"The Play's the Thing": Mathematization as Dramatization

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Mobilizing prevalent themes in the fields of mathematics education, literary criticism, and philosophy, this paper contextualizes 'the mathematical', 'mathematical thinking', and 'mathematical pedagogy' with respect to ancient Greek concept of mathesis, modern notions of mathematical agency, the Keatsian concept of negative capability, and the analogy of 'staging' a dramatic/mathematical 'play'. Its central claim is that mathematization is dramatization-that learning mathematics (indeed, learning to learn, which is what the Greek mathesis actually means) is an activity of setting things up and (in this 'set' or 'setting') allowing things to play out (e-ducere). Beginning with Paul Ernest's identification of the difference between absolutism and fallibilism in the philosophy of math education, and incorporating concepts from Pythagoras, Hippasus, Heraclitus (the 'ancients'), Descartes, Kant, Keats (the 'moderns'), as well as Freud, Heidegger, and Badiou ('nos prochains', to quote Klossowskiⁱ), we argue that 'mathematical knowledge' cannot be understood simply within the framework of logicism, formalism, or even simply as an epistemological articulation. Rather, we endeavour to show that the process of learning mathematically' allows us to gain insight into the foundations of being' itself (i.e. ontology). Learning to learn (mathesis) proceeds, as such, by way of staging and playing-out the half-known or unknown (the ill-seen and ill-said) in the hopes of uncovering the mystery (Greek myesis) at the heart of things.

Number Befor(e)word: The Question of 'the Mathematical'

The interdependence between mathematics and social forces has become a subject of vital importance in the field of mathematics education, made possible in no small part by the emergence of critical discourse about the constructed, political and linguistic nature of mathematical knowledge. Scholars have argued that there are at least two contending paradigms with regard to the nature of mathematical knowledge: the *absolutist* and the *fallibilist* perspectives. The former views mathematics as an "absolute, certain and incorrigible body of knowledge";² the latter views mathematical knowledge as culturally constructed and therefore subject to historical change. In the absolutist paradigm, exemplified by such 20th-century movements as Logicism and Formalism, although new theories and truths may be added to the repertoire of human discovery, mathematics is ultimately universally valid, value-free, and as

¹ Pierre Klossowski, Sade: mon prochain (Paris: Éditions du Seuil, 1947).

² Paul Ernest, 'What is the Philosophy of Mathematics Education?' Philosophy of Mathematics Journal 18 (2004), 8.

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such, "[it] is a body of absolute and certain knowledge."³ In contrast, the fallibilist paradigm, influenced largely by Social Constructivism, "does not reject the role of logic and structure in mathematics, just the notion that there is a unique, fixed and permanently enduring hierarchical structure."⁴ Mathematical thinking is as much subject to human fallibility and linguistic construction as any other realm of human activity: "mathematical language is a part of our language-games, which themselves are contingent and based upon our linguistic conventions that have developed as forms of community agreement."⁵

The idea that mathematical knowledge is not independent of letters and words was a fundamental insight of western antiquity as well. For the ancient Greeks, *ta mathemata*, 'the mathematical', meant the same thing as 'learning' in general.⁶ As Thomas McFarlane notes,

In many ancient languages (e.g. both Hebrew and Greek) the same symbols were traditionally used for both numbers and letters. Thus, the sacred scriptures were not only expressions of vowels and consonants. They were also expressions of number and order. The vibrational qualities of their sounds have a corresponding numerical quantity that represents its particular order or logic. The *logos* is both qualitative and quantitative meaning. Thus, the *poesis* of the world is not a random creation, but is an ordered arrangement and adornment, a *cosmos*. And the ordering principle of this *cosmos* is the *logos*, the Word that is also Number. Thus, it is said in the Pythagorean tradition that 'Number is the principle, source, and root of all things', which is to emphasize the ordering aspect of the Word.⁷

