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TWO NOVEL ROBUST NETWORK DATA ENVELOPMENT ANALYSIS MODELS TO OBTAIN THE PERFORMANCE SCORE INTERVAL OF MULTI-STAGE SERIES SYSTEMS

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The main goal of this paper is to present a new approach for measuring the performance of n-stage series systems in the presence of uncertain data, which are two challenging issues in evaluating the efficiency of Decision-Making Units (DMUs) using traditional Data Envelopment Analysis (DEA) models. By using a Network DEA model and its dual, as well as using Bertsimas *et al*.'s robustness technique, two Robust Network DEA have been presented. These models can display a range of DMUs performance with appropriate accuracy. Proposed models were used to determine the efficiency range of Iranian dairy companies' supply chain with three stages. The results show that the proposed models are applicable and effective. Total efficiency bounds are obtained with percentage deviations of 20%, 10% and 5%. The lower bounds have relative errors of 0.39, 0.23 and 0.12 and a correlation coefficient of more than 97%, and the upper bounds have relative error of 1.1, 0.84 and 0.62 and a correlation coefficient of about 90%. Therefore, the proposed model for calculating the lower bound is more accurate. The calculation of the efficiency bounds of the sub-stages also confirms this issue. Finally, the obtained results have been compared with the values obtained through a fuzzy three-stage DEA model, our results have a higher correlation coefficient and more accurate upper bounds.

Keywords: Efficiency Measurement, Multi-Stage Systems, Network DEA, Robust Optimization, Bertsimas *et al*. Approach.

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1. INTRODUCTION

To measure the relative effectiveness of Decision-Making Units (DMUs), Charnes *et al*. (1978) developed Data Envelopment Analysis (DEA), a non-parametric linear Programming (LP) based method. This method generalizes Farrell's (1957) single-input, single-output technical efficiency measure for multiple-input, multiple-output situations. By comparing the weighted sum of outputs to the weighted sum of inputs and performing operations using mathematical programming, DEA provides relational performance efficiency. DEA does not call for an explicit functional form linking inputs and outputs, in contrast to parametric approaches. The DEA constant returns to scale (CRS) model, which assumes that all businesses function at their ideal scale, is a technique for measuring efficiency. The variable Return to Scale (VRS) was later added to DEA by Banker *et al*. (2004).

Although DEA has gained widespread acceptance as a reliable way to gauge a system's efficiency, in many real-world scenarios, DMUs have a network structure and are composed of numerous interconnected divisions. The inputs and outputs were specific to each division. According to Färe and Grosskopf (2000), Castelli *et al*. (2001), and Liu and Tone (2008), an intermediate output from one division becomes an intermediate input for another division. According to Biresh *et al*. (2014), traditional DEA models view DMUs as "black boxes" that use a set of inputs and outputs to calculate an efficiency score.

Researchers like Färe and Grosskopf (2000), Lewis and Sexton (2004), Sexton and Lewis (2003), Tone and Tsutsui (2009), and Färe *et al*. (2013) have generalized the Network DEA (NDEA) model to analyze the network structure of systems. This has helped open the "black box" and gain more insight into the production process. The network DEA model was first developed by Färe and Grosskopf (2000) and subsequently enhanced and expanded by various researchers. An expansion of the two-stage DEA model proposed by Sexton and Lewis (2003, 2004) presented a network DEA model for a multistage

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system. Their research separately resolved the DEA model for each node. To evaluate efficiency when inputs and outputs may not change proportionally, Tone and Tsutsui (2009) proposed a network slack-based measure model (NSBM) (Huang *et al*., 2014).

Kao and Hwang (2008) created a DEA model to quantify the efficiency of the system and component processes simultaneously for systems made up of two processes connected in series. The efficiency of the system is a product of these two processes, which is an interesting discovery. They could expand their model to include more than two processes. Kao (2014) examined research on network DEA by examining the models applied and the network system structures of the problem under study. From a methodological standpoint, this study identifies certain possibilities for future research, and from an empirical standpoint, it serves as a motivation to investigate new areas of application. Tsihrintzis *et al*. (2019) explained the essential principles of network DEA approaches and their advantages over traditional DEA approaches. Additionally, they conducted a critical analysis of contemporary techniques in the discipline and offered a comprehensive, uniform categorization of a sizable body of network DEA literature. For more information, refer to Kao (2014) and Koronakos (2019).

Data uncertainty is one of the most important problems in DEA. Because the DEA technique produces a problem formulation in the form of linear programming, it is difficult to handle uncertainty using conventional methods when the input data are uncertain.

Three general approaches to deal with data uncertainty in mathematical optimization have been developed so far: stochastic optimization, fuzzy optimization and robust optimization.

In stochastic optimization, we need enough historical data to fit the distribution function. Therefore, it is difficult to achieve the true distribution function. In stochastic optimization, the answer is justified with probability, and it may be unjustified for some real situations. Although the probability of this is low, if it happens, it will impose a high cost.

In fuzzy optimization, determining the shape of the membership function faces similar challenges. In fact, random and fuzzy optimization have a soft approach to constraints. Also, in these approaches, the complexity of the problem has also increased, and even in the scenario-based stochastic method, this happens with the increase in the number of scenarios. (Rouyendegh B.D. *et al*., 2020; Demir E., 2014).

Robust optimization is a method in which the problem is optimized for the worst case. In robust optimization, the solution is feasible for all scenarios. In fact, in robust optimization, the best solution is selected among the solutions that are justified for all scenarios. This method is a hard and precise approach. In this method, the general certainty set is a closed and convex set that determines the parameter change limits. In this way, we do not need to determine the shape of the distribution function or the membership function, and only knowing the range of changes or Support in the interpretation of the fuzzy literature is sufficient. Therefore, it can be applied to almost all real DEA problems.

Robust optimization was suggested by Soyster (1972) to deal with uncertainty. To further explore inexact linear programs, Falk (1976) conducted more studies. Ben-Tal and Nemirovski (1999) presented a unique strategy based on the art of cone programming. El-Ghaoui and Lebret (1997) and Ben-Tal and Nemirovski (1998, 1999, and 2000) proposed a new model for uncertain data based on ellipsoidal uncertainty sets. Bertsimas and Sim (2003, 2004, and 2006) and Bertsimas *et al*. (2004) developed robust optimization methods based on polyhedral uncertainty sets. This approach can preserve the class of problems under consideration. Recently, some studies on DEA have used robust optimization to deal with data uncertainty, called robust data envelopment analysis (RDEA) (Wu *et al*., 2017; Lu *et al*., 2019; Tavana *et al*., 2021).

