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RETAIL OR COMMISSIONED LIVE-STREAMING? MODE CHOICE OF A PLATFORM SUPPLY CHAIN CONSIDERING CONSUMERS' PREFERENCES

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"live-streaming + e-commerce" mode emerged as time required. In this paper, we aim to investigate the manufacturer's pricing and mode choice of cooperation with a live streamer in a dual-channel live-streaming supply chain consisting of a single manufacturer, a KOL(Key Opinion Leader) live streamer, and a live-streaming platform, considering different consumers' preferences. We depict two scenarios for the KOL streamer, retail live-streaming and commissioned live-streaming modes, in the presence of a manufacturer's self-live-streaming and investigate the optimal mode choice with the Stackelberg game. The paper discovers that under the commissioned live-streaming mode, the price of KOL live-streaming is positively (negatively) correlated with the commission ratio (consumers' preferences for the manufacturer's self-live-streaming) and lower than that under manufacturer self-live-streaming under a low commission ratio (a high consumers' preferences for manufacturer's self-live-streaming). In both scenarios, the KOL live-streaming's sales effort is consistently lower than that of the manufacturer's self-live-streaming channel. Additionally, the consumer's sensitivity coefficient, the trust degree, the impact of KOL streamers, and the proportion of impulsive consumers are positively correlated with both channels' price, sales effort, and profit.

Keywords: Dual Channel Supply Chain, KOL Live-Streaming, Manufacturer Self-Live-Streaming, Game Theory, Consumers Preferences

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1. INTRODUCTION

With the continuous development of the e-commerce industry, the number of online shopping gradually increased, and a single online shopping format could no longer satisfy the needs of consumers, The "live streaming + e-commerce" mode emerged as time required. Until June 2021, there were 384 million active e-commerce consumers in China. Li Jiaqi, as the representative of the top streamer, is still refreshing the new data for live-streaming selling products. Taobao, TikTok, Kuaishou, and other significant APPs constantly invite companies and Internet celebrities into the live platform, utilizing the "short video + live" to attract viewers. These examples show that live-streaming platforms develop rapidly and can effectively promote the development of the e-commerce industry (Chen *et al*., 2023).

In practice, with increasing manufacturers offering self-live-streaming and inviting Key Opinion Leaders (KOL, also named Internet Celebrity) to sell goods in live-streaming, a dual-channel supply chain of e-commerce Internet casting has been formed. The fans and exposure of live-streaming platforms give KOL streamers tremendous selling power. However, the top streamers may possess "bargaining power" over the goods and even take the original market of the manufacturer's live-streaming due to their influence and dominant advantage. Meanwhile, many manufacturers are discouraged by prohibitive "booth fees". Unlike top streamers, moderately popular streamers retain the ability to broaden sales channels and increase product awareness with moderate booth fees. Because of this, cooperating with moderate KOL streamers has become a popular new option for many manufacturers (Kang *et al*., 2021; Liu *et al*., 2024). Nevertheless, inviting a KOL to establish

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another live-streaming channel also leads to channel competition, which may challenge manufacturers' profitability. Consequently, there is an urgent need to formulate pricing strategies to maximize the overall profits of both channels when cooperating with a moderately popular streamer.

Based on the above considerations, we will answer the following questions:

(1) How do supply chain members set their prices under different cooperation modes (i.e., the retail live-streaming and commissioned live-streaming modes in the presence of manufacturer self-live-streaming)?
(2) Which cooperation mode should the manufacturer and KOL streamer choose?
(3) How do the consumer preferences affect the equilibrium results?

To answer these questions, we consider a dual-channel live-streaming supply chain consisting of a single manufacturer with a self-live-streaming channel, a moderately popular live streamer, and a live-streaming platform. Given the sequential nature of decision-making among involved parties, we employ Stackelberg games, a two-stage full-information dynamic game framework and utilize backward deduction to resolve the game. This approach enables us to scrutinize the manufacturer's optimal mode selection between retail live-streaming and commissioned live-streaming mode. This requires different pricing decisions for the manufacturer and the live streamer. Specifically, the manufacturer is the leader in the supply chain and always the first one to make decisions. In the commissioned live-streaming (retail live-streaming) mode, the manufacturer is always responsible for initiating decisions regarding sales effort and price within its self-live-streaming channel and determining the price (wholesale price) within the live streamer's streaming channel. Based on the manufacturer's decisions, the live streamer acts as the follower, making decisions regarding sales effort (both sales effort and price) within the streaming channel in the commissioned live-streaming (retail live-streaming) mode as a response.

We show that the price of KOL live-streaming is less expensive than that under manufacturer self-live-streaming within a specific commission ratio and is proportionate to the commission ratio under the commissioned live-streaming mode, whereas only when consumers' preferences are more concentrated on the manufacturer self-live-streaming, the price of KOL live-streaming is less than the manufacturer self-live streaming channel under the retail live-streaming mode. Furthermore, the consumer's sensitivity coefficient and the trust degree in KOL streamers, the impact of KOL streamers, and the proportion of impulsive consumers positively correlate with the price, sales effort, and profit of both channels.

This paper makes several contributions. First, we consider the cooperation mode between the manufacturer and the live streamer in a dual-channel live-streaming supply chain, which, to our knowledge, has not been examined before. Second, by analyzing and comparing the optimal results under different scenarios, we obtain the manufacturer's optimal cooperation mode and further find that the consumer's sensitivity coefficient and the trust degree in KOL streamers, the impact of KOL streamers, and the proportion of impulsive consumers positively correlate with the price, sales effort, and profit of both channels. These findings can offer valuable management insights for firms in making strategic decisions.

2. LITERATURE REVIEW

There are three streams of related literature: pricing decision, channel strategy and live-streaming selling.

2.1 Pricing decision

Firstly, the current pricing decision issue in the e-commerce supply chain has received extensive scholarly discussion. Zhou *et al*. (2018) investigate the price decision in a dual channel structure consisting of a manufacturer and a downstream retailer whose offline sales service may be free-ridden by the manufacturer's online channel. Their results show that contrast to the differential pricing scenario, the non-differential pricing scenario benefits the retailer but harms the manufacturer and the whole supply chain. Guo *et al.* (2019) studied the showroom effect on the price, profit, and service effort for the views of each supply chain member. The paper divides service effort into two stages, ex-ante and ex-post service, contrasting no-service strategies and demonstrates that manufacturers prefer to negotiate lower wholesale pricing for retailers in the ex-post service strategy and that the showroom effect can result in the best benefits for firms using this strategy. Yang *et al*. (2021) examine how online commentary will affect supply chain online channels. The research concludes that posting internet reviews is not always profitable unless they are sufficiently positive. In contrast to centralized decision-making, the manufacturer in a decentralized system is more likely to increase the online price.

Lin *et al*. (2021) explore a new sales mode, "online shopping-offline pickup" (BOPS), based on the Stackelberg game theory and explore the impact of opening a BOPS channel on product quality, price and profitability of manufacturers and retailers from three perspectives: transaction cost for consumers, transportation cost, and handling cost for retail stores. Wei and Chang (2022) examine the impact of implementing a price-matching approach when multichannel retailers open online channels. They discover that using this strategy only benefits the multichannel retailer. Only if the retailer uses the online platform where from e-retailer can the price matching approach increase the profitability of multichannel retailers. The above literature on pricing includes traditional channels, not pure live channels, which do not consider the typical dual live-streaming channel structure in practice. To fill this gap, this paper focuses on the more complex pricing of a manufacturer who operates two live-streaming channels and examines how the price decision is affected by some critical factors in the live-streaming channel, such as the live-streaming platform's commission ratio, consumers' preferences for a live-streaming channel, and consumers' trust degree to the streamer.

2.2 Channel strategy

The second stream of related literature is channel strategy. For example, Sarkar & Pal (2021) construct a multichannel supply chain framework with traditional and direct channels and explore their competitive pricing and service decisions. They also analyze how service cost coefficients and cross-channel price coefficients affect the profitability of each member and the whole supply chain. According to Zhou *et al*. (2019), a manufacturer-dominated dual-channel supply chain system based on the Stackelberg model is considered. In this system, the store operates both a physical and an online channel, each offering substitute products. It is discovered that the producer should determine the price of the dual channel on elements like demand specificity and the sensitivity of demand to price, regardless of the channel. Matsui (2022) discusses whether the retailer should haggle over wholesale prices with the supplier. The research finding runs counter to intuition, and they show that accepting the wholesale price established by the manufacturer without haggling with them might result in higher profits for the retailer if they have enough negotiating leverage and the consumer's substitutability between channels is vital. Barman *et al*. (2021) consider the impact of greenness on price according to the scenario of the manufacturer produced green merchandise and sell them directly through online and retail channels. According to the study, when the price is higher in both channels, the product's greenness is strengthened in a centralized decision. In our paper, we consider the three parties in live-streaming channels, the manufacturer, the live streamer, and the live-streaming platforms, and the two leading live-streaming selling models, retail live-streaming and commissioned live-streaming, to find their equilibrium strategy. Given the sequential behavior of three parties and different decision variables for parties in different modes, we conduct a Stackelberg game (Khanjari *et al*., 2014; Zhang and Xin, 2023; Xin *et al*., 2023; Zhang *et al*., 2022) and investigate the manufacturer's optimal live selling mode in the additional live-streaming channel.