If mathematical thinking can be (and historically has been) understood more generally as learning and as a fundamental human activity, then what implications might this have for mathematics education and pedagogy? Indeed, what exactly does one learn when one learns mathematics (in both its general and specific senses)? And are the basic elements or building-blocks of mathematics (numbers) akin to linguistic elements or building-blocks (letters)? If we assume that 'letter' is prior to 'word' (the condition for any 'statement'), then could 'number' not be considered to be *prior* to any 'mathematical statement'? And if so, what *are* these basic elements ('letters' and 'numbers')? What, if not 'statements', do they express? In what follows, we will argue that the mathematical (qua *mathesis*) is ontologically prior to any epistemological statement expressed in numerical formulations or alphabetical phrasings. Learning at the most basic level (*mathesis*) proceeds by way of that which is only *half-known*, *barely-known*, *near-unknowable*.

Negative Capability: ta Mathemata and the theatrum of Mathesis

In his famous letter of 21-12-1817, the celebrated English poet John Keats described the capacity to "go on" in the face of *partial knowledge* (i.e. of *partial ignorance*), to "proceed by aporia" as Samuel Beckett would say (in his aptly-titled *L'Innommable: The Unnamable*), as being the *signature characteristic* of *poetic sensibility*. Keats called this capacity "*negative capability*—that is, when a man is capable of being in uncertainties, mysteries, doubts, without any irritable reaching after 'fact' and 'reason'." Linking Keats' concept to the philosophical tradition of the ancient Greek Pre-Socratics, the poet and scholar Charles

³ Paul Ernest, 'Social Constructivism as a Philosophy of Mathematics: Radical Constructivism Rehabilitated?' http://www.ex.ac.uk/~PErnest/soccon.htm.

⁴ Paul Ernest, 'What is the Philosophy of Mathematics Education?' Philosophy of Mathematics Journal 18 (2004), 8, 9.

⁵ Michael Peters, 'Wittgenstein, Education and the Philosophy of Mathematics', *Theory and Science* 3.2 (Winter 2002) *http://theoryandscience.icaap.org/content/vol003.002/peters.html.*

⁶ For an extensive commentary on the notion of 'mathematical thinking' in the ancient Greek natural philosophers, see Martin Heidegger's seminal study, *What is a Thing?* (Chicago: Henry Regnery, 1967).

⁷ Thomas McFarlane 'The Mathematical Poetics of Enlightenment', *IntegralScience.org* (2000), *http://www.integralscience.org/mathpoetics.html*.

Olson associates negative capability with the dictum of Heraclitus that "man is estranged from that with which he is most familiar."⁸

By 'negative capability', Keats did not intend to signify that which cut off intellectual enterprise, but rather, "an intention to make room in the life of the mind."⁹ The notion of 'negative capability', as such, seems to be significant for the study of learning and teaching: in order to no longer be estranged from that with which we are most familiar, we have to be negatively capable—that is, we must be able to function without the full picture. Pippa Carter and Norman Jackson even posit a possible connection between Keats' poetic notion of negative capability and Immanuel Kant's philosophical notion of 'negative presentation', arguing that "the pursuit of certainty within a framework of the possibility of achieving it leads invariably to inflexibility and to the failure to recognize opportunities, potential benefits, other solutions to problems, and solutions to other problems, offered by the very ambiguity and uncertainty that are so feared."¹⁰

Foreshadowing Whitehead's famous dictum that 'ignorance of ignorance is the death of knowledge', Keats argued that the ability to work with the *half-known*, with *partial knowledge*, without taking such negativity—the unknown 'facts' or 'reasons'—as a positive *incapacity*, was the royal road to genius. Freud would have called this road the path of *durcharbeiten*: literally a "working through" difficult, repressed, traumatic "blockages" (cf. 'Errinern, Wiederholen und Durcharbeiten', 'Remembering, Repeating and Working-Through', in the *Internationale Zeitschrift für ärtztliche Psychoanalyse* 2, 1914:485-490).

