Radical Nonlocality

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Abstract: This article points out a nonlocality of quantum mechanics that is significantly more radical than that implied by violations of Bell locality or Einstein locality. It consists in the fact that the spatiotemporal differentiation of the physical world is incomplete. The so-called parts of space only exist to the extent that they are physically realized, and arbitrarily small parts cannot be physically realized. Further it is shown that intrinsically all fundamental particles are identical in the radical sense of numerical identity. Hence it is impossible to model reality "from the bottom up," whether on the basis of an intrinsically and completely differentiated space or spacetime or out of a multitude of intrinsically distinct building blocks. Quantum theory's explanatory arrow points in the opposite direction — from unity to multiplicity. In addition to establishing these conclusions, the article examines their implications for the enterprise called physics, illuminates these conclusions and their implications in a quintessential Indian philosophical context, and points out that while the radical nonlocality of the quantum world renders intelligible the possibility of paranormal correlations, quantum mechanics offers no help in explaining how paranormal phenomena come about.

Keywords: quantum mechanics, interpretation (quantum theory), nonlocality, space and time, ultimate reality, fundamental particles, Sri Aurobindo, ontology, metaphysics.

1. Introduction

This article points out a nonlocality of quantum mechanics that is significantly more radical than that implied by violations of Bell locality or Einstein locality. It consists in the fact that the spatiotemporal differentiation of the physical world is incomplete; it does not go "all the way down." The so-called parts of space only exist to the extent that they are physically realized, and arbitrarily small parts cannot be physically realized. If we conceptually partition the physical world into smaller and smaller regions, we therefore reach a point where the distinctions we make between regions no longer correspond to anything in the physical world.

By the same token, if we go on dividing material objects, their components lose their distinctive properties and, along with them, their separate identities. This leads to the conclusion that intrinsically all fundamental particles are identical in the radical sense of numerical identity. It is therefore impossible to model reality "from the bottom up," whether on the basis of an intrinsically and completely differentiated space or spacetime or out of a multitude of intrinsically distinct building blocks. Quantum theory's explanatory arrow points in the opposite direction — from unity to multiplicity.

In addition to establishing these conclusions, the article examines their implications for the enterprise called physics. In a more speculative vein, it illuminates these conclusions and their implications in a quintessential Indian philosophical context.

The article is organized as follows. Section 2 draws a distinction between the real quantum measurement problem and a fictitious one, which is generally mistaken for the real one. The real measurement problem has to answer two questions: (i) Why is the fundamental theoretical framework of contemporary physics a *probability calculus*? (ii) Why does this assign probabilities to *measurement outcomes*? The view according to which a quantum state collapses (or appears to collapse) upon measurement, along with the ensuing fictitious problem regarding the cause of the collapse, arises from a misinterpretation of the time on which a quantum state functionally depends.

Section 3 addresses the questions that are raised by the real measurement problem in the context of a double-slit experiment. Because it is possible for a particle to go through two slits without being divided into parts that go through different slits, the slits cannot be different parts of space nor, therefore, can space be something that by itself has parts. If at all we think of space as an independently existing expanse, rather than as a set of more or less fuzzy spatial relations or as a quality to which such relations owe their spatial character, we must think of it as intrinsically undivided.

This prompts us to ask what it is that furnishes space with its so-called parts. We shall find that space owes its parts to detectors in the broadest sense of the word. By realizing (making real) a particular region of space, a detector makes it possible to attribute to a particle the property of being in that region. In more general terms, the measurement apparatus is needed not only to indicate the answer to a question but also, and in the first place, to define a question by making its possible answers available for attribution. This solves the second part of the measurement problem.

But if it is impossible to attribute to a material object the property of being in a region of space unless this region is realized by a detector, then the spatiotemporal differentiation of the physical world cannot be complete — it cannot go "all the way down." A quantum state therefore cannot be an evolving physical state, for such a state, existing as it does at every instant of time, requires for its existence a completely differentiated time continuum. But if a quantum state cannot be an evolving physical state, then what else could it be than what it manifestly is — a probability algorithm? This solves the first part of the measurement problem.

Differently put, if we conceptually partition the physical world into smaller and smaller regions, we reach a point where the distinctions we make between regions no longer correspond to anything in the physical world. By the same token, if we go on dividing material objects, their components lose their distinctive properties and, along with them, their separate identities. This leads to the conclusion, in Sec. 4, that every fundamental particle is intrinsically identical (in the strong sense of numerical identity) with every "other" fundamental particle. It therefore is impossible to model reality "from the bottom up," whether on the basis of an intrinsically and completely differentiated space or spacetime or out of a multitude of intrinsically distinct building blocks. As will be shown in Sec. 5, quantum theory's explanatory arrow points in the opposite direction — from unity to multiplicity.

As long as we keep thinking of macroscopic objects as composed of microscopic ones, quantum mechanics confronts us with an apparent circularity: while macroscopic objects are made of microscopic ones, microscopic objects can only be described in terms of macroscopic states or events. This circularity disappears as soon as we realize that quantum mechanics is concerned with the emergence of multiplicity out of unity, a process appropriately described by the term "manifestation." (In the context of spiritual cosmologies the term refers to the emergence of the Many out of the One.) The stages of this process being to varying degrees indefinite and indistinguishable, they can only be described in terms of probability distributions over events that are definite and distinguishable, and such events only exist in the macroworld.

Section 6 aims to dispel the misconception that the only way of making sense of the quantum-mechanical correlation laws is to repeat the intellectual sleight of hand that made classical physics seem consistent with local realism, and Sec. 7 clarifies the sense in which local conservation laws are local.