Sadjadi and Omrani (2008) were also the first to propose the concept of a robust two-stage NDEA model to deal with uncertainty in data. They compared the performance of two robust approaches established by Ben-Tal and Nemirovski (1999) and Bertsimas and Sim (2003) to address uncertainty in two applications from the energy and telecom industries.

Peykani *et al*. (2020) examined milestone approaches to dealing with uncertainty in DEA. Full classifications of robust data envelopment analysis (RDEA) are presented in this study. It included 73 studies from 2008 to 2019. The report concludes with recommendations for further research on RDEA. To investigate regional efficiency in China, Chen *et al*. (2020) constructed a two-stage Environmental Sustainability index consisting of a Production Efficiency index and an eco-efficiency index. They used a multiplicative relational network data envelopment analysis model and window analysis.

Radsar *et al*. (2022) proposed a robust three-stage model based on Bertsimas and sim (2004) approach in the field of efficiency evaluation in conditions of uncertainty and having an undesirable output. Also, Peykani *et al*. (2022) applied a two-stage robust network DEA for performance evaluation and ranking of 15 Mutual funds in the presence of uncertain data. They showed that the discriminatory power in the robust NDEA approach is more than in deterministic NDEA models.

In most of the previous studies, according to the need and simplicity of the problems, two or three-stage models of network data envelopment analysis have been used. Also, previous researchers have only obtained one value to approximate the efficiency score of DMUs. Therefore, according to the development of science and the more complexity of problems in the future world, the need to use models with more stages is strongly felt. In this research, two values have been produced to approximate the efficiency score of the units, which can be low approximation and high approximation of the efficiency score of the units. Therefore, in this study, a new general approach for robust network data envelopment analysis (RNDEA) was presented based on the development of Kao and Hwang's two-stage DEA model and its dual model and the robustness method of Bertsimas *et al*. This model helps researchers to provide the appropriate approximate efficiency range for each DMU for all series network systems to evaluate the efficiency and ranking of DMUs with respect to internal structure and data uncertainty. Also, the application and efficiency of the proposed approach was shown by measuring the three-stage supply chain performance of 40 dairy companies active in the capital market of Iran.

The structure of this paper is organized as follows. The literature review classification and literature gaps are introduced in Section 1. The theoretical background, including the structure of series network systems and NDEA modeling based on Kao and Hwang (2008), as well as the formulation of the robust model (RNDEA) based on the Bertsimas *et al*. technique to evaluate the performance of DMUs under uncertainty, is presented in Section 2. In Section 3, a new robust network DEA (RNDEA) model is proposed. The applicability of the proposed approach is shown by a real case study in Section 4, and the results are compared with the three-stage fuzzy data envelopment analysis (TSFDEA) model. Finally, the conclusions and directions for future research are presented in Section 5.

2. LITERATURE

In this section, the history of the research will be presented. In the first subsection, the multi-stage DEA model and its dual are presented to calculate the total efficiency score and the efficiency score of each stage. In the second subsection, Bertsimas *et al*.'s robustness technique is introduced and applied to the traditional DEA model.

2.1 Multi-Stage DEA Models

Consider an h-process series system. Let $x_{ij}^{t}, i = 1,\dots,M^{t}$ and $y_{rj}^{t}, r = 1,\dots,S^{t}$, be defined as the inputs and outputs of process t, which $x_{ij}^{1}, i = 1,\dots,M^{1}$ are initial process inputs and $y_{rj}^{h}, r = 1,\dots,S^{h}$, final process outputs, respectively. For DMU j, $z_{dj}^{t}, d = 1,\dots,D^{t}$, denote the d^{th} intermediate product of process t, $t = 1,\dots, h-1$. The intermediate products of process t are both the process t outputs and the process $t+1$ inputs. It should be noted that the final outputs of the system are the intermediate products of the last process h. Each process may have a varied number of input, intermediate, and output products, M^{t}, S^{t} and D^{t}. It is assumed here that they are the same for all processes, $M^{t} = M$, $S^{t} = S$, $D^{t} = D$, $t = 1,\dots,h$, only to simplify notation. The mechanism is depicted graphically in Figure 1.

Let us introduce the following basic notation:

$j \in J = \{1,\dots,n\}$: The index set of the n DMUs.

$k \in \{1,\dots,n\}$: Denotes the index of under evaluation DMU.

$X_{j}^{t} = \left(x_{ij}^{t}; i = 1,\dots,M^{t}, t = 1,\dots,h\right)$: The vector of stage t external inputs used by DMU j.

$Z_{j}^{t} = \left(z_{dj}^{t}, d = 1,\dots,D^{t}; t = 1,\dots,h-1\right)$: The vector of stage t intermediate outputs produced by DMUj.

$Y_{j}^{t} = \left(y_{lj}^{t}; l = 1,\dots,S^{t}, t = 1,\dots,h\right)$: The vector of stage t final outputs produced by DMU j.

$V^{t} = \left(v_{i}^{t}; i = 1,\dots,M^{t}, t = 1,\dots,h\right)$: The vector of weights for the stage t external inputs.

$W^{t} = \left(w_{d}^{t}, d =,\dots,D^{t}; t = 1,\dots,h-1\right)$: The vector of weights for stage t intermediate outputs.

$U^{t} = \left(u_{r}^{t}; r = 1,\dots,S^{t}, t = 1,\dots,h\right)$: The vector of weights for the stage t external outputs.

$\lambda^{t} = \left(\lambda_{j}^{t}, j = 1,\dots,n; t = 1,\dots,h\right)$: The vector of weights for DMU j in stage t.

E_{k}: The overall efficiency of DMU k.

$E_{k}^{t}(t = 1,\dots,h-1)$: The stage t efficiency of DMU k.