More recently, the philosopher Alain Badiou has suggested that such a power as that of Keatsian *negative capability* or Freudian *working through* can "perhaps [be] recognized" in the mathematical concept of "forcing," which was Paul Cohen's great contribution to mathematical theory. "Forcing," explains Badiou, "is the point at which a [half-known/partially-known] truth, although incomplete, authorizes anticipations of knowledge".¹¹ Through the force or "forcing" of *durcharbeiten* (Keats's "negative capability"), the *unknown* or *unfamiliar*—the *uncertain*, the *mysterious*, that which is *riddled with doubt*—is brought into *play*, allowed onto the *stage* or into the *theatre* of "theory and practice" precisely "as" this *theatre*, *stage*, or *setting*. The unknown is thus acknowledged as the *ground* (the 'road' again, or 'path') of *ta mathemata*, the (open) *stage* or *setting* of 'learning' in general. To stride this stage, to perform upon this stage, is to be an actor: one who acts 'as if' the (unknown/uncertain) world's a stage, one who 'is' (or as Keats says, "is capable of being") "in uncertainties, mysteries, [and] doubts, without any irritable reaching after 'fact' and 'reason'."

Ta mathemata—and *mathesis* understood as 'learning' *in general*, coming to know things that 'matter' or 'count' *in particular*—necessitates *drama (dramatization)*, in other words. 'Drama' itself simply means an 'act' or 'performance' (the Greek word for 'play', 'deed', or 'action'). Descartes, who distinguished "learning in general" from the specific discipline of "mathematics" precisely by way of the word *mathesis*¹² (using *mathesis* and/or *mathesis universalis* as his term for general *disciplina*,¹³ understood as

⁸ Charles Olson, *The Special View of History: Lectures at Black Mountain*, 1956 (Berkeley: Oyez, 1970), 29. Olson is referring to the Diels-Kranz fragment 72, Bywater fragment 93, Marcovich fragment 4.

⁹ Nathan Scott Jr., Negative Capability (New Haven: Yale UP, 1969), xiii.

¹⁰ Pippa Carter and Norman Jackson, 'Negation and Impotence', *In the Realm of Organization: Essays for Robert Cooper* (London: Routledge, 1998), 205-206. "[T]he very ambiguity and uncertainty that are so feared" are the impetus of philosophy (the love/pursuit of wisdom) according to the *Theaetetus* (155^d3; also cf. Aristotle in the *Metaphysics*, 2.982^b12-13), this "fear" being the *ground mood* or *grundstimmung* of *thaumazein* ("awe" and "wonder"). Oliver Ranner notes that, philosophically speaking, *thaumazein* is to be understood as the condition of "being in aporia": "being in uncertainties, mysteries, doubts, without any irritable reaching after 'fact' and 'reason'," as Keats would say ("to be in aporia": cf. Ranner's abstract, 'Plato and Aristotle on the Origin of Philosophy', *http://www.apaclassics.org/AnnualMeeting/03mtg/abstracts/ranner.html*).

¹¹ Alain Badiou, 'Truth, Forcing, and the Unnamable', Theoretical Writings (London: Continuum, 2004), 127.

¹² "A late sixteenth-century work that Descartes had certainly read, Clavius's book on the *Elements* of Euclid (1589; 2nd edition: 1603), speaks of the need to know 'the mathematical sciences'," explains David Sepper in his

the regulae ad directionem ingenii:¹⁴ the 'rules of the game,' so to speak, by which one learns how to learn), went as far as to say "*ut comœdi, moniti ne in fronte appareat pudor, personam induunt, sic ego hoc mundi teatrum conscensurus, larvatus prodeo*": just as actors, in order to conceal their shame (the shame—*pudor*—of 'acting', of putting on an 'act'), are advised to put on a mask (*personam*), so likewise, striding out upon the *Theatre of the World*'s great Stage (*mundi teatrum conscensurus*), "I advance masked" (*larvatus prodeo*).¹⁵