2.3 Live-streaming selling.

The third stream of related literature concerns live-streaming selling. In recent years, the vigorous development of live-streaming has attracted the attention of many academics. Pan *et al*. (2022) discuss the effects of live streamers' sales ability, consumer preferences, and consumption cost on prices and profits. Live streamers' high sales abilities might result in profit loss, and that consumers' preferences with a negative correlation may be more profitable. Fan *et al*. (2022) argue that the live-streaming service of streamers affects both the sales of traditional channel manufacturers and the return rate when selling products live. Comparing the best course of action for both channels, they show that the live commerce spillover effect increases producer profit but decreases streamer profit. When the return rate is higher, the e-commerce streamer needs to increase sales service to increase profit. He *et al*. (2022) focus on three modes for inviting streamers to sell commodities: commission-only, fixed-fee, and a combination, and indicate that retailers prefer to work with streamers with high sales capacity in a pure commission mode. When streamers charge a large, fixed price, retailers prefer to join with streamers with lesser sales capacity but only charge a commission; when commission rates are low and fixed fees are high, the hybrid mode is the more appealing option for retailers.

Peng *et al*. (2021) observe the phenomenon where viewers can randomly reward in live-streaming platforms and combine it with live e-commerce agricultural products to determine whether random rewards in live-streaming platforms can maximize the sales revenue of agricultural products in live e-commerce. The findings demonstrate that random awards optimize farmers' live-streaming sales revenue. Additionally, when random rewards reach a particular level, they encourage farmers to offer discounts. Zhang *et al*. (2022) investigate whether multinational corporations should open live shopping channels on overseas e-commerce platforms. Considering both company's overseas online retail and third-party e-tailer channels, it is found that opening a live shopping sales channel will benefit multinational companies in terms of after-tax profits and sales volume but will hurt the profits of multinational companies' overseas online retail divisions and third-party e-tailers. Three popular live commerce sales modes were examined by Yang *et al*. (2022), including the transfer mode (live-streaming and transaction on both platforms), the live platform mode (live-streaming and transaction based on the live platform), and the e-commerce platform mode (both live-streaming and transaction based on the e-commerce platform). Profits for platforms and sellers depend more on sales than on the mode used, whereas the transfer mode will never be the optimal option for

Internet celebrities. For investigating the equilibrium strategies of supply chain participants under various sales agreements, Wang *et al*. (2022) construct a multi-level supply chain in which upstream suppliers can sell their products through online platforms and live-streaming sales channels, and online platforms can choose to sign resale agreements or agency sales agreements with suppliers. The selling price of the platform is inversely proportional to the commission ratio in the resale agreement. In contrast, the selling price of the live channel is positively proportional to the commission ratio.

Huang *et al*. (2024) examine the impacts and strategies of live-streaming introduction for competing retailers with consumer switch. It shows that live streaming may not increase the introducer's demand or benefit the retailer that offers free rides. The equilibrium strategies regarding who introduces the live-streaming, i.e., NN (no retailer introduces), LN/NL (only one retailer introduces), and LL (both retailers introduce), depending on the commission rate and mismatch cost. The above articles study whether enterprises should introduce live channels and the optimal sales mode for online e-commerce platforms or suppliers. Ji *et al*. (2023) investigate the retailer's selling format between agency selling and reselling when hiring a 3P live streamer to promote items under different price discount strategies. Introducing a live-streaming channel may encourage the retailer to apply the agency selling format. Xin *et al*. (2023) consider the manufacturer's optimal decision on live-streaming product showcasing modes among brand self-live-streaming, influencer-led live-streaming mixture, and influencer-led special live-streaming and find that the fixed participation fee plays a decisive role in the manufacturer's decision. However, we consider a more general situation: the manufacturer not only launches live-streaming but also cooperates with a streamer to distribute items.

In summary, the current academic research on the e-commerce live supply chain focuses on the decision problem of the three-level e-commerce supply chain of live streamers, platforms, and manufacturers, or the optimal decision when the live channel and other channels are adopted simultaneously in a comprehensive manner. In conjunction with the issue of pricing decisions, the literature considers not only prices and profits in different channels but also the services provided by the manufacturer or retailer. However, the issue of determining the price when both the manufacturer's live-streaming and KOL live-streaming channels exist has not received much attention from scholars. To differ from the previous studies, this paper focuses on an online dual-channel live-streaming supply chain consisting of a live-streaming channel by the manufacturer and live-streaming channel in two selling modes, one is the retail live-streaming mode, and the other is the commissioned live-streaming mode. Furthermore, the paper builds the profit models for each member, respectively, and uses a Stackelberg game to assess each member's best choices under the two live-streaming modes.

3. PROBLEM DESCRIPTION AND HYPOTHESIS

3.1. Model description

We consider a supply chain consisting of three parties: a live-streaming platform, a manufacturer, and a KOL. The manufacturer sells products on the live-streaming platform through a self-living stream channel. Moreover, the manufacturer also cooperates with a KOL streamer and utilizes its fame to sell items. In this KOL live-streaming channel, there are two living selling modes for the manufacturer to choose, one is retail live-streaming, and the other is the commissioned live-streaming mode. Their main difference between two modes are from the price decision power controlled by the manufacturer and KOL streamer.

Under the retail live-streaming, the manufacturer determines the live sales price $\boldsymbol{p_{(s)1}}$ and sales effort $\boldsymbol{f_{(s)1}}$ in the self-live-streaming channel, and the wholesale price $\boldsymbol{w}$ in the KOL live-streaming channel. Then, the KOL streamers wholesale the goods from the manufacturer and determine his sales price $\boldsymbol{p_{(s)}}$ and sales effort $\boldsymbol{f_{(s)}}$ in the KOL live-streaming channel. A KOL streamer is equivalent to acting as a retailer. In TikTok, Kuaishou, it is typical for many stores to support streamers to sell one piece.

Under the commissioned live-streaming mode, the manufacturer invites the streamer to join live-streaming with a "booth fee" and pays $\boldsymbol{\lambda}$ proportion of unit revenue as commissions to the streamer. However, unlike retail live-streaming, the KOL live-streaming under the commissioned live-streaming mode is regarded as the manufacturer's "brand salesman," which is responsible for the sale of the items but does not have the authority to determine the price. On the Taobao platform, manufacturers commonly publish relevant live-streaming tasks in the "Ali V Task," after which KOLs will take orders and fulfill the tasks. Under both live-streaming selling modes, it is assumed that the cost function of the manufacturer's or the streamer's sales effort follows an increasing convex pattern, indicating a rise in marginal cost as effort increases. Specifically, the sales effort cost for sale effort level $\boldsymbol{f_{(s)i}}$ is given by $\boldsymbol{\frac{1}{2}f^2_{(s)i}}$, where $\boldsymbol{i=1}$ or $\boldsymbol{2}$ (Chen and Xiao 2012; Xin *et al*. 2023). For simplification, we assume the manufacturer's production cost, distribution cost, or fixed "booth fees" to be zero.

The platform will levy a specific technical service cost $\boldsymbol{c}$ for the live-streaming and, in addition, will charge a specific commission rate $\boldsymbol{\varepsilon}$ for any Internet celebrities who use the platform to take orders and carry out live-streaming tasks. Set the product's market demand as $\boldsymbol{a}$. There will be competition between two live channels; this paper defines $\boldsymbol{\mu}$ as the price competition coefficient and sets range, where $\mathbf{0} \leq \boldsymbol{\mu} \leq \mathbf{1}$ (Zhang *et al*., 2023). Given the nature of live-streaming, the live sales effort affects sales volume of the streaming daily. The paper defines $\boldsymbol{f}$ as the sales effort of the live streamer and $\boldsymbol{\beta}$ as the sensitivity coefficient of consumers to the sales effort of the live streamer, where $\mathbf{0} \leq \boldsymbol{\beta} \leq \mathbf{1}$ (Huang *et al*., 2024). Meanwhile, for the peculiarities of KOL live-streaming, the streamer typically has his impact, which can draw viewers. The essay makes the following assumptions: $\boldsymbol{n}$ represents the influence of the streamer; $\boldsymbol{k}$ represents consumers' trust degree in KOL streamer, where $\mathbf{0} \leq \boldsymbol{k} \leq \mathbf{1}$ (Note that $\boldsymbol{k} = \mathbf{0}$ means complete distrust, and $\boldsymbol{k} = \mathbf{1}$ means complete trust); and $\boldsymbol{\sigma}$ represents the proportion of impulsive customers in KOL live-streaming, where $\mathbf{0} \leq \boldsymbol{\sigma} \leq \mathbf{1}$.