Section 8 proceeds to examine the validity of several locality principles in the context of spatially separated yet statistically non-separable ("entangled") quantum systems. As it turns out, none of them is violated by the quantum-mechanical probability calculus or the equivalent correlation laws. Violations of locality only occur if one ventures beyond the theory's testable predictions, with the hope of explaining them in terms of underlying natural processes, by reifying some calculational tool, with or without postulating physical quantities that cannot be measured or that exist without actually being measured. So what exactly is violated? What is spooky about "spooky actions at a distance"? The answer is that the offending correlations cannot be construed as actions. In other words, they cannot be explained in terms of causes and effects, neither by reifying calculational tools nor or in any other way.

The failure of the quantum-mechanical correlation laws to yield to the explanatory strategies at our disposal highlights a conflict between the ontological implications of quantum theory's testable predictions and certain all but incorrigible ways of thinking about the physical world. Section 9, which is inspired by the integral monism of Sri Aurobindo, throws light on the origin of this conflict.

According to Sri Aurobindo, mind is a secondary, limiting and dividing action of the original creative principle, supermind. As long as mind is separated from its supramental parent, as it is in us, it not only divides *ad infinitum* but also takes the resulting multiplicity for the original truth or fact. This is why we tend to construct reality from the bottom up, on an intrinsically and completely differentiated space or spacetime, out of locally instantiated physical properties, or else by aggregation, out of a multitude of individual building blocks. On the other hand, if mind is employed by supermind as part of the creative action supporting the cosmos, its tendency to divide *ad infinitum* is checked, and this is why there are limitations on the objectification of our mental distinctions.

Section 10 summarizes and concludes.

2. The quantum measurement problem

The mathematical formalism of quantum mechanics is a probability calculus. Its algorithms (state vectors, wave functions, or density operators, here collectively referred to as "quantum states") serve to assign probabilities to possible measurement outcomes on the basis of actual measurement outcomes. Any statement that goes beyond this characterization of the quantum formalism is "not even wrong," to use Wolfgang Pauli's epithet for hypotheses that cannot be empirically falsified.

This does not mean that it is idle to look for an interpretation that goes beyond the formalism's testable predictions. One certainly has a right to ask why the general theoretical framework of contemporary physics should be exclusively concerned with statistical correlations between measurement outcomes. This question is the quantum measurement problem proper, unadulterated by untestable assumptions.

According to the mathematical formalism (supplemented by the minimal instrumentalist interpretation that renders it applicable to the physical world), the time on which a quantum state functionally depends is the time of the measurement to the possible outcomes of which the quantum state serves to assign probabilities. As soon as one thinks of this event-specific time dependence as the continuous time-dependence of an evolving physical state, one is faced with the mother of all quantum-mechanical pseudo-problems: why does a quantum state cease to evolve continuously — why does it *collapse* (apparently if not really) — at the time of a measurement? This is how the quantum measurement problem is usually stated, thanks chiefly to von Neumann (1955).

One of the earliest proposed solutions to this pseudo-problem was to implicate the consciousness of the observer. A physical system cannot collapse the quantum state of another physical system. If the quantum state of a physical system can be affected in this manner, then what so affects it has to be something nonphysical like consciousness. This conclusion would be quite convincing if the premise (viz., that quantum states are evolving physical states) were correct.

When two quantum systems interact, they get entangled, in the sense that subsequently the probabilities of the possible outcomes of a measurement performed on one of the systems are correlated with the probabilities of the possible outcomes of a measurement performed on the other system. That much is testable. What is not testable is the notion that the measurement apparatus can be treated as just another quantum system. If this is done all the same, the apparatus gets entangled with the measured system, and we go down von Neumann's garden path of infinite regress — unless we are stopped by something nonphysical like consciousness. This line of reasoning adds a second false premise to the first, viz., the assumption that it is legitimate to treat the outcome-indicating property of the apparatus as just another quantum-mechanical observable.

Here is why the measurement apparatus is not just another quantum system. The mathematical formalism of quantum mechanics being a probability calculus, it presupposes the events to which it serves to assign probabilities. Without the incontestable factuality of measurement outcomes, quantum theory would be disconnected from reality and hence irrelevant to physics. The question therefore is not how far quantum superpositions propagate before they collapse, nor what it is that causes a superposition to collapse. What we need to ask instead is: why is the fundamental theoretical framework of contemporary physics a probability calculus, and why does this assign probabilities to measurement outcomes?

3. The heart of quantum mechanics

When Feynman said that the double-slit experiment "has in it the heart of quantum mechanics" (Feynman et al, 1965), he may have been more right than he knew. The double-slit experiment owes its well-deserved fame to the fact that if there are no events or states of affairs from which the slits taken can be inferred, then it is inconsistent to assume that each particle nevertheless went through a single slit — either the left one (*L*) or the right one (*R*). (David Bohm has found a way to avoid this conclusion, though at a price that few physicists are willing to pay.)

If a particle has passed the slit plate without going through a particular slit, it must have gone through both slits. But how is that possible? It would indeed be impossible if *L* and *R* were different parts of space. Since it is, in fact, possible for a particle to go through both slits, *L* and *R* cannot be different parts of space. Nor, therefore, can space be something that by itself has parts.

We are inclined to think that *L* and *R* are different. But how are they different? They are cutouts in a slit plate — things that have been removed, things that are *not* there. What difference do they leave behind after they have been removed? The difference between the positions they previously occupied? But positions are properties, and properties exist only if they are possessed. Or do they?

There is something fishy about the way we tend to think about space. We all more or less readily agree that red, round, or a smile cannot exist without a red or round object or a smiling face. That's why the Cheshire cat strikes us as funny. Why then do we tend to believe that positions exist by themselves, without being possessed?

It has not always been so. Influential thinkers from Aristotle to Kant and Gauss have insisted that potential infinities, such as the possibility of conceptually dividing space *ad infinitum*, be thought of as just that — possibilities rather than actualities. Kant (1929) wrote that the so-called parts of space

> cannot precede the one all-embracing space, as being, as it were, constituents out of which it can be composed; on the contrary, they can be thought only as *in* it. Space is essentially one; the manifold in it ... depends solely on the introduction of boundaries [*Einschränkungen*].