According to Descartes, learning is, it seems, a rather shameful (shame-faced) activity: a 'playingaround', a childish or child-like 'pretense' (pretending). The theoretical 'theatre' here—the *theatrum* of *mathesis*—is thus both a kind of stage-play and something along the lines of child's play. What is 'acted out' (this phrase itself being a workable translation of *e-ducere*, 'education') is the *actus* (Latin 'doing') or *agere* (Latin 'to do' or 'to set in motion') derived from the Greek *agon* ('conflict' or 'stuggle') and *agogos* (one who 'leads' or 'forges' the way), or more specifically and pointedly the *pais-* or *paidos-agogos:* the *agogos* (the 'forging ahead') of the *pais* or *paidos* ('youth', 'youthful' or 'child-like' one). *Mathesis*—learning in general, and *mathematics* in particular—is pedagogical: it comes down to the act (the actions) of a child at play or of committed play-acting (role-playing, if you will; "a child playing," as Heraclitus says¹⁶).

The link between the Cartesian insight regarding learning as an 'acting out' or 'playing out' and the Keatsian insight regarding the importance of 'negative capability' is also something the pedagogue and playwright John Mighton acknowledged when we had proposed it to him several years ago; indeed, "partial understanding is not a bad thing in mathematics," he explained. "There's an idea now, which I think is widespread, that if you introduce a kid to something that they don't understand completely, can't explain completely, deduce all the consequences of, 'discover', then they're not developmentally 'ready' for that, and shouldn't even start. Not only does this not reflect actual mathematical practice, but I don't think it reflects how kids actually *learn*, because quite often kids only have a partial understanding of things, and they work at it and work at it and work at it and one day the understanding emerges. [...] This is what I call '*emergent intelligence*' [...]; "sometimes actual practice with the rule and using it without the full understanding not only gets kids excited about being able to do the thing, but mysteriously 'opens up' the understanding. This is why I think understanding 'emerges': you don't suddenly understand mathematics—especially at higher levels. Perhaps when you get to higher levels the things at the lower levels become more coherent and complete, but you always realize that there is a mystery at the heart of things."¹⁷

study Descartes's Imagination: Proportion, Images, and the Activity of Thinking. "Descartes's 'mathesis' in fact radicalizes the admonition: learn not 'mathematics' but the foundation of everything that is learnable" (Berkeley: University of California Press, 1996), 150n.22.

¹³ "Descartes notes that 'mathesis' amounts to the same thing as 'disciplina', *discipline*," writes Sepper in the study previously footnoted (151); and in the fourth of his *Regulae ad directionem ingenii* he explained that "[the] demand, made by ancient thinkers, that one learn 'mathesis' -- the way of *cognitive discipline* -- as a prerequisite to the study of wisdom" (indeed the demand that was carved above the entrance to the Platonic Academy, "radicalized" by Descartes, *op.cit.*) was put forth precisely "because it is the simplest and most necessary [*regulum*] of all for preparing the *ingenium*" (150).

¹⁴ cf. René Descartes, Regulae ad directionem ingenii, Oeuvres de Descartes, Vol. X (Paris: J. Vrin, 1956) and Regulae ad directionem ingenii: Règles utiles et claires pour la direction de l'esprit en la recherche de la vérité (The Hague: Martinus Nijhoff, 1977), under 'References', below.

¹⁵ René Descartes, 'Praeambula' (Cogitationes Privatae), Oeuvres de Descartes, Vol. X, (Paris: J. Vrin, 1956), 213.

¹⁶ The "child playing [...] has a kingly (sovereign) power," he explains in Diels-Kranz fragment 52, Bywater fragment 79, Marcovich fragment 93.

¹⁷ Dan Mellamphy and Nandita Biswas Mellamphy, 'trialogue' with John Mighton, Fields Institute for Mathematical Research, October 2005.