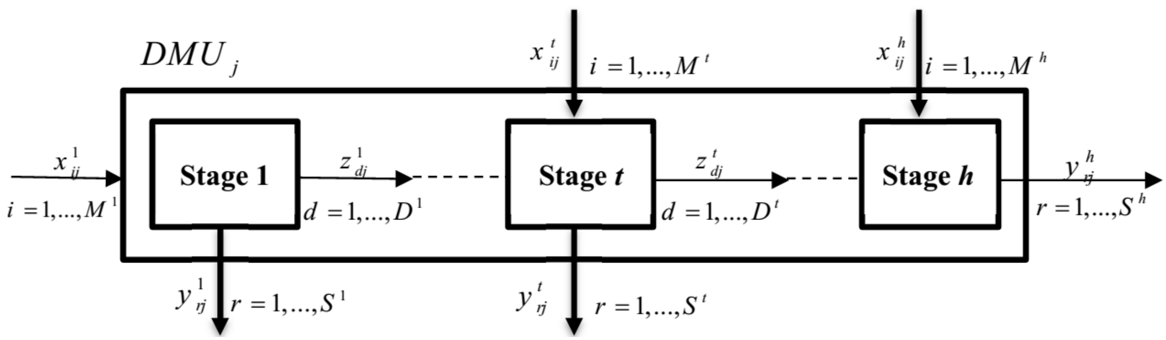

Figure 1. The multi-stage series system with intermediate products

The following model based on the generalized of Kao and Hwang (2008) is used to calculate the system efficiency of DMU k:

$$E_k = max \sum_{t=1}^{h} \sum_{r=1}^{S} u_r^t y_{rk}^t \quad (1.1)$$
$$s.t. \quad \sum_{t=1}^{h} \sum_{i=1}^{M} v_i^t x_{ik}^t = 1, \quad (1.2)$$
$$\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{r=1}^{S} u_r^1 y_{rj}^1 - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \qquad j = 1, \dots, n \quad (1.3)$$
$$\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, j = 1, \dots, n \;\; t = 2, \dots, h-1 \quad (1.4)$$
$$\sum_{r=1}^{S} u_r^h y_{rj}^h - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \qquad j = 1, \dots, n \quad (1.5)$$
$$\sum_{t=1}^{h} \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{t=1}^{h} \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, \quad (1.6)$$
$$u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \quad r = 1, \dots, S, \; i = 1, \dots, M, d = 1, \dots, D, t = 1, \dots, h. \quad (1)$$

As can be observed, constraint (1.6) equals the sum of constraints (1.2) until (1.5). As a result, it can be ignored. As a result, model (1) may be summarized as follows:

$$E_k = max \sum_{t=1}^{h} \sum_{r=1}^{S} u_r^t y_{rk}^t \quad (2.1)$$
$$s.t. \quad \sum_{t=1}^{h} \sum_{i=1}^{M} v_i^t x_{ik}^t = 1, \quad (2.2)$$
$$\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{r=1}^{S} u_r^1 y_{rj}^1 - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \qquad j = 1, \dots, n \quad (2.3)$$
$$\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, j = 1, \dots, n \;\; t = 2, \dots, h-1 \quad (2.4)$$
$$\sum_{r=1}^{S} u_r^h y_{rj}^h - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \qquad j = 1, \dots, n \quad (2.5)$$
$$u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \qquad r = 1, \dots, S, \quad i = 1, \dots, M, \quad d = 1, \dots, D, \quad t = 1, \dots, h. \quad (2)$$

The efficiency of each process for DMU k is calculated by replacing the following relations for relations (2.1) and (2.2) in the preceding model, along with the same additional constraints:

$$E_k^1 = max \sum_{d=1}^{D} w_d^1 z_{dk}^1 + \sum_{r=1}^{S} u_r^1 y_{rk}^1 \quad (3.1)$$
$$s.t. \quad \sum_{i=1}^{M} v_i^1 x_{ik}^1 = 1, \quad (3.2)$$
$$\begin{aligned}
&\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{r=1}^{S} u_r^1 y_{rj}^1 - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \qquad j = 1, \dots, n \\
&\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, j = 1, \dots, n, t = 2, \dots, h-1 \\
&\sum_{r=1}^{S} u_r^h y_{rj}^h - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \qquad j = 1, \dots, n \\
&u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \qquad r = 1, \dots, S, \quad i = 1, \dots, M, \\
&d = 1, \dots, D, \quad t = 1, \dots, h.
\end{aligned} \quad (3)$$

The efficiency score for each stage T, $(T = 2, \dots, h-1)$, is as follows.

$$E_k^T = max \sum_{d=1}^{D} w_d^T z_{dk}^T + \sum_{r=1}^{S} u_r^T y_{rk}^T \quad (4.1)$$
$$s.t. \quad \sum_{d=1}^{D} w_d^{T-1} z_{dk}^{T-1} + \sum_{i=1}^{M} v_i^T x_{ik}^T = 1, \quad (4.2)$$
$$\begin{aligned}
&\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{r=1}^{S} u_r^1 y_{rj}^1 - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \qquad j = 1, \dots, n \\
&\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, j = 1, \dots, n \\
&t = 2, \dots, h-1 \\
&\sum_{r=1}^{S} u_r^h y_{rj}^h - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \qquad j = 1, \dots, n \\
&u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \qquad r = 1, \dots, S, \quad i = 1, \dots, M, \\
&d = 1, \dots, D, \quad t = 1, \dots, h
\end{aligned} \quad (4)$$

and the following model is used to calculate the efficiency score for the final stage.

$$E_k^h = max \sum_{r=1}^{S} u_r^h y_{rk}^h \quad (5.1)$$
$$s.t. \quad \sum_{d=1}^{D} w_d^{h-1} z_{dk}^{h-1} + \sum_{i=1}^{M} v_i^h x_{ik}^h = 1, \quad (5.2)$$
$$\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \sum_{r=1}^{S} u_r^1 y_{rj}^1 - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \qquad j = 1, \dots, n \quad (5)$$
$$\sum_{d=1}^{D} w_d^t z_{dj}^t + \sum_{r=1}^{S} u_r^t y_{rj}^t - \sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, j = 1, \dots, n$$
$$= 2, \dots, h-1$$
$$\sum_{r=1}^{S} u_r^h y_{rj}^h - \sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \qquad j = 1, \dots, n$$

$u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \qquad r = 1, \dots, S, \quad i = 1, \dots, M,$
$d = 1, \dots, D, \quad t = 1, \dots, h.$

On the other hand, the dual of model (2) can be represented as follows.

$$
\begin{aligned}
& E_k = \min \theta \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq \theta x_{ik}^t, \qquad i = 1, \dots, m, \ \ t = 1, \dots, h \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t \geq 0, d = 1, \dots, D, \quad t = 1, \dots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq y_{rk}^t, \qquad r = 1, \dots, s, \quad t = 1, \dots, h \\
& \lambda_j^t \geq 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h-1
\end{aligned}
\tag{6}
$$

As the same way, the dual of model (3), the efficiency of the first stage, is as follows.