What Kant says about the parts of space applies, a fortiori, to the so-called points of space. Insofar as the concept of composition and the concept of a point relate to the space in which physical experiments are performed, they derive their meanings from our immediate, nonverbal knowledge (intuition, *Anschauung*) of space. They presuppose space and thus cannot be its constituents. In the second half of the nineteenth century mathematics nevertheless shifted to dealing with the continuum as a set of points. "So successful has this shift been," von Weizsäcker

(1980, Sec. IV.4) remarked, "that it is nearly impossible to disabuse the contemporary student of mathematics of the superstition that this conception is the only possible, indeed 'the' theory of ... 'the' continuum."

If one calls a self-adjoint operator "elephant" and a spectral decomposition "trunk," one can prove that every elephant has a trunk. Likewise, if one calls a real number "point" and the transfinite manifold of real numbers "continuum," one can think of the continuum as composed of points. But has this mathematical continuum any more in common with physical space than the spectral theorem has with certain pachyderms? Von Weizsäcker (1980, p. 130) did not think so:

> The conception of the continuum as potential, which originated with Aristotle, appears to be more suitable for the quantum theoretical way of thinking than is the set theoretical conception of an actually existing transfinite manifold of "real numbers," or of the spatial points they designate. The "real number" is a free creation of the human mind and perhaps not conformable to reality.

If proof is needed that the set-theoretic conception of space is not conformable to reality, it is the ability of a particle to go through more slits than one (as a whole, without being divided into parts that go through different slits), which implies that space cannot be something that by itself has parts. If at all we think of space as a self-existent (substantial) expanse, rather than as a set of more or less fuzzy spatial relations or as a quality to which such relations owe their spatial character, we need to think of it as intrinsically undivided.

To what, then, does space owe its so-called parts? Recall that the question “Through which slit did the particle go?” has an answer if and only if there is an actual event or state of affairs from which the answer can be inferred. For this to be the case, the setup must include the equivalent of two detectors, one for each slit. The first and more obvious function these detectors fulfill is to indicate the slit through which the particle went. The second is just as important: since in their absence the two slits form an undivided whole, it also falls to them to make the slits distinct, to realize them as separate regions. By realizing (making real) a particular region of space, a detector (in the broadest sense of the word) makes it possible to attribute to a particle the property of being in that region.

This answers the question of why the events to which quantum mechanics serves to assign probabilities are measurement outcomes. The measurement apparatus is needed not only to indicate the answer to a question but also, and in the first place, to define a question by making its possible answers available for attribution. This is precisely what Niels Bohr tried to convey by stressing that, out of relation to experimental arrangements, the properties of quantum systems are undefined (Jammer, 1974; Petersen, 1968).

But if it is impossible to attribute to a material object the property of being in a region of space unless this region is realized by a macroscopic device, then no material object can have a sharp (pointlike) position (relative to another material object), for no macroscopic device can realize such a position. This means that we can conceive of a partition of space into finite regions so small that none of them is realized. None of them, therefore, exists. From this it follows that the spatial differentiation of the physical world is incomplete — it does not go "all the way down."

The same applies to the temporal differentiation of the physical world, for two reasons. The first is the relativistic interdependence of distances and durations. If space is not differentiated "all the way down," spacetime cannot be so differentiated, and if spacetime is not differentiated "all the way down," time cannot be so differentiated. The second reason is that just as properties or values need to be realized by macroscopic devices, so the times at which properties or values are possessed need to be realized by macroscopic clocks. And just as macroscopic devices cannot realize sharp positions, so macroscopic clocks cannot realize sharp (instant-like) times. The uncertainty principle for energy and time forbids it, for it implies that a transition from one time-indicating state to another cannot occur at an exact time (Hilgevoord, 1998). Time, therefore, cannot be a set of instants. If at all we think of physical time as an expanse, rather than a set of temporal relations or a quality to which such relations owe their temporal character, we have to think of it as intrinsically undivided.

But if neither the spatial nor the temporal differentiation of the physical world goes "all the way down," then determinism is out the window — not only the unbroken determinism of the past and the respective cryptodeterminisms of Bohm and Everett but also the determinism-between-measurements posited by collapse interpretations. Quantum states therefore cannot be evolving physical states, for an evolving physical state exists at every instant of time and requires for its existence a completely differentiated time (which is to say, a time *continuum*). This is the reason why the fundamental theoretical framework of contemporary physics is a probability calculus. If the spatiotemporal differentiation of the physical world is incomplete, a quantum state cannot be anything but a probability algorithm.

The incompleteness of the spatiotemporal differentiation of the physical world is a direct consequence of quantum theory's testable predictions. No prior metaphysics is needed to conclude that the world's spatiotemporal aspects are not differentiated "all the way down." Why is this all-important fact about the physical world not universally recognized? Is it not strange that the ontological and/or epistemological status of the wave function has been the focus of a lively controversy for nearly a century, while the ontological status of the points and instants on which a wave function depends has hardly ever been called in question? There appear to be deeper reasons that have something to do with how the spatiotemporal aspects of the world are perceived by us, as distinct from how they are described by quantum mechanics (Mohrhoff, 2006, 2007). We will return to this topic in Sec. 9.

4. A central mystery of physics

There is one notion that is decidedly at odds with the incomplete spatial differentiation of the physical world. It is the notion that fundamental particles — according to the standard model of fundamental particles and forces, the quarks and the leptons — are pointlike. In reality, what characterizes a fundamental particle is its lack of internal structure. This could mean that it has a pointlike form, but it could also mean that it has no form at all.

The notion that a fundamental particle is literally pointlike is unwarranted on both theoretical and experimental grounds. In addition, it explains nothing. Specifically, it does not explain why a composite object — be it a nucleon, a molecule, or a galaxy — has the form that it does, inasmuch as all empirically accessible forms are fully accounted for by the relative positions and orientations of their material constituents.

