zur Phys. d. fr. Atmos., IX, 99–113.

Georgii, W., 1947, Segelflug Aerologie: Alpine Wellen, Schweizer Aero Revue 22, 283–284.

Hess, S. L. and **Wagner, H.,** 1948, Atmospheric waves in the Northwestern United States, Jour. Met. 5, 1–19.

Klanke, G., 1938, Mitteilungen über Wellensegelflugversuche über der Kurischen Nehrung, Bericht 102, Lilienthal Ges. Luftfahrtforschung, 21–22.

Koch, H., 1927.28, Graphische Analyse der Strömung im Luv einer Düne, Jahrbuch des Forschung-Institutes der Rhon-Rossitten-Gesellschaft e.v., 17–22.

Krug-Pielsticker, U., 1942, Beobachtungen der hohen Föhnwelle an den Ostalpen, Beit. z. Phys. d. fr. Atmos. 27, 140–164.

Kuettner, J., 1939, Moazagotl und Fohnwelle, Beit. z. Phys. d. fr. Atmos. 25, 79–114.

Landsberg, H. and Riley, N. A., 1943, Wind influences on the transportation of sand over a Michigan sand dune, Proc. Sec. Hydraulics Conf. Bulletin 27, Univ. Iowa Studies in Eng., 342–352.

Lyra, G., 1943, Theorie der stationaren Leewellenstromung in freien Atmosphare, Zeit. angew. Math. u. Mech., 23, No. 1.

Pockels, F., 1901, Zur Theorie Niederschlagbildung an Gebirgen, Ann. der Physik, 4, 459–480.

Queney, P., 1947, Theory of perturbations in stratified currents with applications to airflow over mountain barriers, Misc. Report No. 23, Univ. Chaicago, 81 pp., 1948, The Problem of air flow over mountains: A summary of theoretical studies, Bull. A.M.S., 29, 16–26.

Raethjen, P., 1926, Das Zweidimensionale, Atmosphärische Stromfeld um ein Hindernis, Zeit. Flugtechnik u. Motorluftschiffahrt, 17, 185–190.

Raspet, A., 1948, The sailplane as a meteorological probe, Trans. Amer. Geophys. Union, 29, 531–637.

Ringleb, F. O., 1948, Basic atmospheric flow problems, Report to Office of Naval Research, Contract N7onr-400.

Ross, H. C., 1948, Soaring to the stratosphere, Soaring, 12, 9–10.

Sekera, Z., 1938, Zur Wellenbewegung in Flüssigkeitsschichten mit vertikal verändlichbarer Geschwindigkeit, Astrophysica Norvegica, 3, No. 1, 1–69.

Wagner, F., 1926, Die Stromfeldbestimmung im Luv der Düne von Rossitten, Zeit. Flugtechnik u. Motorluftschiffahrt, 17, 190–192.