Table 1. Symbolic description of relevant parameters.

Symbol	Description
i	Index for live-streaming selling mode. $i = 1$ represents commissioned live-streaming mode, $i = 2$ represents retail live-streaming mode.
$p_{(s)i}$	Product sales price in the KOL live-streaming channel (or self-live-streaming channel) under live mode i.
$f_{(s)i}$	Sales effort for live-streaming in the KOL live-streaming channel (or self-live-streaming channel) under live mode i.
a	Market demand for products in live-streaming.
β	The sensitivity coefficient of consumers to the sales effort.
μ	The price competition factor of the product.
λ	The commission proportion of KOL live-streaming.
k	Consumers' trust degree in KOL streamer.
N	The viewer in KOL live-streaming.
w	Wholesale price.
c	Technical service cost for unit products of the platform.
ε	The commission rate levied by the platform for KOL live-streaming.
n	The impact of KOL streamer.
θ	Consumers' preference for manufacturer's self-live-streaming, hereafter referred to as consumer preference.
σ	The proportion of impulsive customers in KOL live-streaming.

3.2. Commissioned Live-streaming Mode

As shown in Figure 1, the manufacturer is the leader in the Stackelberg game and contracts with KOL streamers. In contrast, the KOL streamer is responsible for selling the merchandise and is not engaged in determining the pricing decision of the products. The manufacturer decides the price $\boldsymbol{p_{s1}}$ of the manufacturer's self-live-streaming channel and the sales effort $\boldsymbol{f_{s1}}$ and the price $\boldsymbol{p_1}$ of the KOL live-streaming channel. KOL streamers, who is a follower, determines the sales efforts of KOL live-streaming $\boldsymbol{f_1}$, and the platform determines the commission ratio $\boldsymbol{\varepsilon}$ and the cost of the unit service $\boldsymbol{c}$. The demands in the commissioned live-streaming mode are displayed as follows.

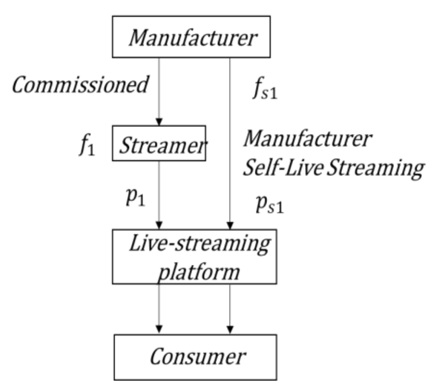

Figure 1. Commissioned live-streaming mode.

The demand for KOL live-streaming channel is

$$\boldsymbol{D_1 = (1-\theta)a - p_1 + \mu p_{s1} + \beta f_1 + k(n+\sigma N)}. \tag{1}$$

The demand for manufacturer self-live-streaming channels is

$$\boldsymbol{D_{s1} = \theta a - p_{s1} + \mu p_1 + \beta f_{s1}} \tag{2}$$

In the commissioned live-streaming mode, the manufacturer must give the KOL streamer a set amount in commission. Considering that the platform must charge service cost, and the commission ratio charged by different products differs, we set the commission ratio range as (0.1,0.5). The profits of the manufacturer and the streamer are shown as follows.

The profit function of the manufacturer is

$$\boldsymbol{\Pi_{s1} = (1-\lambda)\, p_1 D_1 + (1-c) p_{s1} D_{s1} - \frac{1}{2} f_{s1}^2} \tag{3}$$

The profit function of KOL live-streaming is

$$\boldsymbol{\Pi_1 = (\lambda - c)(1-\varepsilon) p_1 D_1 - \frac{1}{2} f_1^2} \tag{4}$$

The profit function of the platform is

$$\boldsymbol{\Pi_e^h = c(p_{s1} D_{s1} + p_1 D_1) + (\lambda - c)\varepsilon p_1 D_1}. \tag{5}$$

The problem is solved by backward deduction. We first calculate the sales effort decision of KOL live-streaming. The second-order derivatives of $\boldsymbol{f_1}$ in equation (4) is as follows.

Since $\boldsymbol{\frac{\partial^2 \Pi_1}{\partial (f_1)^2} = -1 < 0}$, there exists an optimal solution. Leting $\boldsymbol{\frac{\partial \Pi_1}{\partial f_1} = 0}$, we can get $\boldsymbol{f_1 = \beta p_1 (\lambda - c)(1-\varepsilon)}$; Substituting into formula (3), we can get

$$\boldsymbol{\Pi_{s1} = (1-\lambda)\, p_1 \left((1-\theta)a + p_1\left(-1 + \beta^2(\varepsilon - 1)(\mathrm{c} - \lambda)\right) + p_{s1} + k(n + N\sigma)\right)} \\ \boldsymbol{+ p_{s1}(1-c)(\theta a - p_{s1} + \mu p_1 + \beta f_{s1}) - \frac{1}{2} f_{s1}^2} \tag{6}$$

Taking partial derivatives of the $\boldsymbol{p_1}$, $\boldsymbol{p_{s1}}$, $\boldsymbol{f_1}$ and setting them equal to 0, we can get the optimal solution is obtained simultaneously.

$$\boldsymbol{p_1^* = \frac{(-1+c)(a(2+\beta^2(-1+c))(-1+\lambda)(-1+\theta) - a(-2+c+\lambda)\theta\mu - (2+\beta^2(-1+c))k(-1+\lambda)(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda) + (-2+c+\lambda)^2\mu^2}},$$

$$\boldsymbol{p_{s1}^* = \frac{-(-1+\lambda)(-2a(-1+c)(-1+\beta^2(-1+e)(c-\lambda))\theta a(-2+c+\lambda)(-1+\theta)\mu + k(-2+c+\lambda)u(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda) + (-2+c+\lambda)^2\mu^2}},$$

$$\boldsymbol{f_{s1}^* = \frac{-b(-1+c)(-1+\lambda)(2a(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))\theta + a(-2+c+\lambda)(-1+\theta)\mu - k(-2+c+\lambda)\mu(n+N\sigma)}{2(2+\beta^2(-1+c))(-1+c)(-1+b^2(-1+\varepsilon)(c-\lambda))(-1+\lambda) + (-2+c+\lambda)^2\mu^2}},$$

$$f_1^* = \frac{\beta(-1+c)(-1+\varepsilon)(c-\lambda)(a(2+\beta^2(-1+c))(-1+\lambda)(-1+\theta) - a(-2+c+\lambda)\theta\mu - (2+\beta^2(-1+c))k(-1+\lambda)(n+N\sigma))}{2(2+\beta^2(-1+c))(-1+c)(-1+\beta^2(-1+\varepsilon)(c-\lambda))(-1+\lambda)+(-2+c+\lambda)^2\mu^2},$$

To substitute the optimal solutions $p_1^*,\ p_{s1}^*,\ f_{s1}^*$, into Eqs. (4), (5), (6) yields.

$$\Pi_1^* = \frac{\left\{\begin{matrix} -(-1+c)^2(\varepsilon-1)(c-\lambda)(a\,A_1(\lambda-1)(\theta-1) - a\,A_4\theta\mu - k\,A_1(\lambda-1)(n+N\sigma)) \\ (a(\,A_1\,A_2(\lambda-1)(\theta-1) + \,A_3(c-\lambda)\theta\mu + 2\,A_4(\theta-1)\mu^2) - k(\,A_1\,A_2(\lambda-1) + 2\,A_4\mu^2)(n+N\sigma)) \end{matrix}\right\}}{2\left(2\,A_1\,A_5(c-1)(\lambda-1) + \,{A_4}^2\mu^2\right)^2},$$

$$\Pi_{s1}^* = \frac{\left\{\begin{matrix} (c-1)(\lambda-1)(a^2(2-2\lambda+4(\lambda-1)\theta-2\,A_4s^2+\beta^2(c-1)\left(1+\theta(-2+\theta+2c(\varepsilon-1)\theta)+\lambda\left(-1+\theta(2+\theta-2\varepsilon\theta)\right)\right) \\ +2\,A_4(\theta-1)\theta\mu) + 2ak(\,A_1(\lambda-1)(\theta-1) - \,A_4\theta\mu)(n+Nz) - k^2\,A_1(\lambda-1)(n+Nz)^2) \end{matrix}\right\}}{2\left(2\,A_1\,A_5(c-1)(\lambda-1) + \,{A_4}^2\mu^2\right)^2},$$