Is there *any* property that a fundamental particle might possess "by itself" — any property that does not merely characterize its relations to the rest of the world? The answer is a resounding No. Positions and momenta are kinematical relations, coupling parameters characterize dynamical relations, and the physical significance of mass is confined to mass ratios. But if there is not property that a fundamental particle possesses "by itself," then there is no property by which one fundamental particle is intrinsically distinct from another.

We arrive at the same conclusion by considering a physical system consisting of two particles. Suppose that there are four non-overlapping regions A, B, C, and D, that initially one particle is found in A and one in B, and that subsequently one particle is found in C and one in D. We will abbreviate "the particle in region X" to p_X. Two kinds of situation occur. In situations of the first kind, the two particles carry "identity tags," which makes it possible to identify them across time. Such particles are said to be distinguishable. In this case either p_C is identical with p_A (and p_D with p_B) or p_C is identical with p_B (and p_D with p_A).

In situations of the second kind, the particles do not carry "identity tags." Such particles are said to be indistinguishable or "of the same type." In this case there are no events or states of affairs from which the answer to the following question can be inferred: "Which of the two particles present at the initial time is identical with which of the two particles present at the final time?" Can we nevertheless assume that the question has an answer, albeit one not known? Emphatically not, for doing so leads to predictions that are in conflict with both quantum mechanics and the experimental data. About this "miraculous identity of particles of the same type" Misner et al (1973, p. 1215) wrote that it "must be regarded, not as a triviality, but as a central mystery of physics."

Quantum mechanics challenges us to think in ways that do not raise unanswerable questions. If we take it for granted that space is an intrinsically differentiated expanse, we are led to ask the unanswerable question "Through which slit did the particle go?" If we take it for granted that initially there are two things, p_A and p_B, and that subsequently there are the same two things, p_C and p_D, we are similarly led to ask an unanswerable question. We can prevent this question from arising by proceeding instead from the following assumption: there is but *one* thing; initially it is present in both A and B; thereafter it is present in both C and D.

This is how quantum mechanics settles a question that has been debated for centuries. Suppose that in front of you there are two exactly similar things. The only difference between them is that they are in different places. Is the fact that they are in different places the *only* reason why they are *two* things, or is there another reason? If it is the only reason — and this is what the quantum-mechanical predictions imply — then what there is in front of you is not two things in two places — this is one "two" too many — but one and the same thing in two places.

What holds for two particles holds equally for any number of particles. Intrinsically, therefore, all fundamental constituents of matter are identical in the strong sense of numerical identity. This also holds if the number N_F of particles present at the final time differs from the number N_I of particles present at the initial time. There is but one thing, initially present N_I times in N_I different regions or moving in N_I different directions, and subsequently present N_F times in N_F different regions or moving in N_F different directions.

5. The explanatory arrow of quantum mechanics

To recap, if we conceptually partition the physical world into smaller and smaller regions, we reach a point where the distinctions we make between regions no longer correspond to anything in the physical world, and if we go on dividing material objects, their components lose their distinctive properties and, along with them, their separate identities. This makes it impossible to model reality "from the bottom up," whether on the basis of an intrinsically and completely differentiated space or spacetime or out of a multitude of ultimate building blocks. Quantum theory's explanatory arrow points in the opposite direction: from unity to multiplicity, from an Ultimate Reality intrinsically beyond description to a world of forms — forms that ultimately resolve themselves into spatial relations between formless particles — relations that ultimately are self-relations, particles that ultimately are identical in the strong sense of numerical identity, each being intrinsically the selfsame Ultimate Reality.

The transition from unity to multiplicity is effected by a progressive differentiation of the undifferentiated, leading from an undifferentiated Reality via increasingly differentiated structures — numerically identical particles, non-visualizable atoms, partly visualizable molecules — to the most differentiated structure, the macroworld. As long as we keep thinking of macroscopic objects as composed of microscopic ones, we are confronted with an apparent circularity: while macroscopic objects are made of microscopic ones, microscopic objects can only be described in terms of macroscopic states or events. This apparent circularity has occasionally been remarked upon, for instance by Landau and Lifshitz (1977), who wrote that "quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation."

The apparent circularity disappears once we realize that quantum mechanics, rather than being about things that are made of other things, is concerned with the emergence of multiplicity out of unity, a process appropriately described by the term "manifestation." The so-called microscopic objects are instrumental in the manifestation of the macroworld. They are stages in the transition from an undifferentiated Reality to the macroworld, stages that are characterized by varying degrees of indefiniteness and indistinguishability. How then do we describe, with mathematical rigor, the indefinite and indistinguishable? We must resort to probability distributions over events that are definite and distinguishable, and such events only exist in the macroworld. What is instrumental in the world's manifestation can only be described in terms of the final result, the manifested world. (For a rigorous definition of the term "macroworld" see Section 8 of my 2009b and Chapter 19 of my 2011).

6. Classical illusions

The problem of nonlocality is as old as Newton's theory of gravity. Newton's (1729) stance with regard to the instantaneous and apparently unmediated action at a distance implied by his theory is well known:

> I have not been able to discover the cause of those properties of gravity from phænomena, and I frame no hypotheses.... to us it is enough, that gravity does really exist, and act according to the laws which we have explained.

The relativistic delay between causes and effects in classical electrodynamics and in the Einstein's theory of gravity subsequently made it seem as if actions at a distance could be reduced to local actions. In reality, what made it possible to entertain this belief was something else, viz., "our habit of inappropriately reifying our successful abstractions" (Mermin, 2009), which Whitehead (1997/1925) has dubbed "the fallacy of misplaced concreteness." Mermin recalls:

> When I was an undergraduate learning classical electromagnetism, I was enchanted by the revelation that electromagnetic fields were real. Far from being a clever calculational device for how some charged particles push around other charged particles, they were just as real as the particles themselves, most dramatically in the form of electromagnetic waves, which have energy and momentum of their own and can propagate long after the source that gave rise to them has vanished.
>
> That lovely vision of the reality of the classical electromagnetic field ended when I learned as a graduate student that what Maxwell's equations actually describe are fields of operators on Hilbert space. Those operators are quantum fields, which most people agree are not real but merely spectacularly successful calculational devices. So real classical electromagnetic fields are nothing more (or less) than a simplification in a particular asymptotic regime (the classical limit) of a clever calculational device.