Mathesis and Myesis: The Mystery at the Heart of 'the Mathematical'

The idea that understanding "emerges" and "opens up" in the process of following a rule or role, of playing [by] the rules of the game or [by] the roles of the drama (dramathematics), is one reflected in the notions of *negative capability* (as opposed to *positive incapacity*), of *durcharbeiten* (Beckettian "proceeding by aporia"), of *forring* in *set-theory*, and (according to scholars such as Charles Olson, Nathan Scott, Tilottama Rajan and Christopher Bamford¹⁸) Heraclitean/Heideggerian *ontology*. "It is this"—the idea that *mathesis* "emerges" from "acting things out" (*agere, e-ducere*)—"that Heraclitus meant when he laid down the law that was vitiated by [Plato and] Socrates and only restored by [Keats and] Rimbaud," that we are estranged from that which is most familiar to us.¹⁹ Making the unfamiliar familiar, treating the unfamiliar as if it were familiar—"forcing" it, if you will, but not by one's *own* force. Rather, by the force of *the unfamiliar* (the "estranged") *as such*—allowing the *strange, uncertain* and *unknown (unfamiliar*) to 'be', we are brought to the brink of the *ontos* (ontology), the mystery at the heart of things (Heidegger would hearken back to Meister Eckhart's *abgeschiedenheit* and *gelassenheit* in order to explain this process, this uncanny 'active passivity').²⁰

If ontology concerns itself with the question of what counts as (and what accounts for) 'being', then it is indeed a *mathesis* (qua *ta mathemata*). What manifests itself on the stage or in the setting of *mathesis*, through the very pedagogy of *ta mathemata*, is "that which is" (that which "counts for" or "counts as" something, *nescioquid*) <u>prior to</u> "what that is" (*i.e.* the *quod* or *quoddity* prior to the *quid* or *quiddity*; the *quod* of the *quid*, or *nesciquid*—the "*je ne sais quoi*" of Vladimir Jankélévitch²¹). "That which is" (the *nescioquid*) is the numerable *noumenon* encountered in the mathe[ma]tical opera[tion], something (*nescioquid*) which functions *prior to* epistemology. Epistemology, in this sense, is the *gnomon* or *shadow*²² of the numerable *noumenon* or mathe[ma]tical *ontos*—it takes place in the aftermath[esis]. Hence the "mystery at the heart of" *mathesis* is precisely *mysterious: a myesis*—that which the mathematicians of antiquity (and Pythagoras in particular) described as *alogos* ('unspeakable'; *absurdum* in Latin; *sourd* in French; *silent*—deaf and *mute*—in English).

That which is unspeakable (alogos, absurdum), that without what (the nescioquid or je-ne-sais-quoi), is the pure matheme, as Badiou points out in a recent interview;²³ mathesis in this sense precedes physis (ta mathemata is distinguished from ta physika). And yet—as Pythagoras insisted and as Ernst Chladni (amongst others) has so convincingly exemplified—physis (the physical/phenomenal) is the very face/surface of mathesis (the noumenal/mathematical): the mask (larva/persona) with which, through which, and by which that which counts is actually encountered. Physis kryptesthai, wrote Heraclitus; "nature encrypts" (indeed, physis kryptesthai philei: nature loves encryption).²⁴ Alles was tief ist liebt die Maske, wrote Nietzsche; "everything that is deep loves masks" (by extension, mathesis loves physis; physis loves to mask; physis is the mask of mathesis).²⁵ The physical personam, the physical phenomenon, qua physis, qua personam, is precisely what the noumenal note or number "sounds through" (per-sonare).

The pedagogue-understood as the one who playfully, youthfully forges ahead-gives voice via

²⁰ Martin Heidegger, Gelassenheit (Pfullingen: Nske, 1960).

¹⁸ Charles Olson, *The Special View of History: Lectures at Black Mountain, 1956* (Berkeley: Oyez, 1970); Nathan Scott Jr., *Negative Capability* (New Haven: Yale UP, 1969); Tolottama Rajan, 'Keats, Poetry, and The Absence of the Work' (*Modern Philology* 95.3, 1998), 334-351; Christopher Bamford, 'Negative Capability' (*Parabola: Tradition, Myth, and the Search for Meaning* 30.2, Summer 2005), 14-20.