$$\Pi_e^* = \frac{\left\{\begin{matrix} c(-1+\lambda)^2\left(2a(c-1)\,A_5\theta + a\,A_4(\theta-1)\mu - kA_4\mu(n+N\sigma)\right)\left(a\left(2(c-1)\,A_5\theta - \,A_6(\theta-1)\mu + \,A_4\theta\mu^2\right) + k\,A_5\mu(n+N\sigma)\right) \\ -(c-1)^2c(aA_1(\lambda-1)(\theta-1) - aA_4\theta\mu - A_1k(\lambda-1)(n+N\sigma))(a(A_1A_5(\lambda-1)(\theta-1) + (\lambda-c)A_5\theta\mu + A_4(\theta-1)\mu^2) - \\ k(A_1A_5(\lambda-1) + A_4\mu^2)(n+N\sigma)) + (c-1)^2\varepsilon(c-\lambda)(aA_1(\lambda-1)(\theta-1) - aA_4\theta\mu - A_1k(\lambda-1)(n+N\sigma)) \\ (a(A_1A_5(\lambda-1)(\theta-1) + A_5(\lambda-c1)\theta\mu + A_4(\theta-1)\mu^2) - k(A_1A_5(\lambda-1) + A_4\mu^2)(n+N\sigma)) \end{matrix}\right\}}{2\left(2\,A_1\,A_5(c-1)(\lambda-1) + \,{A_4}^2\mu^2\right)^2}$$

where $A_1 = 2+\beta^2(c-1)$; $A_2 = -2+3\beta^2(\varepsilon-1)(c-\lambda)$; $A_3 = -2+\beta^2(\varepsilon-1)(2+c-3\lambda)$; $A_4 = c-2+\lambda$; $A_5 = -1+\beta^2(\varepsilon-1)(c-\lambda)$; $A_6 = -c+b^2(c-1)(\lambda-1)+\lambda$.

3.3. Retail live-streaming mode

Figure 2 depicts the retail live-streaming mode. The manufacturer remains the leader in the Stackelberg game. First, the manufacturer sells the product to the KOL streamer at the wholesale price w; sells the product to the consumers at the sales price p_{s2} of the manufacturer's self-live-streaming while determining the sales effort f_{s2}. Then, depending on the wholesale price established by the manufacturer, the streamer establishes his live-streaming sales price p_2 and sales effort f_2. The platform determines the unit service cost c for the two channels. The demands and profits under the retail live-streaming mode are as follows.

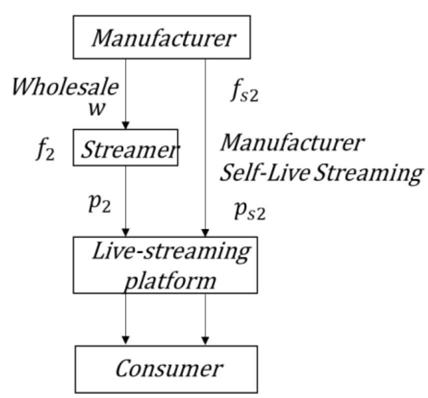

Figure 2. Retail live-streaming mode.

The demand for KOL live-streaming channel is

$$D_2 = (1-\theta)a - p_2 + \mu p_{s2} + \beta f_2 + k(n+\sigma N). \tag{7}$$

The demand for manufacturer self-live-streaming channel is

$$\boldsymbol{D_{s2} = \theta a - p_{s2} + \mu p_2 + \beta f_{s2}}. \tag{8}$$

The profit function of the manufacturer is

$$\boldsymbol{\Pi_{s2} = wD_2 + p_{s2}(1-c)D_{s2} - \frac{1}{2}f_{s2}^2} \tag{9}$$

The profit function of KOL live-streaming is

$$\boldsymbol{\Pi_2 = ((1-c)p_2 - w)D_2 - \frac{1}{2}f_2^2}. \tag{10}$$

The profit function of the platform is

$$\boldsymbol{\Pi_e^l = c(p_2D_2 + p_{s2}D_{s2})}. \tag{11}$$

The paper uses the inverse solution method. Firstly, we consider the sales effort decision of KOL live-streaming. The second partial derivatives of p_2 and f_2 are obtained according to formula (10).

$\frac{\partial^2\Pi_2}{\partial(p_2)^2} = 2c - 2<0$, $\frac{\partial^2\Pi_2}{\partial(f_2)^2} = -1 < 0$, so there exists an optimal solution.

Let $\frac{\partial\Pi_2}{\partial p_2} = 0, \frac{\partial\Pi_2}{\partial f_2} = 0$,

we can get: $p_2 = \frac{a(c-1+\theta-c\theta)+(c-1)p_{s2}\mu+(-1+\beta^2-\beta^2c)w+(c-1)k(n+N\sigma)}{(2+\beta^2(c-1))(c-1)}$

and

$f_2 = \frac{\beta(a(c-1)(\theta-1)+p_{s2}\mu-cp_{s2}\mu-w-(c-1)k(n+N\sigma))}{2+\beta^2(c-1)}$.

By substituting into formula (9), we can find the partial derivatives of p_{s2}, w, f_{s2} and make them equal to 0, the optimal solution can be obtained.

$$p_2^* = \frac{-a(6+\beta^4(c-1)^2(1+\theta(\mu-1))-2\mu^2+2\theta(\mu-1)(3+\mu)+\beta^2(c-1)(5+\theta(-5+3\mu)))+k(6+\beta^2(5+\beta^2(c-1))(c-1)-2\mu^2)(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$p_{s2}^* = \frac{2a(-2-\beta^2(c-1))\theta+a(4+\beta^2(c-1))(\theta-1)\mu-(4+\beta^2(c-1))k\mu(n+N\sigma)}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$w^* = \frac{(c-1)(a(-\left(2+\beta^2(c-1)\right)^2(\theta-1)+\left(2+\beta^2(c-1)\right)^2\theta\mu-\beta^2(c-1)(-1+\theta)\mu^2)+k(4+\beta^2(c-1)(4+\beta^2(c-1)+\mu^2))(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$f_2^* = \frac{\beta(c-1)(a(2(\theta-1)(-1+\mu^2)+\beta^2(c-1)(1-\mu^2+\theta((-1+\mu)\mu-1)))-(2+\beta^2(c-1))k(-1+\mu^2)(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)},$$

$$f_{s2}^* = \frac{-\beta(c-1)(-2aA_1\theta+a(4+\beta^2(c-1))(\theta-1)\mu-(4+\beta^2(c-1))k\mu(n+N\sigma))}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)}.$$

Substituting p_2^*, p_{s2}^*, w^*, f_2^*, f_{s2}^* into equation (9), equation (10), and equation (11), we can get the optimal profit.

$$\Pi_2^* = -\frac{A_1(c-1)\begin{Bmatrix}(a(2(\theta-1)(\mu^2-1)+\beta^2(c-1)(1-\mu^2+\theta((-1+\mu)\mu-1)))\\-A_1k(-1+\mu^2)(n+N\sigma))\end{Bmatrix}^2}{2(\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2))^2},$$

$$\Pi_{s2}^{*}=\frac{\left\{\begin{array}{c}(1-c)(a^2(-A_1(1+\theta(-2+3\theta))+2(4+\beta^2(c-1))(\theta-1)\theta\mu-2(\theta-1)^2\mu^2)\\ -2ak\left(\beta^2(c-1)\left(1+\theta(\mu-1)\right)+2\left(1-\theta(\mu-1)^2+\mu^2\right)\right)(n+N\sigma)\\ -k^2(\beta^2(c-1)+2(1+\mu^2))(n+N\sigma)^2)\end{array}\right\}}{2(\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2))},$$

$$\Pi_e^{l*}=\frac{\left\{\begin{array}{c}c(a^2\left(-4\left(\mu^2-1\right)A_7+4\beta^2(c-1)\left(\mu^2-1\right)A_8+\beta^6(c-1)^3A_9+\beta^4(c-1)^2A_{10}\right)+\\ ak\left(-8\left(\mu^2-1\right)A_{11}+4\beta^2(c-1)\left(\mu^2-1\right)\left(8+\theta(-8+5\mu)\right)+\beta^6(c-1)^3A_{12}+2\beta^4(c-1)^2A_{13}\right)\\ (n+N\sigma)-k^2(\mu^2-1)(n+N\sigma)^2A_{14})\end{array}\right\}}{(\beta^2(4+\beta^2(c-1))(c-1)(u^2-2)+8(u^2-1))^2},$$

where $A_7=3+s^2(-7+u)(-1+u)-2s(-3+u)(-1+u)+u^2$; $A_8=-4+(-1+s)s(-8+5u)$; $A_9=1+s(-1+u))(1-u^2+s(-1+(-1+u)u)$; $A_{10}=-3+s(-3+u)(-1+u)-u^2$; $A_{11}=7-7u^2+2s(-7+u+7u^2-3u^3)+s^2(11+2u(-1+3(-2+u)u))$; $A_{12}=2(-1+u^2)+s(2+(-2+u)u^2)$; $A_{13}=-7+7u^2+s\left(7+u\left(-1+u(-7+3u)\right)\right)$; $A_{14}=(2+b^2(-1+c1))^2(3+b^2(-1+c1))+4u^2$.