Incidentally, since it is more profitable for science journalists and popularizers of quantum physics, not to speak of certain woolly masters (Kaiser, 2011), to showcase interpretations championed by vocal minorities, the fact that "most people agree" goes woefully underreported.

The testable predictions of classical physics are based on correlations, not on any story that purports to explain how causes produce effects. I jiggle the electrons in this aerial, and in due course electrons in that aerial begin to jiggle as a result. Based on how I jiggle the electrons here, Maxwell's equation and the Lorentz force law allow me to calculate how the electrons will jiggle there. However, because the testable predictions of classical physics are based on deterministic correlations, rather than statistical ones, they seem to admit of causal interpretations, and this makes it possible to invent such a story. This is how the electromagnetic field — a calculational tool —came to be thought of as a physical entity in its own right, which is locally acted upon by charges, which locally acts on charges, and which mediates the action of charges on charges by locally acting on itself. The principle at the heart of this story has been felicitously articulated by Dewitt and Graham (1971):

> physicists are, at bottom, a naive breed, forever trying to come to terms with the "world out there" by methods which, however imaginative and refined, involve in essence the same element of contact as a well-placed kick.

The erroneous impression that local action is intelligible derives from the familiarity of experiences involving pushing or pulling, which a closer look reveals to be based on interatomic and intermolecular forces that act at a distance. (Besides, even if we granted that the classical laws describes local cause–effect relations, they do not explain how these relations are physically realized.)

In the classical limit, the quantum-mechanical probability algorithms degenerate into trivial probability algorithms, which only assigns trivial probabilities (either 0 or 1). A trivial probability algorithm — represented by a point in some phase space — can be interpreted as a state in the classical sense of the word: a collection of possessed properties. Hence it may be said that the quantum laws, which correlate the probabilities of measurement outcomes statistically, degenerate in the classical limit into laws that deterministically correlate intrinsically possessed properties or values. And since deterministic correlations lend themselves to causal interpretations, it may be said that, as a result, the quantum-mechanical probability algorithms degenerate into algorithms that serve to compute the effects that matter has on matter. They do not degenerate into descriptions of physical mechanisms or natural processes by which matter acts on matter.

7. Local conservation laws

Every relativistic field theory is defined by means of a function that is known as the Lagrangian. According to Emmy Noether's famous theorem, every symmetry of the Lagrangian implies a local conservation law. In other words, if the Lagrangian is invariant under a continuous transformation of the fields on which it depends, there is a corresponding physical quantity that is locally conserved on account of this invariance. Noether's theorem serves not only to identify the conservation laws that exist in any given theory but also to *define* the theory's conserved quantities. Thus while it would not be wrong to say that Noether's theorem implies the local conservation of energy–momentum if the Lagrangian is invariant under translations in spacetime, it would be more to the point to say that it implies the existence of a conserved quantity; to this we give the name "energy-momentum." If a theory is not invariant under translations in spacetime, this quantity does not merely fail to be conserved; rather, it is ill-defined (as it actually is in Einstein's theory of gravity, except in regions of spacetime where the curvature of the free-fall geodesics can be ignored).

Local conservation laws can be expressed as equations of continuity. What does "continuity" designate in this context? Needless to say, it would be simple-minded in the extreme to construe a physical quantity whose existence is implied by the invariance of a Lagrangian under spacetime translations as a continuously distributed and continuously moving stuff of some kind. The same applies to charges — physical quantities whose existence is implied by the invariance of a Lagrangian under gauge transformations. If quantum fields are, as Mermin put it, "merely spectacularly successful calculational devices" (in which case classical fields are simplifications of such devices useful in a particular asymptotic regime) then the question reduces itself to this: what effect does an equation of continuity have on the testable predictions of a theory?

Relativistic quantum field theories predict correlations between the in-states and the out-states in collision experiments involving particles, and they are tested by measuring these correlations.

"Local conservation of energy-momentum" simply means that the total energy-momentum of the outgoing particles (including gauge bosons like photons) will be equal to the total energy-momentum of the incoming particles. Likewise, "local conservation of electric charge" simply means that the total electric charge of the outgoing particles will be equal to the total electric charge of the incoming particles. We know zilch about what happens between the preparation of the in-state and the detection of the out-state. All we can predict and test is the (diachronic) correlations between in-states and out-states. We know as little about the physical mechanisms or natural processes by which in-states are transformed into out-states as we know about the physical mechanisms or natural processes underlying the (synchronic) correlations between entangled quantum systems.

8. Quantum nonlocality

The non-separability of spatially separated quantum systems consists in the existence of correlations between the probabilities of the possible outcomes of a measurement performed on one of two or more such systems and the probabilities of the possible outcomes of measurements performed on the other system or systems. Redhead (1987) has listed five variations of a locality principle (L) that might be violated by these correlations. But are they? Let us find out.

(L): Elements of reality pertaining to one system cannot be affected by measurements performed "at a distance" on another system.

"At a distance" here has two possible readings. A violation of *Bell locality* would mean that elements of reality pertaining to one system can be affected by measurements performed on another system in the absence of causal influences recognized by current physical theories. A violation of *Einstein locality* would mean that elements of reality can be affected by measurements performed on another system even if the changes they undergo as a result are simultaneous with the measurements by which they are affected. However, all this is purely academic, for there are no elements of reality in the intended sense.

Considering the reason for quantum theory's inevitable reference to measurements, this should not surprise us. The measurement apparatus, you will recall, is needed not only to indicate an outcome but also, and in the first place, to define the question to which the possible outcomes are possible answers. Out of relation to experimental arrangements there are no properties that can be attributed to quantum systems.