$$
\begin{aligned}
& E_k^1 = \min \theta^1 \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq \theta^1 x_{ik}^1, \qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq 0, \qquad i = 1, \dots, m, \ \ t = 2, \dots, h \\
& \sum_{j=1}^{n} \left(\lambda_j^1 - \lambda_j^2\right) z_{dj}^1 \geq z_{dk}^1, d = 1, \dots, D, \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t \geq 0, d = 1, \dots, D, \quad t = 2, \dots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 \geq y_{rk}^1, \qquad r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq 0, \qquad r = 1, \dots, s, \quad t = 2, \dots, h \\
& \lambda_j^t \geq 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h
\end{aligned}
\tag{7}
$$

For each intermediate stage, $T = 2, \dots, h-1$, dual model of (4) is equal to

$$
\begin{aligned}
& E_k^T = \min \theta^T \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^T x_{ij}^T \leq \theta^T x_{ik}^T, \qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq 0, \qquad i = 1, \dots, m, \ \ t = 1, \dots, h, t \neq T \\
& \sum_{j=1}^{n} \left(\lambda_j^{T-1} - \lambda_j^T\right) z_{dj}^{T-1} + \theta^T z_{dk}^{T-1} \geq 0, \ d = 1, \dots, D, \\
& \sum_{j=1}^{n} \left(\lambda_j^T - \lambda_j^{T+1}\right) z_{dj}^T \geq z_{dk}^T, \ d = 1, \dots, D, \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t \geq 0, \ d = 1, \dots, D, \quad t = 1, \dots, h-1, t \neq T \\
& \sum_{j=1}^{n} \lambda_j^T y_{rj}^T \geq y_{rk}^T, \qquad r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq 0, \qquad r = 1, \dots, s, \quad t = 1, \dots, h, t \neq T \\
& \lambda_j^t \geq 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h
\end{aligned}
\tag{8}
$$

and for the last stage, the dual of model (5) is as follows.

$$
\begin{aligned}
& E_k^h = \min \theta^h \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq 0, \qquad i = 1, \dots, m, \ \ t = 1, \dots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^h x_{ij}^h \leq \theta^h x_{ik}^h, \qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t \geq 0, \ d = 1, \dots, D, \quad t = 1, \dots, h-2 \\
& \sum_{j=1}^{n} \left(\lambda_j^{h-1} - \lambda_j^h\right) z_{dj}^{h-1} \geq -\theta^h z_{ik}^{h-1}, \qquad d = 1, \dots, D, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \geq 0, \qquad r = 1, \dots, s, \quad t = 1, \dots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^h y_{rj}^h \geq y_{rk}^h, \qquad r = 1, \dots, s, \\
& \lambda_j^t \geq 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h
\end{aligned}
\tag{9}
$$

2.2 Robust Model with Bertsimas Technique

Consider the following nominal linear optimal problem:

$$\begin{aligned} & max \;\; cx \\ & \text{s.t:} \\ & \tilde{A}x \le b \quad l \le x \le u. \end{aligned} \tag{10}$$

Assume that just the elements of matrix $\tilde{A} = [\tilde{a}_{ij}]$ are uncertain in the previous formulation. Without losing generality, suppose object function c is not uncertain, and we can utilize maximize object z, add constraint $z - cx \le 0$ and incorporate this constraint in $\tilde{A}x \le b$ (Bertsimas and Sim, 2002).

Bertsimas and Sim (2003, 2004, and 2006) and Bertsimas *et al*. (2004) proposed a method for robust linear optimization with complete control over the conservative degree. They used a particular row i of the matrix $\tilde{A}$ and assumed $\tilde{a}_{ij} \in [a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}], j \in J_i$ as a limited and symmetric independent random variable (but with unknown distribution) that $J_i = \{j | \hat{a}_{ij} > 0\}, i = 1, \ldots, m$ and defined the standard deviation of nominal value a_{ij} as $\eta_{ij} = \frac{(\tilde{a}_{ij} - a_{ij})}{\hat{a}_{ij}}$. Where $\tilde{a}_{ij}$ and a_{ij} are uncertain and nominal values, respectively, and $\hat{a}_{ij}$ represents the approximation's correctness. Clearly, η_{ij} has an uneven but unknown distribution with values in the range $[-1,1]$. The total scaled deviation of the ith constraint, on the other hand, can be any amount in the interval $[-n, n]$, but is restricted to $\sum_{j=1}^{n} \eta_{ij} = \Gamma_i$.

The job of the constraint parameter Γ_i is to adjust the suggested method's stability to the conservative level of the problem, which does not have to be an integer for any constraint i and takes a value in the interval $[0, |J_i|]$. The cost of the constraint's uncertainty is denoted by Γ_i, where:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j = \sum_{j=1}^{n} (a_{ij} + \eta_{ij} \hat{a}_{ij}) x_j = \sum_{j=1}^{n} a_{ij} x_j + \sum_{j \in J_i} \hat{a}_{ij} \eta_{ij} x_j \,, \quad i = 1, \ldots, m \tag{10}$$

Now, model (10) is formulated as follows

$$\begin{aligned} & min \;\; cx \\ & \text{s.t.} \\ & A_i x + max_{\eta_{ij} \in [-1,1]} \sum_{j \in J_i} \hat{a}_{ij} x_j \eta_{ij} \le b_i, \;\; i = 1, \ldots, m \\ & l \le x \le u \end{aligned}$$

Then, the robust counterpart model for model (10) is given as follows (Bertsimas and Sim, 2004).

$$\begin{aligned} & min \;\; cx \\ & \text{s.t.} \quad \mathrm{A}_i x + \Gamma_i p_i + \sum_{j \in J_i} q_{ij} \le b_i \qquad i = 1, \ldots, m \\ & p_i + q_{ij} \ge \hat{a}_{ij_j} |x_j| \qquad j \in J_i, i = 1, \ldots, m \\ & p_i, q_{ij} \ge 0, \qquad j \in J_i, i = 1, \ldots, m \\ & x \in X. \end{aligned} \tag{11}$$

2.2.1 DEA Counterpart Models Based on Bertsimas and Sim Method

Consider an input-oriented DEA model with uncertain output, $\tilde{y}_{rj} = y_{rj} + \eta_{rj}^{y} \hat{y}_{rj}, \hat{y}_{rj} \ge 0, \eta_{rj}^{y} \in [-1,1]$, such as follow:

$$\begin{aligned} & E_k = max \;\; w \\ & \text{s.t.} \;\; w - \sum_{r=1}^{s} u_r \tilde{y}_{rk} \le 0 \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \ldots, n \\ & \sum_{i=1}^{m} v_i x_{ik} = 1 \\ & v_r, u_i \ge 0 \quad i = 1, \ldots, m \quad \mathrm{r} = 1, \ldots, s \end{aligned}$$