¹⁹ Charles Olson, The Special View of History: Lectures at Black Mountain, 1956 (Berkeley: Oyez, 1970), 29.

²¹ cf. Vladimir Jankélévitch, Le Je-ne-sais-quoi et le Presque-rien -- The I-Know-Not-What and the Almost-Nothing -- (Paris: Presses Universitaires de France, 1957).

²² (ombre du nombre)

²³ Alain Badiou, 'Philosophy, Science, Mathematics', Collapse: An Independent Journal of Philosophical Research and Development 1 (September 2006), 19.

²⁴ Heraclitus, Diels-Kranz fragment 123, Bywater fragment 10, Marcovich fragment 8.

²⁵ Nietzsche, Jenseits von Gut und Böse (Leipzig: C.G. Naumann, 1895), §40.

playful masks and is *attuned* to that which "sounds through" masks (the *per-sonare* of *persona/phenomena*): "the mystery" that is hidden in and as the "depth" or "heart" of things. Hence the *tetractys* or *fourfold* of *mathesis* according to Heidegger: in the subsection of *What is a Thing* entitled '*Mathesis* and the Mathematical', Heidegger explains that "the Greeks identify the mathematical, *ta mathemata*, with the following determinations": that of *ta physika* (natural things), that of *ta poioumena* (artificial things), that of *ta chremata* (things in use) and that of *ta pragmata* (useful things). Things are "learnable" (*mathemata*) insofar as they are natural, artificial, in use, or useful (he uses, as an example, three chairs—*i.e.* 'threeness' encountered in the form of three chairs—in *What is a Thing*?; "[w]hat 'three' is the three chairs do not tell us," he notes; "nor three *apples*, three *cats*, nor any *other* three things. [...] Things do not help us to grasp 'three' as such, i.e. *threeness*"²⁶): these are the phenomenal aspects or "masks" of *mathesis*—its epistemological epidermis, so to speak. What lies at and as the heart of such phenomena (poioumena, pragmata, physika, chremata) is a *fifth-point*, *pempte-ouisia*, or *quinta-essentia*: "the characterization running through these four,"²⁷ or their "fundamental condition" qua "fundamental presupposition,"²⁸ namely "a domain (*Spielraum*) where things [can] show themselves";²⁹ a 'set' or 'setting', in other words—a 'stage' or 'screen' for (dis)play or projection.

This fifth point or place of fourfold frames or phenomena (the heart of phenomena's *quincunx*) is the set or setting in which and from which all that may be learned can be learned: it is the prologue to all possible pedagogy, and, as prologue (*pre-logos*, *proto-logos*), prior to all subsequent logics and logistics (*logos*), prior to *davar* (the word and worded world, in Hebrew), *logos* (the word and worded world, in Greek), *verbum* (the word and worded world, in Latin). It is, again, *alogos*: mute, deaf, dumb the mute, deaf, dumb number prior to the wor[1]d. "Therefore," writes Heidegger (who distinguishes between the earthly ground and the worded world in his works), "we do not first get it *[i.e.* number, *ta mathemata*] out of things, but, in a certain way, we bring it already with us": it is already there, as the condition of our knowing.³⁰