4. PROPERTY ANALYSIS OF EQUILIBRIUM SOLUTION

According to the equilibrium solutions of commissioned live-streaming mode and retail live-streaming mode in Section 2, we can get the following conclusions from the comparative analysis.

Proposition 1. In the commissioned live-streaming mode, (1) comparing the sales effort of the manufacturer's self-live-streaming channel and KOL live-streaming channel, there exists $\boldsymbol{f_{s1}>f_1}$*; (2) contrasting the price of them, there exists a threshold value* $\boldsymbol{\lambda_0}$*, when* $\boldsymbol{\lambda<\lambda_0}$*,* $\boldsymbol{p_1<p_{s1}}$*; (3) and the price, sales effort and profit of KOL live-streaming are positively correlated with the commission ratio* $\boldsymbol{\lambda}$*.*

Proof: Making a comparison between the price and sale effort of the two live-streaming channels, respectively, we can get

$$\boldsymbol{f_1-f_{s1}=\frac{\left\{\begin{array}{c}\beta(c-1)(a(c(\lambda-1)(2+2\varepsilon(\theta-1)-4\theta-\beta^2(\varepsilon-1)(1+\lambda)(3\theta-1))+\\ (-1+\lambda)\left(2\theta+(\varepsilon-1)\lambda\left(2-2\theta+\beta^2(3\theta-1)\right)\right)+c(1-\lambda+(-3+2\varepsilon+\lambda)\theta)\mu+\\ (-2+\lambda)(1-\theta+\lambda(\varepsilon\theta-1))\mu+c^2(\varepsilon-1)(\beta^2(\lambda-1)(3\theta-1)-\sigma\mu))\\ -k(\lambda-1)((2+\beta^2(c-1))(\varepsilon-1)(c-\lambda)+(-2+c+\lambda)\mu)(n+N\sigma))\end{array}\right\}}{2\left(2+\beta^2(c-1)\right)(c-1)\left(1+\beta^2(\varepsilon-1)(c-\lambda)\right)(\lambda-1)-(-2+c+\lambda)^2\mu^2}},$$

As we know that, $\boldsymbol{0.1<\lambda<0.5}$, $\boldsymbol{0<\mu<1}$. So, we can figure out $\boldsymbol{f_1-f_{s1}<0}$;

$$\boldsymbol{p_1-p_{s1}=\frac{\begin{array}{c}-a(c-1)(\lambda-1)\left(2-4\theta+\beta^2(-1+c+\theta+c(-3+2\varepsilon)\theta-2(\varepsilon-1)\lambda\theta)\right)-a(-2+c+\lambda)(1-\lambda+(-2+c+\lambda)\theta)\mu\\ +k(\lambda-1)(2-\beta^2(c-1)^2-2c+(-2+c+\lambda)\mu)(n+N\sigma)\end{array}}{2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(-2+c+\lambda)^2\mu^2}}.$$

Existence of threshold $\boldsymbol{\lambda_0}$, $$\boldsymbol{\lambda_0=\frac{\begin{array}{c}(a(c-1)(2-4\theta+\beta^2(c-1+(-1-3c+2(1+c)\varepsilon)\theta))+a(3-c+2(c-2)\theta)\mu\\ +k\left(-2+\beta^2(c-1)^2-c(\mu-2)+3\mu\right)(n+N\sigma)-(1-c)\sqrt{(B_1+B_2)}\end{array}}{4a\beta^2(c-1)(\varepsilon-1)\theta-2a(\theta-1)\mu+2k\mu(n+N\sigma)}},$$

where

$$\boldsymbol{B_1=a^2\left(\beta^4(c-1)^2(1+(-3+2\varepsilon)\theta)^2+2\beta^2(c-1)(2+2\theta^2(6+4\varepsilon(\mu-1)-5\mu)-(-5+2\varepsilon)\theta(\mu-2)-\mu)+(\mu-2)^2+16\theta(\mu-1)-16\theta^2(\mu-1)\right)},$$

$\boldsymbol{B_2=2ak(\beta^4(c-1)^2(1+(-3+2\varepsilon)\theta)-\beta^2(c-1)(2+(-5+2\varepsilon)\theta)(\mu-2)+(\mu-2)^2+8\theta(\mu-1))(n+N+k^2(2+\beta^2(c-1)-\mu)^2(n+N\sigma)^2}$, and $\boldsymbol{\lambda<\lambda_0}$, $\boldsymbol{p_1<p_{s1}}$.

$$\frac{\partial p_1}{\partial \lambda}=\frac{\left\{\begin{array}{c}(-1+c)(a(2\beta^2\left(2+\beta^2(c-1)\right)^2(c-1)(\varepsilon-1)(\lambda-1)^2(\theta-1)+2(2+\beta^2(c-1))(c-1)\\ \left(1-c+\beta^2(\varepsilon-1)(-2+c^2-2c\lambda-(-4+\lambda)\lambda)\right)\theta\mu+\left(2+\beta^2(c-1)\right)(c-\lambda)(-2+c+\lambda)(\theta-1)\mu^2+\\ (-2+c+\lambda)^2\theta\mu^3)-(2+\beta^2(c-1))k(2\beta^2(2+\beta^2(c-1))(c-1)(\varepsilon-1)(\lambda-1)^2+(c-\lambda)(-2+c+\lambda)\mu^2)(n+N\sigma))\end{array}\right\}}{(2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(-2+c+\lambda)^2\mu^2)^2},\ \frac{\partial p_1}{\partial \lambda}\geq 0,$$

and the same method can be used to confirm the other conclusions.

Proposition 1 indicates that under the commissioned live-streaming mode, there is a threshold value of the commission ratio. The price of KOL live-streaming will be more advantageous when the ratio is below this limit. Because the products are typically only on sale in KOL live-streaming for a limited time or even just one day, the manufacturer prefers to sell at a concessional price during the live-streaming and hopes that KOL streamers would boost sales. However, as the commission ratio rises, the more products are sold during KOL live-streaming, the more profits of KOLs will be made. As a result, the streamer selects to step up sales efforts to increase revenues. Due to cost increases, the manufacturer will increase the price of KOL live-streaming, rendering the commodity price of KOL live-streaming uncompetitive. Therefore, the manufacturer will only invite KOL streamers to sell stuff in live-streaming when the commission range is appropriate. With the help of the existing traffic of KOL streamers and the gimmick of low prices to attract consumers, the manufacturer can not only increase sales quickly but also promote commodities and enhance the visibility of brands.

Additionally, consumers can purchase the goods for less money. While both channels use live-streaming to sell goods, KOL streamers have a wide range of products and a group of fans. By contrast, the manufacturer only sells products from their shop, and the manufacturer's live self-live-streaming channel typically has longer time and higher frequencies; they typically put more effort into increasing sales. These conclusions are consistent with business practice. For example, on live-streaming platforms, when manufacturers invite KOLs to help sell their products (i.e., the commissioned live-streaming mode), they are often willing to use more favorable prices to attract more consumers.

Proposition 2. In the retail live-streaming mode, (1) Contrasted the sales efforts of manufacturer self-live-streaming and KOL live-streaming channels, there exists $\boldsymbol{f_{s2}>f_2}$. (2) Compared with the price of live-streaming channels of the manufacturer and KOL streamer, there is a threshold $\boldsymbol{\theta}$, when $\boldsymbol{\theta>\theta_0}$, $\boldsymbol{p_{s2}>p_2>w}$. (3) While the pricing and sales effort of the manufacturer are positively related to $\boldsymbol{\theta}$, the price, sales effort, and wholesale price of KOL live-streaming are negatively correlated with $\boldsymbol{\theta}$.

Proof: Making a subtraction between the sales effort of KOL live-streaming and the sales effort of manufacturer self-live-streaming and making a comparison between the price of KOL live-streaming and the price of manufacturer self-live-streaming and the wholesale price, respectively, we can get

$$f_2-f_{s2}=\frac{\begin{array}{c}-b(c-1)(a(2-2\mu(2+\mu)+2\theta(\mu-1)(3+\mu)+\beta^2(c-1)(1-\mu(1+\mu)+\theta(-3+\mu^2)))\\ -k(-2+2\mu(2+\mu)+\beta^2(c-1)(-1+\mu+\mu^2))(n+N\sigma))\end{array}}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)}<0,$$

$$p_{s2}-w=\frac{\begin{array}{c}a(-\beta^4(c-1)^3(1+\theta(\mu-1))-4(-1+c+2\theta+c\theta(\mu-1)+\mu-2\theta\mu)+\\ \beta^2(c-1)(4+4c(\theta-1)-6\theta+(-1+(5-4c)\theta)\mu+(c-1)(\theta-1)\mu^2))-\\ k(4(-1+c+\mu)+\beta^2(c-1)((4+\beta^2(c-1))(c-1)+\mu+(c-1)\mu^2))(n+N\sigma)\end{array}}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)};$$

$$p_2-p_{s2}=\frac{\begin{array}{c}a\left(\beta^4(c-1)^2\left(1+\theta(\mu-1)\right)+2(\mu-1)(-3+5\theta+(\theta-1)\mu)+\beta^2(c-1)(5-7\theta-\mu+4\theta\mu)\right)\\ +k(2+\beta^2(c-1)-2\mu)(3+\beta^2(c-1)+\mu)(n+N\sigma)\end{array}}{\beta^2(4+\beta^2(c-1))(c-1)(-2+\mu^2)+8(-1+\mu^2)};$$

Existence of threshold value $\boldsymbol{\theta_0}$, $\boldsymbol{\theta_0}=\frac{-(2+\beta^2(c-1)-2\mu)(3+\beta^2(c-1)+\mu)(a+k(n+N\sigma))}{a(\beta^4(c-1)^2(\mu-1)+2(\mu-1)(5+\mu)+\beta^2(c-1)(-7+4\mu))}$ satisfying $\boldsymbol{\theta>\theta_0, p_{s2}>p_2>w}$.