According to Bertsimas *et al*. method, we must have

$$E_k = max \;\; w$$

$$\begin{aligned}
&\text{s.t.} \quad w - \left(\sum_{r=1}^{s} u_r y_{rk} + min \sum_{r=1}^{s} u_r \eta_{rk}^{y} \hat{y}_{rk}\right) \leq 0 \\
&\left(\sum_{r=1}^{s} u_r y_{rj} + max \sum_{r=1}^{s} u_r \eta_{rj}^{y} \hat{y}_{rj}\right) - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \; j = 1, \dots, n \\
&\sum_{i=1}^{m} v_i x_{ik} = 1 \\
&v_r, u_i \geq 0 \qquad i = 1, \dots, m, \qquad \mathrm{r} = 1, \dots, s
\end{aligned}$$

In the first constraint, $min \sum_{r=1}^{s} u_r \eta_{rk}^{y} \hat{y}_{rk}$is equal to $-max \sum_{r=1}^{s} u_r \left(-\eta_{rk}^{y}\right) \hat{y}_{rk}$. We define $\xi_{rk}^{y} = -\eta_{rk}^{y}, \xi_{rk}^{y} \in [-1,1]$ and so it will be equal to $-max \sum_{r=1}^{s} u_r \xi_{rk}^{y} \hat{y}_{rk}$, $\xi_{rk}^{y} \in [-1,1]$. So, we'll have the following relationship.

$$min \sum_{r=1}^{s} u_r \eta_{rk}^{y} \hat{y}_{rk} = -max \sum_{r=1}^{s} u_r \eta_{rk}^{y} \hat{y}_{rk} \, , \eta_{rk}^{y} \in [-1,1].$$

Also $u_r \geq 0$ implies $|u_r| = u_r$. As maintained above, the Bertsimas *et al*. The robust DEA (BRDEA) model based on uncertainty in outputs is as follows:

$$\begin{aligned}
&E_k = max \;\; w \\
&\text{s.t.} \quad \mathrm{w} - \left(\sum_{r=1}^{s} u_r y_{rk} - p_k \Gamma_k - \sum_{r-1}^{s} q_{rk}\right) \leq 0 \\
&\sum_{r=1}^{s} u_r y_{rj} + p_j \Gamma_j + \sum_{r-1}^{s} q_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n \\
&\sum_{i=1}^{m} v_i x_{ik} = 1 \\
&p_j + q_{rj} \geq \hat{y}_{rj} u_r, \qquad r = 1, \dots, s \qquad j = 1, \dots, n \\
&q_{rj}, v_r, p_j, u_i \geq 0 \quad i = 1, \dots, m \;\; r = 1, \dots, s \;\; j = 1, \dots, n.
\end{aligned} \tag{12}$$

Bertsimas *et al*. (2004) developed a counterpart model based on linear programming difficulties. In other words, by employing this strategy, the primary problem classification can be preserved. For example, the robust counterpart of a linear programming problem, like DEA, is a linear programming problem, such as model (12).

3. BERTSIMAS ROBUST NETWORK DEA (BRNDEA) MODELS

In this section, using the network DEA models presented in section 2.1 and the robustness technique presented in section 2.2, we will build our robust network DEA models.

First, assume that the external input data, x_{ij}^{t}, are definite values, but the values of intermediate products, z_{dj}^{t}, and output products,y_{rj}^{t}, are subject to uncertain data, that's mean

$$\begin{aligned}
&\tilde{y}_{rj}^{t} = y_{rj}^{t} + \eta_{rj}^{y^t} \hat{y}_{rj}^{t} \;\; , \quad \hat{y}_{dj}^{t} \geq 0, \eta_{rj}^{y^t} \in [-1,1] \\
&\tilde{z}_{dj}^{t} = z_{dj}^{t} + \eta_{dj}^{z^t} \hat{z}_{dj}^{t} \;\; , \quad \hat{z}_{dj}^{t} \geq 0, \eta_{dj}^{z^t} \in [-1,1].
\end{aligned} \tag{*}$$

We remember that $\hat{y}_{rj}^{t}$ and $\hat{z}_{dj}^{t}$ are devotions from nominal data and so $\hat{y}_{rj}^{t} < y_{rj}^{t}$ and $\hat{z}_{dj}^{t} < z_{dj}^{t}$. Then $\tilde{y}_{rj}^{t} \geq 0$ and $\tilde{z}_{dj}^{t} \geq 0, t = 1, \dots, h$.

3.1 Multi-Stage Robust Model based on Kao and Hwang Model (model (2))

In model (2), we use $max \;\; w$and add constraint $w - \sum_{t=1}^{h} \sum_{r=1}^{S} u_r^t y_{rk}^t \leq 0$. By substituting uncertain data in (*), the modal will be as follows

$$\begin{aligned}
&E_k = max \, w \\
&s.t. \;\; \sum_{t=1}^{h} \sum_{i=1}^{M} v_i^t x_{ik}^t = 1, \\
&\mathrm{w}\text{-} \sum_{t=1}^{h} \sum_{r=1}^{S} \left(u_r^t y_{rk}^t + u_r^t \eta_{rk}^{y^t} \hat{y}_{rk}^t\right) \leq 0 \\
&\sum_{d=1}^{D} \left(w_d^1 z_{dj}^1 + w_d^1 \eta_{dj}^{z^1} \hat{z}_{dj}^1\right) + \sum_{r=1}^{S} \left(u_r^1 y_{rj}^1 + u_r^1 \eta_{rj}^{y^1} \hat{y}_{rj}^1\right) - \sum_{i=1}^{M} v_i^1 x_{ij}^1 \leq 0, \; j = 1, \dots, n \\
&\sum_{d=1}^{D} \left(w_d^t z_{dj}^t + w_d^t \eta_{dj}^{z^1} \hat{z}_{dj}^t\right) + \sum_{r=1}^{S} \left(u_r^t y_{rj}^t + u_r^t \eta_{rj}^{y^t} \hat{y}_{rj}^t\right) - \sum_{d=1}^{D} \left(w_d^{t-1} z_{dj}^{t-1} + w_d^{t-1} \eta_{dj}^{z^{1-1}} \hat{z}_{dj}^{t-1}\right) \\
&- \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, \quad j = 1, \dots, n, t = 2, \dots, h-1 \\
&\sum_{r=1}^{S} \left(u_r^h y_{rj}^h + u_r^h \eta_{rj}^{y^h} \hat{y}_{rj}^h\right) - \sum_{d=1}^{D} \left(w_d^{h-1} z_{dj}^{h-1} + w_d^{h-1} \eta_{dj}^{z^{h-1}} \hat{z}_{dj}^{h-1}\right) - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, j = 1, \dots, n
\end{aligned}$$