X Marks the Spot

X marks the spot, then: whatever one comes across (whatever one encounters) is at the point of its crossing, at the point of its encounter, something already *there*—something that *counts*, something that *matters*—regardless of what this thing (this matter) 'is' in the wor[l]d (*i.e.* according to the logic and logistics of the wor[l]d). "X is also the letter most commonly used in the sciences for the unknown," Mighton reminded us in our 'trialogue' of 2005; whereas on the grade-sheet it might stand as a marker of 'failure' in the sense of a 'positive incapacity', in the theatre of mathematics (mathematical opera[tion]s) it "[stands] for a different kind of ignorance: a positive and healthy ignorance—a sense of mystery." What's more, X marks the spot on the stage where the actor "proceeds by aporia": "blocks" his or her progression and progresses by way of this blocking. At this suggestion, the playwright and mathematician nodded in agreement; "we're always led by the X-factor—the mysteries—of the questions we can't answer," he responded. "In mathematics, obviously, but in playwrighting too. My favourite playwrights are writers like Chekhov who were able to capture in even the most banal events the profound mysteries of existence and of human nature [...]: that feeling for, and that wonderful sense of, the mystery at the heart of things." X marks the spot where one *stands* and where, *standing there*, one can (by way of one's *negative capability, abgeshiedenheit, gelassenheit, durcharbeiten, what-have-you*) understand.

²⁶ Martin Heidegger, What is a Thing? (Chicago: Henry Regnery, 1967), 74.

²⁷ Martin Heidegger, What is a Thing? (Chicago: Henry Regnery, 1967), 70.

²⁸ Martin Heidegger, What is a Thing? (Chicago: Henry Regnery, 1967), 75.

²⁹ Martin Heidegger, What is a Thing? (Chicago: Henry Regnery, 1967), 92.

³⁰ Martin Heidegger, What is a Thing? (Chicago: Henry Regnery, 1967), 74.

The underwhelming, here, can be overwhelming ("awe inspiring").³¹ Chekhov in the absence of checkmarks (Chekhov in a field of 'X's, *quincunxes*³²). And this spot is always already there (the prologue, the *prologos*): to discern it is to discern—to learn—what one already has (one's 'place', one's 'set', one's 'setting'); it is a matter of *recognition*, or as Olson suggests, of *refamiliarization*.

For Mighton, "the key" to such fundamental insights (to this recognition or refamiliarization) "is to reduce any given problem to its simplest or most elementary (elemental, essential) aspect, which usually comes down to an action. Strip the problem down to the level of action: what there is to do." Mathematization always, again, comes down to a dramatization (as we say), to an (en)action, a playing out (e-ducere). "The act, the action, is its essential feature. Anything and everything (no matter what you're looking at) can be reduced to an action. If you're going to teach reducing, start with fractions that will always divide by two, three or five, so that if the kids can count on their fingers by those they can do all their divisions and stuff. And if I introduce long division," he admits, in Thomas-Brownian (quincunxian/fifth-part) fashion, "I start with the number five because every kid can count to five. The interesting thing is, in terms of getting kids to recognize what they can do, or take steps by themselves, that you don't even have to talk about the concepts to have them see how these operations work and extend the application of their principles. If you work in these small steps sometimes the brain will suddenly organize itself to the point where it will recognize things it has never actually seen before. You start on your fingers and then, quite naturally, the fingers will point you in all sorts of different directions and allow you to grasp all sorts of different conceptions. Even with more complex problems, you can allow that recognition or that understanding to emerge, you can harness what they can do and what they can understand about the world, and sometimes (often times) there are mysterious leaps that occur, or they'll go from something that appears more mechanical to the sudden manifestation of a general insight or some kind of ability that seems beyond what you are teachingand that's what I don't think we've accounted for in math education. Math educators have done a great job of developing some very good problems and activities and such, but the question is: how do you get the kids there if they're not ready for it?---that's where we have to do more work. How do you allow kids to 'recognize' and to allow their understanding to emerge?"