$\frac{\partial p_{s2}}{\partial \theta}=\frac{a\left(b^2(-1+c1)(-2+u)+4(-1+u)\right)}{b^2\left(4+b^2(-1+c1)\right)(-1+c1)(-2+u^2)+8(-1+u^2)}$, and the other conclusions can be drawn in the same way.

Proposition 2 indicates that the retail live-streaming mode differs from the commissioned live-streaming mode. Taking into consideration the wholesale cost, the streamer will raise the price for the commodities to maximize profits. Customers favor the live channel of the streamer when $\boldsymbol{\theta}$ is low. For the manufacturer, the sales volume is concentrated in KOL live-streaming channels, and the sales volume of the manufacturer's self-live-streaming channel is bleak. To lure customers with low pricing and capture the market, the manufacturer will raise the wholesale price while lowering that of the manufacturer's self-live-streaming. Naturally, the price of KOL live-streaming is more expensive than the price of the manufacturer's self-live-streaming. When θ grows gradually, the manufacturer is willing to offer the live streamer a lower wholesale price for the manufacturer's channel that already dominates most of the market for the products and anticipates that broadening the live channel would boost sales.

Additionally, KOL Streamer offers its products for a low price to draw in more consumers. Ultimately, the price of KOL live-streaming commodities is less than that of the manufacturer's channel. Proposition 2 implies that in the retail live-streaming mode when setting their prices in the two live channels, the manufacturer needs to carefully consider the channel preferences of consumers.

Proposition 3. Compared that in the two modes, the price and sales effort of KOL live-streaming, the price and sales effort of manufacturer self-live-streaming, and the wholesale price positively correlate with the sensitivity coefficient $\boldsymbol{\beta}$ *of consumers to sales effort, the trust degree of KOL streamer*$\boldsymbol{k}$, *KOL streamer* $\boldsymbol{n}$, *and the proportion of impulsive consumers* $\boldsymbol{\sigma}$.

Proof: $\frac{\partial p_1}{\partial k} = \frac{-(2+\beta^2(c-1))(c-1)(\lambda-1)(n+N\sigma)}{2(2+\beta^2(c-1))(c-1)(-1+\beta^2(\varepsilon-1)(c-\lambda))(\lambda-1)+(c+\lambda-2)^2\mu^2} > 0$.The consistent approach can be used to draw other conclusions.

Proposition 3 illustrates that as consumers become more sensitive to sales efforts, KOL streamers and the manufacturer will consequently raise the sales effort and, correspondingly, the cost increase, at which point streamers in both modes will appropriately raise their sales prices for their benefit. Nevertheless, the most effective approach for the streamer is still to strive to improve the sales effort. Only when consumers perceive the streamer's sales effort will they choose to purchase commodities, which in turn increase demand and, ultimately, sales revenue of the live-streaming. As the public's trust in KOL streamers grows, so many fans are willing to buy merchandise from KOL live-streaming, and product pricing deciders of both live-stream channels will raise the prices. At the same time, the manufacturer will inevitably increase the amount of sales effort on his live channel to improve competitiveness, and the pricing will reflect this. Because consumers are driven by the external factor of the influence of KOL streamers and will be less price-sensitive, the price of products in each channel will rise in accordance when the influence of KOL streamers is positive and gradually expands. Supplementing the previous papers (e.g., Kang *et al*., 2021 and Huang *et al*., 2024), Proposition 3 shows that in the retail live-streaming mode, the price and sales effort of the two live channels are also influenced by the sensitivity coefficient $\boldsymbol{\beta}$ of consumers to sales effort, the trust degree of KOL streamer $\boldsymbol{k}$, the influence of KOL streamer $\boldsymbol{n}$, and the proportion of impulsive consumers $\boldsymbol{\sigma}$.

5. EXAMPLE ANALYSIS

Since the KOL streamer investigated in this research is a moderately popular streamer, the default consumers' preference will be overly focused on the manufacturer's self-live-streaming channel, where $\boldsymbol{\theta \geq 0.5}$. Similar to Wei *et al*. (2021), to be able visually to analyze the impact of commission ratio, consumers' sensitivity coefficient to the sales effort, and the trust degree of KOL streamer in two different modes, we make the following assumptions assume $\boldsymbol{a = 10}$; $\boldsymbol{N = 50}$; $\boldsymbol{\mu = 0.2}$; $\boldsymbol{\varepsilon = 10}$%; $\boldsymbol{\sigma = 8}$%; $\boldsymbol{\theta = 0.6}$.

5.1 The impact of the commission ratio on the equilibrium solution.

Under the commissioned live-streaming mode, we first consider the impact of the commission rate on price, sales effort, and profit. Assuming $\boldsymbol{\beta = 0.6}$; $\boldsymbol{k = 0.5}$; $\boldsymbol{n = 1}$. Considering the commission rate charged by the platform, the range of $\boldsymbol{\lambda}$ is (0.1, 0.5); the results are displayed in Fig.3 and Figure 4.

Figure 3 shows that as the commission coefficient increases, both the sale price and the sales effort of KOL streamers significantly rise. With the rising commission ratio, the range of variation of price and sales effort of KOL streamers are more than those of the manufacturer's self-live-streaming channel. Figure 3 further supports the finding of Proposition 1, which states that although an increase in commission improves a KOL streamer's live sales effort, he can close a deal with less effort due to other factors, such as the streamer's influence. The revenues of the KOL streamer will increase when the commission ratio rises, as shown in Figure 4, while the manufacturer's profits will fall. Therefore, under the commissioned live-streaming mode, the manufacturer possesses more discourse power than the moderate streamer and will strictly control the commission ratio within a specific range.

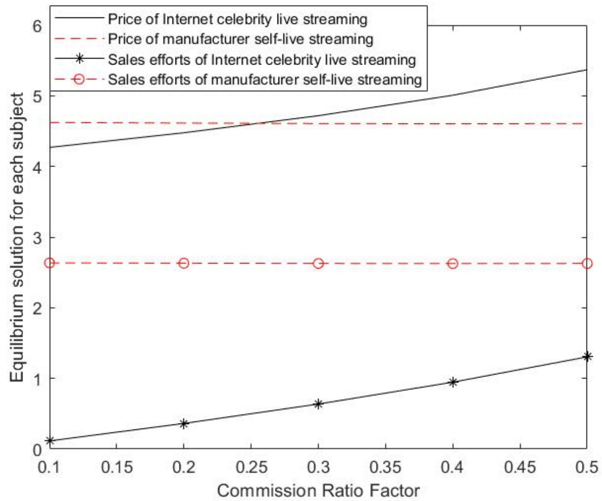

Figure 3. Impact on the equilibrium solution [%]

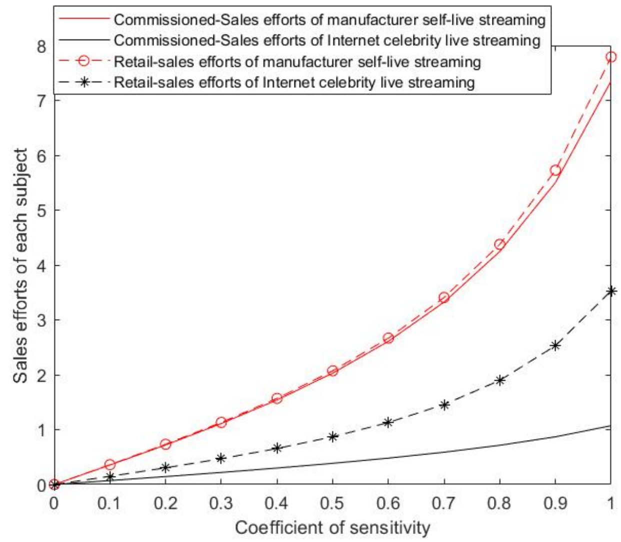

Figure 4. The Impact on profit [%]

5.2 Effect of consumer's sensitivity coefficient on the equilibrium solution.

Secondly, we consider the consumer's sensitivity coefficient on the sales effort of KOL live-streaming. Suppose $\boldsymbol{\lambda = 0.25}$; $\boldsymbol{k = 0.5}$; $\boldsymbol{n = 1}$; $\boldsymbol{\beta}$ is (0, 1), and the results are shown in Figure 5, Figure 6, and Figure 7.