$$u_r^t \geq 0, v_i^t \geq 0, w_d^t \geq 0, \qquad r = 1,\dots,S, \quad i = 1,\dots,M,$$
$$\eta_{rk}^{y^t}, \eta_{dj}^{z^t} \in [-1,1] \qquad d = 1,\dots,D, \quad t = 1,\dots,h.$$

The above uncertain model will be replaced by the following model as a result of Bertsimas *et al*.' appointment for robustness.

$$\begin{aligned}
&E_k = max\, w \\
&\text{s.t.} \quad \textstyle\sum_{t=1}^{h}\sum_{i=1}^{M} v_i^t x_{ik}^t = 1, \\
&\mathrm{w} - \left(\textstyle\sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t y_{rk}^t + \min\limits_{\eta_{rk}^{y^t},\in[-1,1]} \sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t \eta_{rk}^{y^t} \hat{y}_{rk}^t\right) \leq 0, \\
&\left(\sum_{d=1}^{D} w_d^1 z_{dj}^1 + \max_{\eta_{dj}^{z^1}\in[-1,1]} \sum_{d=1}^{D} w_d^1 \eta_{dj}^{z^1} \hat{z}_{dj}^1\right) + \left(\sum_{r=1}^{S} u_r^1 y_{rj}^1 + \min_{\eta_{rk}^{y^1}\in[-1,1]} \sum_{r=1}^{S} u_r^1 \eta_{rj}^{y^1} \hat{y}_{rj}^1\right) - \sum_{i=1}^{m} v_i^t x_{ij}^t \leq 0, j = 1,\dots,n \\
&\left(\textstyle\sum_{d=1}^{D} w_d^t z_{dj}^t + \max\limits_{\eta_{dj}^{z^t}\in[-1,1]} \sum_{d=1}^{D} w_d^t \eta_{dj}^{z^t} \hat{z}_{dj}^t\right) + \left(\sum_{r=1}^{S} u_r^t y_{rj}^t + \min\limits_{\eta_{rk}^{y^t}\in[-1,1]} \sum_{r=1}^{S} u_r^t \eta_{rj}^{y^t} \hat{y}_{rj}^t\right) \\
&- \left(\textstyle\sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} + \min\limits_{\eta_{dj}^{z^{t-1}}\in[-1,1]} \sum_{d=1}^{D} w_d^{t-1} \eta_{dj}^{z^t} \hat{z}_{dj}^{t-1}\right) - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0,\ t = 2,\dots,h-1,\ \ j = 1,\dots,n \\
&\left(\sum_{r=1}^{S} u_r^h y_{rj}^h + \max_{\eta_{rk}^{y^h}\in[-1,1]} \sum_{r=1}^{S} u_r^h \eta_{rj}^{y^h} \hat{y}_{rj}^h\right) - \left(\sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} + \min_{\eta_{dj}^{zh-1}\in[-1,1]} \sum_{d=1}^{D} w_d^{h-1} \eta_{dj}^{z^{h-1}} \hat{z}_{dj}^{h-1}\right) - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \\
&j = 1,\dots,n \\
&u_r^t, v_i^t, w_d^t \geq 0, \quad \mathrm{r} = 1,\dots,s, \quad i = 1,\dots,m, \quad d = 1,\dots,D, \quad t = 1,\dots,h, \\
&\eta_{rk}^{y^t}, \eta_{dj}^{z^1} \in [-1,1]
\end{aligned} \tag{13}$$

Now, we must find a robust equivalent for each constraint according to Bertsimas *et al*.'s approach. In the third constraint, statement $\min\limits_{\eta_{rk}^{y^t},\in[-1,1]} \sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t \eta_{rk}^{y^t} \hat{y}_{rk}^t$is equal to $-\max\limits_{\eta_{rk}^{y^t},\in[-1,1]} \sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t \eta_{rk}^{y^t} \hat{y}_{rk}^t$. According Bertsimas *et al*. method, because $u_r^t \geq 0, y_{rk}^t \geq 0$, then we have

$$\begin{aligned}
&max \sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t \eta_{rk}^{y^t} \hat{y}_{yk}^t \\
&s.t. \ \ 0 \leq \eta_{rk}^{y^t} \leq 1, \quad r = 1,\dots,s,\ \ t = 1,\dots,h, \\
&\sum_{r=1}^{S} \eta_{rk}^{y^t} = \Gamma_k^t,\ \ t = 1,\dots,h
\end{aligned}$$

The dual of the above model is expressed as follows

$$\begin{aligned}
&min \quad \textstyle\sum_{t=1}^{h} p_k^{y^t} \Gamma_k^{y^t} + \sum_{t=1}^{h}\sum_{r=1}^{S} q_{rk}^{y^t} \\
&s.t. \quad p_k^{y^t} + \textstyle\sum_{r=1}^{s} q_{rk}^{y^t} \geq \hat{y}_{rk}^t u_r^t, r = 1,\dots,s,\ \ t = 1,\dots,h, \\
&q_{rk}^{y^t} \geq 0, \qquad r = 1,\dots,s,\ \ t = 1,\dots,h, \\
&p_k^{y^t} \geq 0, \qquad t = 1,\dots,h.
\end{aligned}$$

Then, it's robust counterpart constraint will be as follow

$$\begin{aligned}
&w - \left(\textstyle\sum_{t=1}^{h}\sum_{r=1}^{S} u_r^t y_{rk}^t - \sum_{t=1}^{h} p_k^{y^t} \Gamma_k^{y^t} - \sum_{t=1}^{h}\sum_{r=1}^{S} q_{rk}^{y^t}\right) \leq 0 \\
&s.t. \quad p_k^{y^t} + \textstyle\sum_{r=1}^{s} q_{rk}^{y^t} \geq \hat{y}_{rk}^t u_r^t, r = 1,\dots,s,\ \ t = 1,\dots,h, \\
&q_{rk}^{y^t} \geq 0, \qquad r = 1,\dots,s,\ \ t = 1,\dots,h,
\end{aligned}$$