The emergence of this understanding is a special kind of cognition: namely, a *re-cognition*. It is a matter of "recognizing" *portions, proportions, patterns* or *parts*—acknowledging their *familiarity*, even if, at times, it might seem to be *uncanny* (the familiarity of a *déjà-vu*); in any case, it is a matter of no longer being estranged from these *portions, proportions, pattern,s* or *parts*, no longer being estranged (as Heraclitus said) from that with which we should in fact be most familiar. What "emerges" is something that was already there—something hitherto submerged, suppressed, shut out (perhaps by dint of being merely *partial*, a mere *portion*, a *point* or *hole* in the *whole*). Education, *e-ducere*—leading or conducting *out*—seems (etymologically) to reflect this process of emergence or coming-forth. Indeed, education in this sense—or *mathesis*, more precisely—would be the veritable *dramatization* of this emergence: its *enaction*, its *performance*. The key, again, is to reduce any given problem to an *act*, an *action*—to its underlying *drama*, its theoretical/theatrical *performance* and/or *practice*, which is an essential feature of any and every mathe[ma]tical mystery. If (as Mighton said) "behind every action there is the mystery," we should add that the action itself is its *embodiment* (it "embodies" the mystery, in other words). Which

³¹ Cf. the preceding footnote re: the philosophical *thaumazein* (the "awe" and "wonder" *at* and *as* the root – *rhizome* –of *philo-sophia*) and Rainer's related abstract at *http://www.apaclassics.org/AnnualMeeting/03mtg/abstracts/ranner.html*.

³² Cf. sir Thomas Browne's rather Jorge-Luis-Borges-like Quincunxial Garden of Cyrus. In this treatise, X again marks the spot, in the form/formation of quincunxes: "a continued pattern of four [points placed] at the corners of a [square or] parallelogram" with a "fifth point" (spot) at its centre (Frank Huntley, 'Introduction' to sir Thomas Browne, Hydriotaphia [Urn Burial] and The Garden of Cyrus, New York: Appleton-Century-Crofts, 1966, viii). "The addition of two fives" (for instance, counting the fingers of both hands) "is ten, or the Roman X, the perfect number, made of two fives -- Roman Vs -- joined at their apices," notes Huntley (x). The X or quincunx is thus the very symbolon of an encounter, confrontation, and conjunction (this at the very mid-point hidden at the heart or centre of the fourfold/square).

brings us to the concluding statement of our 'trialogue' with Mighton: namely, that *action* is a kind of *embodiment* and that in *every* action there is a *mystery* embodied—something that theatre, in fact, exemplifies wonderfully. "Yes. And that's why, I think, theatre was once a religious event," remarked Mighton. "This explains the awe inherent in mathematics—in the sciences as well as the arts. The same religiousness or sense of mystery is at the heart of both the arts and sciences."³³ In this regard, what is most interesting is that *mystery* itself originally meant *closed-mouthed*: the *mystes* (the Greek word for an initiate, a student who was there to listen quietly, and to silently perform the religious rituals, the mystery's actual actions) was the one who kept his mouth closed, and *mysteries* were therefore (are therefore) things *unspeakable, unspoken (alogos, alogon)*.

To talk about 'mathematics' and 'dramatics' ('mathematization' and 'dramatization') is thus in some respects to speak about the unspeakable (something so fundamental as to be prior to the wor[l]d—its very ontogenesis). The fundaments of learning are there embodied. Embodiment, however, also *encrypts (physis kryptesthai:* the physical encrypts³⁴); and yet this encryption, this occlusion, this obscuring, in no way incapacitates the *paidos-agogos:* the unseen and/or unspeakable (the ill-seen and/or ill-said) is the very *chthonos* (ground) and *hodos* (path) of the *paidos-agogos*—the set and setting of *ta mathemata*, the *theatrum* of *mathesis.* To ignore this (to close one's eyes to blindness, turn a deaf ear to deafness) would amount to a fundamental incapacity with respect to education. Hence we conclude with two quotations (two ontological epigraphs), the first from A. N. Whitehead, the second from J. Lacan: "Not ignorance, but ignorance of ignorance is the death of knowledge" and "Mathematization alone reaches a 'real'."

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³³ Dan Mellamphy and Nandita Biswas Mellamphy, 'trialogue' with John Mighton, Fields Institute for Mathematical Research, Toronto, Canada, October 2005.

³⁴ Heraclitus, Diels-Kranz fragment 123, Bywater fragment 10, Marcovich fragment 8, previously quoted.

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