A different and easily the most efficient demolition of elements of reality is based on an experiment first discussed by Greenberger, Horne, and Zeilinger (1989) and first performed by

Bouwmeester et al (1999). I present it here as re-formulated by Mermin (1990), who imagines three spin-1/2 particles flying apart in different directions in the horizontal plane. The total spin of the three particles is 0. Identifying the x axis with the vertical axis, we define the z axis for each particle to be parallel to the particle's direction of motion, and we define the y axis for each particle to be horizontal and perpendicular to the particle's direction of motion. If we assign the value +1 to the outcome "up" and the value −1 to the outcome "down," the following predictions are certain:

(X) If the x component of each particle's spin is measured, the product of the outcomes will be –1.

(Y) If the x component of the spin of one particle and the y components of the spins of the two other particles are measured, the product of the outcomes will be +1.

Because we know the product of the outcomes of measuring one x component and two y components, we can predict with certainty the outcome of a measurement of the x component of the spin of anyone of the three particles by measuring the y components of the spins of the two other particles. Since the three particles can in principle be light years apart, it stands to reason that the two y-component measurements cannot "disturb" the particle whose x component remains to be measured. The reality criterion of Einstein, Podolsky, and Rosen (1935) thus applies:

> If without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

If Einstein et al are right, there are three elements of physical reality, X_1, X_2, and X_3, each having the value + 1 or −1, each waiting to be revealed by the outcomes of two y-component measurements. In much the same way we can predict with certainty the outcome of measuring the y component of the spin of any particle by measuring one x component and one y component of the spins of the two other particles. There are thus another three elements of reality, Y_1, Y_2, and Y_3, with values + 1 or −1, also waiting to be revealed by far-away measurements. All six must exist whether or not they are actually measured.

Suppose then that Einstein et al are right, and that we have measured the y components of the spins of the three particles. Calling the outcomes Y_1, Y_2, and Y_3 and using (X) and (Y), we can predict with certainty the values of the elements of reality X_1, X_2, and X_3:

$$X_1 = Y_2 Y_3, \quad X_2 = Y_1 Y_3, \quad X_3 = Y_1 Y_2.$$

We can therefore predict with certainty that the product of the three x components will turn out to be

$$X_1 X_2, X_3 = (Y_2 Y_3)(Y_1 Y_3)(Y_1 Y_2) = (Y_1)^2 (Y_2)^2 (Y_3)^2 = +1$$

since the square of each possible outcome equals unity. Yet whenever we measure the x components of the spins of the three particles, the product of the outcomes equals −1! Concludes Mermin (1990):

> So farewell elements of reality! And farewell in a hurry. The compelling hypothesis that they exist can be refuted by a single measurement of the three x components: The elements of reality require the product of the three outcomes invariably to be +1; but invariably the product of the three outcomes is −1.

We now turn to the five variations of (L) listed by Redhead.

(L1) The unsharp value of an observable cannot be changed into a sharp value by a measurement performed at a distance.

Absent elements of reality, an observable has a value only at the time at which it is measured. A measurement therefore does not change a value existing before the measurement into a value existing after the measurement. Instead it realizes (or contributes to realize if several measurements of the same observable are simultaneously performed) a value that only exists at the time at which the measurement is made.

Moreover, in the context in which (L1) is generally discussed — two spin-1/2 particles in the singlet state — a measurement performed on one particle does not qualify as a measurement performed on the other particle. The reason this is so is that the gradient of an inhomogeneous magnetic field is needed to define the measurement axis and thereby to define the values that a measurement can yield. If no such gradient exists at the location of particle 2, no spin measurement can be performed on particle 2. The measurement of a spin component of particle 1 therefore only warrants the conditional prediction that a measurement of a spin component of particle 2, *if performed*, will yield the opposite value with probability $\cos^2(\alpha/2)$, where α is the angle between the axes to which the two components refer.

(L2) A previously undefined value of an observable cannot be defined by a measurement performed at a distance.

Since a measurement performed on one system does not amount to a measurement performed on another system, quantum mechanics violates neither (L1) nor (L2).

(L3) The sharp value of an observable cannot be changed into another sharp value by altering the setting of a remote piece of apparatus.

If quantum mechanics does not violate (L1), a fortiori it does not violate (L3).

(L4) A macroscopic object cannot have its classical state changed by altering the setting of a remote piece of apparatus.

It hardly needs saying that nothing amounting to a negation of (L4) is implied by the theory's testable predictions.

(L5) The relative frequencies of measurement outcomes cannot be altered by performing measurements at a distance.

As an illustration of why quantum mechanics does not violate (L5), consider again the singlet state of two spin-1/2 particles. Let α be the angle between the two components measured, and consider all instances in which the spin measurement on particle 1 yields "up." In this case the probability that the spin measurement on particle 2 yields "down" is $p(2\text{ down}|1\text{ up}) = \cos^2(\alpha/2)$. Now consider all instances in which the spin measurement on particle 1 yields "down." In this case the probability that the spin measurement on particle 2 yields "down" is $p(2\text{ down}|1\text{ down}) = 1 - \cos^2(\alpha/2) = \sin^2(\alpha/2)$. Since the two outcomes of the measurement on particle 1 are equiprobable, the probability that the spin measurement on particle 2 yields "down" is

$$p(2\text{ down}) = p(1\text{ up})\, p(2\text{ down}|1\text{ up}) + p(1\text{ down})\, p(2\text{ down}|1\text{ down})$$
$$= \tfrac{1}{2}\cos^2(\alpha/2) + \tfrac{1}{2}\sin^2(\alpha/2) = 1/2$$

— exactly what it would be if no measurement were performed on particle 1.