$p_k^{y^t} \geq 0, \qquad t = 1, \dots, h.$

Also, in the same way, for each $t = 1, \dots, h$, the robust counterpart of $\max\limits_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{d=1}^{D} w_d^t \eta_{dj}^{z^t} \hat{z}_{dj}^t$ is equal to

$$\begin{aligned}
&min \quad p_j^{z^t} \Gamma_j^{z^t} + \textstyle\sum_{d=1}^{D} q_{dj}^{z^t} \\
&s.t. \quad p_j^{z^t} + q_{rj}^{z^t} \geq \hat{z}_{dj}^t w_d^t, \;\; d = 1, \dots, D \\
&p_j^{z^t}, q_{dj}^{z^t} \geq 0, \qquad d = 1, \dots, D
\end{aligned}$$

and so, the robust counterpart model for $\max\limits_{\eta_{rk}^{y^t} \in [-1,1]} \sum_{r=1}^{s} u_r^t \eta_{rj}^{y^t} \hat{y}_{rj}^t$ is

$$\begin{aligned}
&min \quad p_j^{y^t} \Gamma_j^{y^t} + \textstyle\sum_{r=1}^{s} q_{dj}^{y^t} \\
&s.t. \quad p_j^{y^t} + q_{rj}^{y^t} \geq \hat{y}_{rj}^t u_r^t, \;\; \mathrm{r} = 1, \dots, s \\
&p_j^{y^t}, q_{rj}^{y^t} \geq 0, \qquad \mathrm{r} = 1, \dots, s
\end{aligned}$$

Therefore, a counterpart robust multi-stage model based on model (2) in the Bertsimas *et al*. method will be as follows.

$$\begin{aligned}
&E_k = max\, w \\
&\text{s.t.} \\
&\textstyle\sum_{t=1}^{h} \sum_{i=1}^{M} v_i^t x_{ik}^t = 1, \\
&w - \left(\textstyle\sum_{t=1}^{h} \sum_{r=1}^{S} u_r^t y_{rk}^t - \sum_{t=1}^{h} p_k^{y^t} \Gamma_k^{y^t} - \sum_{t=1}^{h} \sum_{r=1}^{S} q_{rk}^{y^t} \right) \leq 0, \\
&\left(\textstyle\sum_{d=1}^{D} w_d^1 z_{dj}^1 + p_j^{z^1} \Gamma_j^{z^1} + \sum_{d=1}^{D} q_{dj}^{z^1} \right) + \left(\sum_{r=1}^{S} u_r^1 y_{rj}^1 + p_j^{y^1} \Gamma_j^{y^1} + \sum q_{rj}^{y^1} \right) - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \dots, n \\
&\left(\textstyle\sum_{d=1}^{D} w_d^t z_{dj}^t + p_j^{z^t} \Gamma_j^{z^t} + \sum_{d=1}^{D} q_{dj}^{z^t} \right) + \left(\sum_{r=1}^{S} u_r^t y_{rj}^t + p_j^{y^t} \Gamma_j^{y^t} + \sum q_{rj}^{y^t} \right) \\
&- \left(\textstyle\sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - p_j^{z^{t-1}} \Gamma_j^{z^{t-1}} - \sum_{d=1}^{D} q_{dj}^{z^{t-1}} \right) - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, t = 2, \dots, h-1 \quad j = 1, \dots, n \\
&\left(\textstyle\sum_{r=1}^{s} u_r^h y_{rj}^h + p_j^{y^h} \Gamma_j^{y^h} + \sum q_{rj}^{y^h} \right) - \left(\sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - p_j^{z^{h-1}} \Gamma_j^{z^{h-1}} - \sum_{d=1}^{D} q_{dj}^{z^{h-1}} \right) - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \\
&j = 1, \dots, n \\
&p_j^{y^t} + q_{rj}^{y^t} \geq \hat{y}_{rj}^t u_r^t, \qquad j = 1, \dots, n, \quad r = 1, \dots, s, \quad t = 1, \dots, h \\
&p_j^{z^t} + q_{rj}^{z^t} \geq \hat{z}_{dj}^t w_d^t, \qquad j = 1, \dots, n, \quad d = 1, \dots, D, \quad t = 1, \dots, h-1 \\
&p_j^{y^t}, q_{rj}^{y^t} \geq 0, \qquad j = 1, \dots, n, \quad r = 1, \dots, s, \quad t = 1, \dots, h \\
&p_j^{z^t}, q_{dj}^{z^t} \geq 0, \qquad j = 1, \dots, n, \quad d = 1, \dots, D, \quad t = 1, \dots, h-1 \\
&v_i^t, u_r^t, w_d^t \geq 0, \quad i = 1, \dots, m, \quad r = 1, \dots, s, \quad d = 1, \dots, D, \quad t = 1, \dots, h.
\end{aligned} \tag{14}$$

Furthermore, the robust counterpart models for calculating the performance evaluation of each stage *t* for each DMU are generated as follows based on Bertsimas *et al*. achievement and Kao and Hwang's network DEA model and the aforementioned technique.