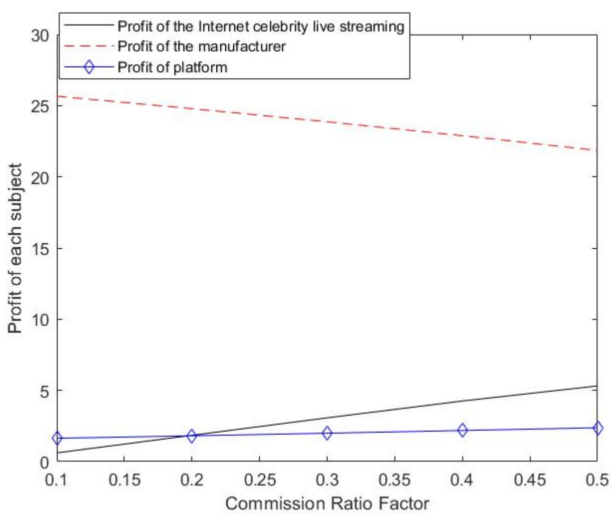

Figure 5. The influence on price [%].

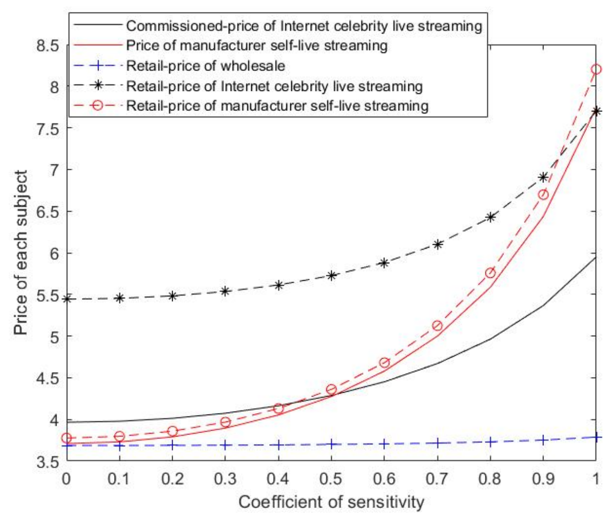

Figure 6. The influence on sales effort [%].

Combined with Figure 5 and Fig.6, the commodity price for each channel in both modes grows with consumers' sensitivity coefficient to the sales effort of the live-streaming, among which the rise of wholesale prices is slower. When consumers are less sensitive to sales efforts, the manufacturer's self-live-streaming channel lacks competition compared with KOL live-streaming, and the manufacturer will decide to lower commodity pricing, even close to wholesale commodity cost, to promote sales. Because KOL live-streaming has the authority to decide the product's price in decentralized decision-making, the cost of the wholesale pricing will result in an increase in the cost of its live-streaming. Ultimately, the price of KOL live-streaming in retail mode is consistently the highest. The profit of KOL live-streaming under the retail live-streaming mode is somewhat higher than that in the commissioned mode when Figure 7 is combined. To increase revenues, KOL live-streaming under the retail live-streaming mode is willing to enhance its sales effort. In conclusion, an increase in sales effort can more effectively entice customers to make purchases than a low price at a low sales effort. The development in sales effort level can also increase profitability for all live-streaming channels.

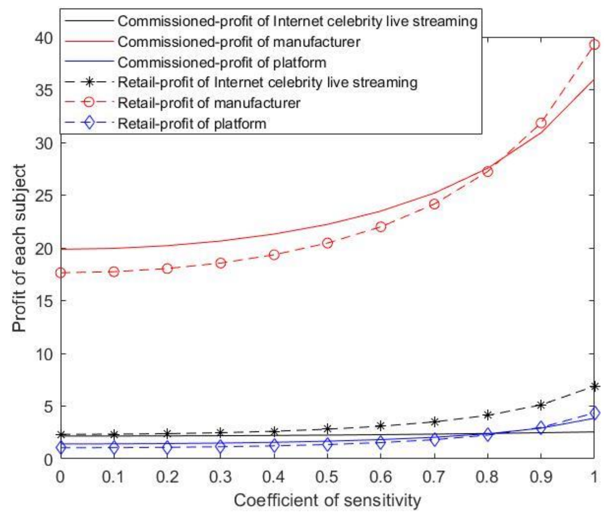

Figure 7. The influence of profit [%].

5.3 The influence of trust degree on equilibrium solution.

Thirdly, this paper considers the influence of consumers' trust degree in KOL streamers. Suppose $\boldsymbol{\mu = 0.2}$; $\boldsymbol{\lambda = 0.25}$; $\boldsymbol{\beta = 0.6}$; $\boldsymbol{n = 1. k}$ takes values in the range of (0,1). The results are shown in Figures 8, 9, and 10, respectively.

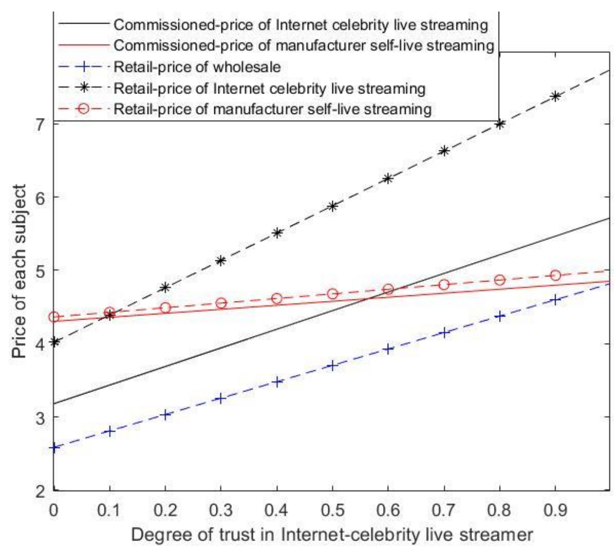

Figure 8. The effect on price [%].

Figures 8-10 reveal that, in both modes, price, sales effort level, and profit growth have in positive correlation with the degree of customer faith in KOL streamers, which is consistent with Proposition 3's findings. Even though the manufacturer's self-live-streaming channel is proportionate to the level of trust, it has little impact on price, sales effort, and profit since the manufacturer's self-live-streaming channel lacks influence. When the manufacturer, under the retail live-streaming mode, considers that the KOL streamer's followers have a high level of trust, the wholesale price will also rise. This is because when customers' trust in KOL live-streaming reaches a specific level, it will cause this type of group to become less price-sensitive and select their preferred channel. They will only purchase stuff from the live streaming of the streamer. These consumer groupings will eventually develop into loyal followers of KOL live-streaming. When the manufacturer considers the price, they will raise the product's price commensurately. Especially when other conditions are specific, and customers fully trust the live streamer, the wholesale pricing under the retail live-streaming mode is highly close to the price of the manufacturer's self-live-streaming channel. Many top streamers, like Li Jiaqi, have a sizable following of devoted followers who place orders immediately in his live studio when making purchases online rather than comparing pricing with other channels.

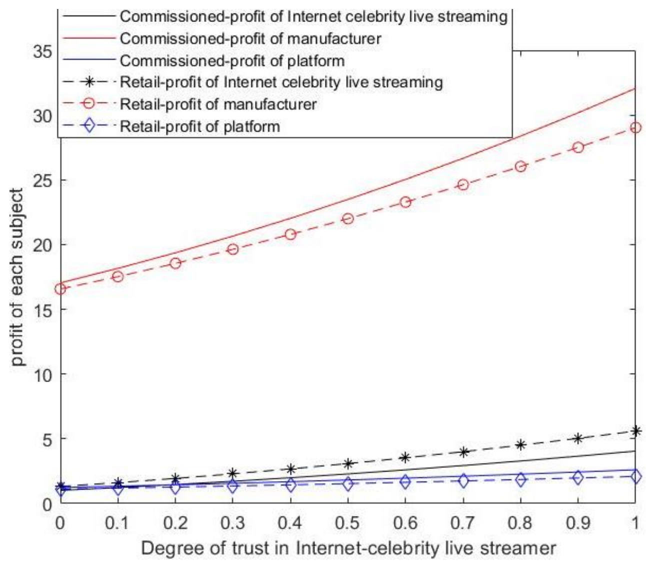

Figure 9. The effect on sales effort [%].