The bottom line: No locality principle is violated by the quantum-mechanical probability calculus or the equivalent correlation laws. Violations of locality only occur if one ventures beyond the theory's testable predictions, with the hope of explaining them in terms of underlying natural processes, by reifying some calculational tool, with or without postulating physical quantities that cannot be measured or that exist without actually being measured. See Redhead (1987, Sec. 4.7) for the respective locality principles violated by a sample of interpretations of quantum mechanics.

But if neither (L) nor any of its variations is violated, what exactly is violated? What is spooky about "spooky actions at a distance"?

What is spooky about "spooky actions at a distance" is that the offending correlations cannot be construed as actions and therefore cannot be explained in terms of causes and effects. One reason why they cannot be so construed is that (L5) is true at the level of the mathematical formalism and therefore necessarily true irrespective of which physical interpretation one adopts. The relative frequencies of measurement outcomes cannot be altered by distant measurements, and so the offending correlations cannot be used to produce measurable effects. Furthermore, to be able to speak of an action, it must be possible to unambiguously distinguish that which acts from that which is acted on. Yet if the measurements whose outcomes are correlated are simultaneous with respect to at least one inertial frame, this possibility does not exist, for then there are inertial frames relative to which one measurement is made before the other as well as inertial frames relative to which the temporal order of the two measurements is reversed. (Speaking of "passion at a distance," as some have done, does not help, for if it there is no matter of fact about what acts, there also is no matter of fact about what is acted on.)

9. A larger picture

The failure of the quantum-mechanical correlation laws to yield to the explanatory strategies at our disposal highlights a profound conflict between the ontological implications of quantum theory's testable predictions and certain all but incorrigible ways of thinking about the physical world (Mohrhoff, 2007). The following is easily the most illuminating passage in all the philosophical literature I have sifted through in search of a clue to the origin of this conflict:

> Mind in its essence is a consciousness which measures, limits, cuts out forms of things from the indivisible whole and contains them as if each were a separate integer. Even with what exists only as obvious parts and fractions, Mind establishes this fiction of its ordinary commerce that they are things with which it can deal separately and not merely as aspects of a whole. For, even when it knows that they are not things in themselves, it is obliged to deal with them as if they were things in themselves; otherwise it could not subject them to its own characteristic activity.

The passage is from a major philosophical exploration, *The Life Divine* by Sri Aurobindo (2005, p. 173). Let me outline the context in as few words as possible. In line with the dominant Indian philosophical tradition, Sri Aurobindo posits an Ultimate Reality, which, though in itself beyond categorization, relates to the world in three mutually irreducible ways: it is the substance that constitutes the world (Sanskrit: *sat*), it is a consciousness that contains the world (Sanskrit: *chit*), and it is an infinite bliss (or quality, or value) that expresses or manifests itself in the world (Sanskrit: *ānanda*). In brief, it is *sachchidānanda* (*sat-chit-ānanda*). For the purpose of denoting the creative principle by which *sat* determines itself, *chit* experiences itself, and *ānanda* expresses itself, Sri Aurobindo has coined the term "supermind." The action of supermind is primarily qualitative and only secondarily quantitative. Mind in Sri Aurobindo's terminology is essentially the supermind's secondary, limiting and dividing (and as a result, limited and divided) working.

To supermind, everything that exists is the one Ultimate Reality, self-extended as undifferentiated space and undifferentiated time to make room for spatial relations, the experience of change, the great adventure called "evolution." All is the one Ultimate Reality entering into relations with itself, presenting itself to itself under a myriad of aspects. All is manifested through self-relations, including the forms of what we call matter. Mind, on the contrary, "limits, cuts out forms of things from the indivisible whole and contains them as if each were a separate integer."

> It is this essential characteristic of Mind which conditions the workings of all its operative powers, whether conception, perception, sensation or the dealings of creative thought. It conceives, perceives, senses things as if rigidly cut out from a background or a mass and employs them as fixed units of the material given to it for creation or possession. (Sri Aurobindo, 2005, pp. 173–174).

This, I believe, is the reason why we readily agree with Einstein (1948) that "things claim an existence independent of one another" whenever they "lie in different parts of space," and why we tend to believe that things can influence each other only by some kind of direct contact, across common boundaries. (Recall DeWitt and Graham's felicitous formulation of the principle of local action.) From the Greek atomists who posited ultimate constituents, which are not merely indivisible but *uncuttable* (*atomos*), to Kant who held that the manifold in space depends on the introduction of *boundaries*, to physical theories based on set-theoretic conceptions of physical space and time (or spacetime), things are conceived "as if rigidly cut out from a background or a mass" and/or "as if each were a separate integer."

When Bohr insisted that the reason why quantum mechanics is mysterious is that it forces us to recognize limitations on the applicability of the familiar concepts of classical physics, he was criticized for setting dogmatic limitations on scientific theorizing on the basis of obscure philosophical preconceptions. Bohr did not know how right he was, nor his critics how wrong they were. The unequivocal message of quantum theory's testable predictions is that there are limitations on the objectification of our mental distinctions. It bears repetition: if we conceptually partition the physical world into smaller and smaller regions, we reach a point where the distinctions we make between regions no longer correspond to anything in the physical world, and if we go on dividing material objects, their components lose their differences and, along with them, their separate identities.

Why this loss of difference and separate identity? As was said, mind is a secondary, limiting and dividing action of the original creative principle, supermind. As yet the mind we are familiar with — a partial evolutionary unfolding of this action — is effectively separated from its source. As long as mind is separated from its supramental parent, as it is in us, it not only divides *ad infinitum* but also takes the resulting multiplicity for the original truth or fact. This is why we tend to construct reality from the bottom up, on an intrinsically and completely differentiated space or spacetime, out of locally instantiated physical properties, or else by aggregation, out of a multitude of individual building blocks. On the other hand, if mind is employed by supermind as part of the creative action supporting the cosmos, its tendency to divide *ad infinitum* is checked, and this is why there are limitations on the objectification of our mental distinctions.

But why should there be limitations on the objectification of our mental distinctions? Suppose that you are the Ultimate Reality, and suppose that you want to experience the joys and excitements of discovery, surprise, conquest, and victory. You will have to sacrifice your omniscience and you omnipotence, for as long as you are omniscient, there is nothing for you to discover, nothing that can surprise you, and as long as you are omnipotent, there is nothing for you to conquer or to vanquish. So you sacrifice your knowledge, you sacrifice your power, and you do it thoroughly — no half measures for you.

In other words, you set the stage for the drama of evolution. You do this by a process Sri Aurobindo (2005) calls "involution," whose end result is a multitude — at any rate, an apparent or effective multitude — of objects that lack spatial extent. With the help of such objects you then have to manifest objects that have spatial extent and neither collapse nor explode as soon as they are formed. For this you need the well-tested laws of contemporary physics. Why? Because nearly every aspect of these laws is implied by (and thus requisite for) the existence of spatially extended, stable objects that are composed of finite numbers of objects lacking spatial extent (Mohrhoff, 2002, 2009a, 2011 Chap. 22). In brief, the well-tested laws of contemporary physics can be characterized as preconditions (conditions of possibility) of Ultimate Reality's adventure of evolution.

On this view, the one force ultimately at work in the universe is infinite. If it works under self-imposed constraints, as it does in the physical world, we need to know why it does so, and we need to know why under these particular constraints — the well-tested laws of physics — rather than others. On the other hand, it would be self-contradictory to try and explain the working of an infinite force in terms of physical mechanisms or natural processes.

One might object to this by calling it another cop-out of the God-of-the-gaps kind, were it not for an ever-growing number of "no-go theorems" (e.g., Bell, 1964; Kochen and Specker, 1967; Greenberger et al, 1989; Conway and Kochen, 2006), which rule out naturalistic explanations of how measurement outcomes determine the probabilities of measurement outcomes. It is high time that the implications of these theorems are recognized and we stop wasting personal and collective resources on contriving gratuitous solutions to fictitious problems.

10. Summary and conclusion

One respect in which the quantum physical laws may be described as local finds expression in the local conservation laws that are implied by Noether's theorem. The locality of these laws, however, merely warrants the consistency of quantum mechanics with the other pillar of contemporary physics, the special theory of relativity. As was explained in Sec. 7, a local conservation law ensures that the total energy-momentum (or the total charge of some kind) associated with a detected final state equals the total energy-momentum (or the total charge) associated with the corresponding prepared initial state. It is a feature of a calculational device, which does nothing to justify the reification of the device.

There is another respect in which the quantum theoretical laws may be described as local, and this consists in the fact that none of the locality principles examined in Sec. 8 is violated by the theory's testable predictions. Violations of some of these principles occur, but only if one attempts to explain the theory's predictions in terms of underlying natural processes. The nonlocality of quantum mechanics that finds expression in such violations is merely a symptom of a more general disease, viz., the failure of the offending correlations — and in the last analysis, the failure of all quantum-mechanical correlations — to yield to causal explanations.

Quantum mechanics, however, is nonlocal in a more radical sense than that suggested by the theory's synchronic correlations, which is almost trivial by comparison. As was exemplified in Sec. 3 with the help of a double-slit experiment, physical space cannot be something that by itself has parts. If at all we think of space as an independently existing expanse, we must think of it as intrinsically undivided. The so-called parts of space only exist to the extent that they are physically realized by detectors (in the broadest sense of the word), and arbitrarily small parts cannot be physically realized. The spatial differentiation of the physical world is therefore incomplete, and so is its temporal differentiation. Add to this the fact (discussed in Sec. 4) that intrinsically all fundamental particles are identical in the radical sense of numerical identity, along with the consequence (discussed in Sec. 5) that quantum theory's explanatory arrow points "from the top down," and you have the radical nonlocality of the quantum world. A possible reason why this nonlocality has hitherto gone unacknowledged has been suggested in Sec. 9.

As long as we take it for granted that physical space is differentiated "all the way down," so that any two conceptually distinct "points of space" correspond to physically distinct locations, we are confronted with the impossibility of understanding how the synchronic correlations between entangled system are possible at all. As soon as we take account of the radical nonlocality of the quantum world, the impossibility disappears. It was another pseudo-problem arising from a false assumption, viz., the assumption that the spatiotemporal differentiation of the physical world is complete. There are several reasons why it is possible for the three spins in the experiment of Greenberger et al (discussed in Sec. 8) to be entangled just the way they are. If we think of space as an independently existing (i.e., substantial) expanse, the possibility exists because this expanse is undifferentiated; it lacks parts. We may say, paradoxically yet to the point, that ultimately there is only one place, and this is everywhere. Space is Ultimate Reality self-extended to make spatial relations possible. This comes remarkably close to what Newton may have suggested by writing (in the General Scholium at the End of the Principia, quoted by Misner et al, 1973, p. 41):

> He is not eternity or infinity, but eternal and infinite; He is not duration or space, but He endures and is present. He endures forever, and is everywhere present; and by existing always and everywhere, He constitutes duration and space.

If instead we think of space as a set of relations, the relations are relations between Ultimate Reality and itself, and what makes it possible for Ultimate Reality to enter into self-relations also makes it possible for Ultimate Reality here to be correlated with Ultimate Reality there.

The radical non-locality of quantum mechanics also explains why the only consistent physical interpretation of the mathematical formalism of quantum mechanics — consistent, that is, with the world's incomplete spatiotemporal differentiation implied by the theory's testable predictions — is that it is a probability calculus. Because an evolving physical state requires for its existence a completely differentiated time continuum, a quantum state cannot be anything but a probability algorithm.

Finally, the radical nonlocality of the quantum world renders intelligible the *possibility* of paranormal correlations. However, the view that quantum mechanics can be of any help in *explaining* how paranormal phenomena come about, entertained by many researchers in the field, is unfounded. If quantum mechanics cannot explain the correlations it predicts, how could it possibly explain correlations that it does not predict?

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