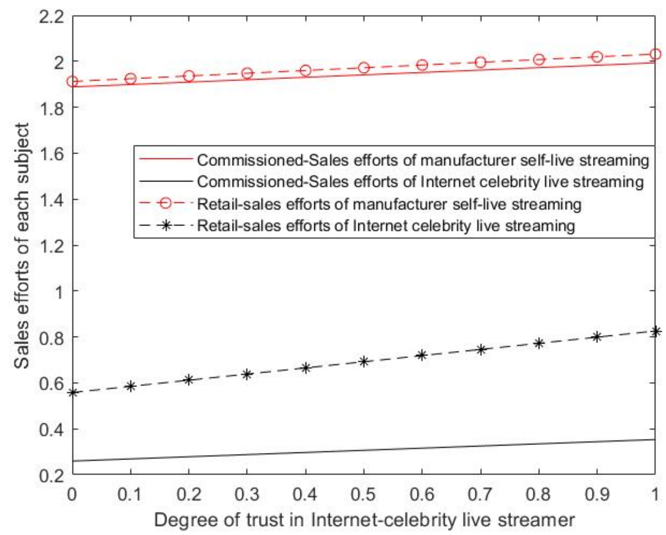

Figure 10. The effect on profit [%].

5.4 The influence of trust on demand and profit.

The influence and consumers' trust degree of KOL live-streaming on the demand and profit of each channel are considered comprehensively. Assume $\boldsymbol{\lambda = 0.25}$, $\boldsymbol{\beta = 0.6}$, $\boldsymbol{n}$ belongs to (0,3), and $\boldsymbol{k}$ belongs to (0,1). The results are displayed in Figures 11-13.

In conjunction with Figure 10-13, it is apparent that when consumers' trust degree is low, the streamer's growing influence on demand and profit is minimal; however, when consumers' trust degree is high, the promotion of influence can significantly increase demand and profit. In this situation, KOL Streamer prefers to expand its influence to boost commodity sales and profit. In a comprehensive view, a rising degree of trust or influence can boost demand and revenue from live streaming. Especially under the commissioned live-streaming mode, although the manufacturer's self-live-streaming channel continues to hold most of the live-streaming channel market. These can partly explain why sellers want to cooperate with streamers with many fans. When there is a high amount of trust and influence, the demand for KOL live-streaming with significant growth can even outpace the manufacturer's self-live-streaming channel. Therefore, compared with the retail live-streaming mode, it is preferable for the manufacturer to collaborate with the streamer and directly invite KOL streamers to promote products. Additionally, the retail live-streaming mode may enable the streamer to generate more revenue at a particular expense.

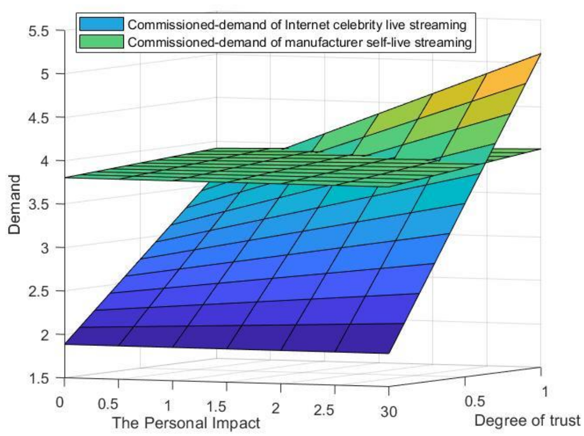

Figure 11. Demand under live commissioned mode [%].

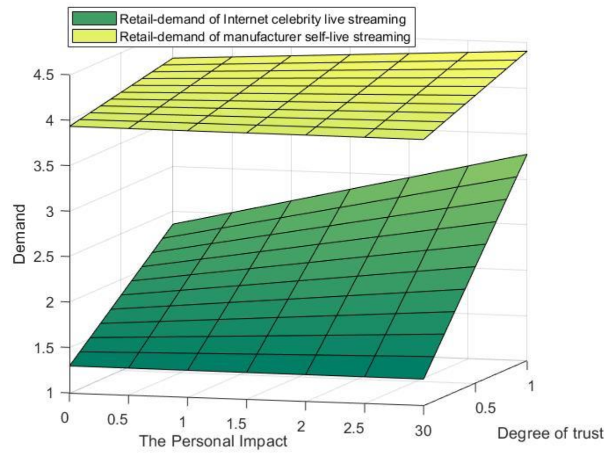

Figure 12. Demand under retail live-streaming mode [%].

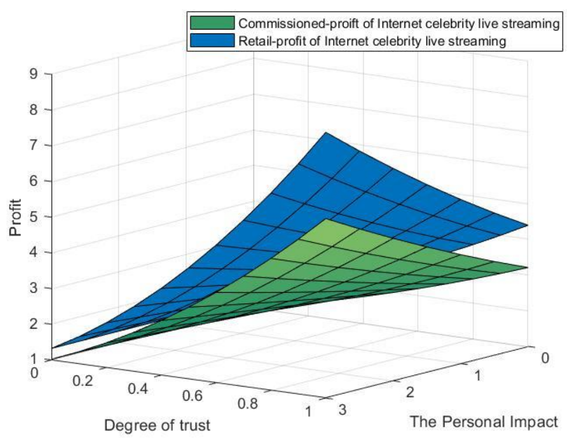

Figure 13. Profit of KOL live-streaming [%].

6. CONCLUSIONS

In this paper, we investigate an online dual-channel supply chain system with a manufacturer, a KOL streamer, and a live platform and consider the optimal decision price, sales effort level, and profitability of the supply chain members. The following results are reached after analyzing the impact of the commission ratio, consumers' sensitivity to the sales effort of the live-streaming, the trust in the KOL streamer, and the streamers' influence on each supply chain member's decision-making.

- Under the commissioned live-streaming mode, KOL streamers' sales efforts is never as strong as those of the manufacturer self-live-streaming. When the commission ratio keeps within a specific range, the price of KOL live-streaming is typically cheaper than that of the manufacturer's self-live-streaming and is proportional to the commission ratio. In contrast to the channel of manufacturer self-live-streaming, the commission ratio is more responsive to fluctuations in pricing and sales efforts on KOL streamers.
- Under the retail live-streaming mode, the amount of sales effort put forth by the KOL streamer fewer sales efforts than that of the manufacturer self-live-streaming, and the price of KOL live-streaming is only lower when consumers' preferences are concentrated more in the manufacturer channel. The price of KOL live-streaming is higher in the retail live mode than in the commissioned live mode.
- The pricing, sales effort, and profit of each subject are positively correlated with the sensitivity of consumers to live sales effort, trust in KOL streamers, the influence of KOL streaming, and the proportion of impulse consumption, according to the two modes. The optimal decision of KOL streamers is inversely proportional to consumers' preferences and directly

proportional to the manufacturer's channel. In contrast, the manufacturer's self-live-streaming is more affected by the sensitivity coefficient than the KOL live-streaming channel.

- For manufacturers, inviting KOL streamers in the way of signing commissioned contracts can promote sales more. Furthermore, for KOL streamers, the way of wholesaling merchandise can obtain more profits.

Our research generates several managerial implications for parties involved in live-streaming selling. For manufacturers, making optimal pricing decisions is intricately tied to understanding consumers' channel preferences, regardless of the cooperation modes in play. Thus, it is imperative for manufacturers to meticulously evaluate the influence of consumer channel preferences when formulating pricing strategies. At the same time, the selection of the optimal cooperation mode by manufacturers is contingent upon consumers' responsiveness to sales efforts. Consequently, manufacturers must consider consumers' sensitivity to sales efforts when determining the most suitable cooperation mode. Specifically, when sensitivity to sales efforts is low, manufacturers tend to adopt the commissioned live-streaming mode; conversely, when sensitivity is high, they are more inclined to apply the retail live-streaming mode.

Regarding Internet celebrities, within the retail live-streaming mode, achieving the optimal degree of sales effort necessitates a level lower than that employed in manufacturer self-live-streaming. This distinction highlights the considerations Internet celebrities must undertake to maximize their effectiveness within different live-streaming modalities.

Finally, live-streaming platforms' profitability is intricately linked to several critical factors, including the commission ratio, consumers' responsiveness to sales efforts during live streamings, trust levels in KOL streamers, and the streamers' influence. Consequently, these platforms must conduct thorough assessments to discern the effects of these business variables and adjust their strategies accordingly to maximize profitability.

Some directions merit further investigation. First, in this paper, we only considered how the moderate streamer and the manufacturer make pricing decisions when the manufacturer is dominant. In the future, we can examine and investigate how manufacturers and top KOL streamers decide on prices (either when both sides are equally powerful or when Internet celebrities are in the dominant position). Second, we use a theoretical model to investigate the optimal pricing decision, sales effort level, and profitability of the supply chain members. In further research, one can check and complement the theoretical findings of this paper by using empirical methods. Finally, this paper focuses on a market consisting of a manufacturer, a KOL streamer, and a platform. A supply chain consisting of one platform and multiple manufacturers is also typical and could be studied in the future.

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