$$\begin{aligned}
&E_k^1 = max\, w \\
&s.t. \quad \textstyle\sum_{i=1}^{M} v_i^1 x_{ik}^1 = 1, \\
&w - \textstyle\sum_{d=1}^{D} w_d^1 z_{dk}^1 + p_k^{z^1} \Gamma_k^{z^1} + \sum_{d=1}^{D} q_{dk}^{z^1} - \sum_{r=1}^{S} u_r^1 y_{rk}^1 + p_k^{y^1} \Gamma_k^{y^1} + \sum_{r=1}^{S} q_{rk}^{y^1} \leq 0 \\
&\left(\textstyle\sum_{d=1}^{D} w_d^1 z_{dj}^1 + p_j^{z^1} \Gamma_j^{z^1} + \sum_{d=1}^{D} q_{dj}^{z^1} \right) + \left(\sum_{r=1}^{S} u_r^1 y_{rj}^1 + p_j^{y^1} \Gamma_j^{y^1} + \sum q_{rj}^{y^1} \right) - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n \\
&\left(\textstyle\sum_{d=1}^{D} w_d^t z_{dj}^t + p_j^{z^t} \Gamma_j^{z^t} + \sum_{d=1}^{D} q_{dj}^{z^t} \right) + \left(\sum_{r=1}^{S} u_r^t y_{rj}^t + p_j^{y^t} \Gamma_j^{y^t} + \sum q_{rj}^{y^t} \right) - \left(\sum_{d=1}^{D} w_d^{t-1} z_{dj}^{t-1} - p_j^{z^{t-1}} \Gamma_j^{z^{t-1}} - \right. \\
&\left. \textstyle\sum_{d=1}^{D} q_{dj}^{z^{t-1}} \right) - \sum_{i=1}^{M} v_i^t x_{ij}^t \leq 0, t = 2, \dots, h-1 \quad j = 1, \dots, n \\
&\left(\textstyle\sum_{r=1}^{s} u_r^h y_{rj}^h + p_j^{y^h} \Gamma_j^{y^h} + \sum q_{rj}^{y^h} \right) - \left(\sum_{d=1}^{D} w_d^{h-1} z_{dj}^{h-1} - p_j^{z^{h-1}} \Gamma_j^{z^{h-1}} - \sum_{d=1}^{D} q_{dj}^{z^{h-1}} \right) - \sum_{i=1}^{M} v_i^h x_{ij}^h \leq 0, \; j = \\
&1, \dots, n
\end{aligned} \tag{15}$$

$$p_j^{y^t}+q_{rj}^{y^t}\ge \hat{y}_{rj}^t u_r^t,\qquad j=1,\dots,n,\quad r=1,\dots,s,\quad t=1,\dots,h$$
$$p_j^{z^t}+q_{rj}^{z^t}\ge \hat{z}_{dj}^t w_d^t,\qquad j=1,\dots,n,\quad d=1,\dots,D,\quad t=1,\dots,h-1$$
$$p_j^{y^t},q_{rj}^{y^t}\ge 0,\qquad j=1,\dots,n,\quad r=1,\dots,s,\quad t=1,\dots,h$$
$$p_j^{z^t},q_{dj}^{z^t}\ge 0,\qquad j=1,\dots,n,\quad d=1,\dots,D,\quad t=1,\dots,h-1$$
$$v_i^t,u_r^t,w_d^t\ge 0,\quad i=1,\dots,m,\quad r=1,\dots,s,\quad d=1,\dots,D,\quad t=1,\dots,h.$$

and for stage T, $T=2,\dots,h-1$

$$\begin{aligned}
&E_k^T=\max w\\
&s.t.\quad \textstyle\sum_{d=1}^{D} w_d^{T-1}z_{dk}^{T-1}+\sum_{i=1}^{M} v_i^T x_{ik}^T=1\\
&w-\textstyle\sum_{d=1}^{D} w_d^T z_{dk}^T+p_k^{z^T}\Gamma_k^{z^t}+\sum_{d=1}^{D} q_{dk}^{z^T}-\sum_{r=1}^{S}u_r^T y_{rk}^T+p_k^{y^T}\Gamma_k^{y^T}+\sum_{r=1}^{S} q_{rk}^{y^T}\le 0,\\
&\left(\textstyle\sum_{d=1}^{D} w_d^1 z_{dj}^1+p_j^{z^1}\Gamma_j^{z^1}+\sum_{d=1}^{D} q_{dj}^{z^1}\right)+\left(\sum_{r=1}^{S}u_r^1 y_{rj}^1+p_j^{y^1}\Gamma_j^{y^1}+\sum q_{rj}^{y^1}\right)-\sum_{i=1}^{m} v_i x_{ij}\le 0, j=1,\dots,n\\
&\left(\textstyle\sum_{d=1}^{D} w_d^t z_{dj}^t+p_j^{z^t}\Gamma_j^{z^t}+\sum_{d=1}^{D} q_{dj}^{z^t}\right)+\left(\sum_{r=1}^{S}u_r^t y_{rj}^t+p_j^{y^t}\Gamma_j^{y^t}+\sum q_{rj}^{y^t}\right)-\left(\sum_{d=1}^{D} w_d^{t-1}z_{dj}^{t-1}-p_j^{z^{t-1}}\Gamma_j^{z^{t-1}}-\right.\\
&\left.\textstyle\sum_{d=1}^{D} q_{dj}^{z^{t-1}}\right)\\
&-\textstyle\sum_{i=1}^{M} v_i^t x_{ij}^t\le 0,\ j=1,\dots,n,\ t=2,\dots,h-1, t\neq T-1,T\\
&\left(\textstyle\sum_{d=1}^{D} w_d^T z_{dj}^T+p_j^{z^T}\Gamma_j^{z^T}+\sum_{d=1}^{D} q_{dj}^{z^T}\right)+\left(\sum_{r=1}^{S}u_r^T y_{rj}^T+p_j^{y^T}\Gamma_j^{y^T}+\sum q_{rj}^{y^T}\right)-\sum_{d=1}^{D} w_d^{T-1}z_{dj}^{T-1}\\
&\textstyle\sum_{i=1}^{M} v_i^{T-1}x_{ij}^{T-1}\le 0,\quad j=1,\dots,n\\
&\textstyle\sum_{d=1}^{D} w_d^{T-1}z_{dj}^{T-1}+\left(\sum_{r=1}^{S}u_r^{T-1}y_{rj}^{T-1}+p_j^{y^{T-1}}\Gamma_j^{y^{T-1}}+\sum q_{rj}^{y^{T-1}}\right)-\left(\sum_{d=1}^{D} w_d^{T-2}z_{dj}^{T-2}+p_j^{z^{T-2}}\Gamma_j^{z^{T-2}}+\right.\\
&\left.\textstyle\sum_{d=1}^{D} q_{dj}^{z^{T-2}}\right)-\sum_{i=1}^{M} v_i^{T-2}x_{ij}^{T-2}\le 0,\quad j=1,\dots,n\\
&\left(\textstyle\sum_{r=1}^{s}u_r^h y_{rj}^h+p_j^{y^h}\Gamma_j^{y^h}+\sum q_{rj}^{y^h}\right)-\left(\sum_{d=1}^{D} w_d^{h-1}z_{dj}^{h-1}-p_j^{z^{h-1}}\Gamma_j^{z^{h-1}}-\sum_{d=1}^{D} q_{dj}^{z^{h-1}}\right)-\sum_{i=1}^{M} v_i^h x_{ij}^h\le 0,\ j=\\
&1,\dots,n\\
&p_j^{y^t}+q_{rj}^{y^t}\ge \hat{y}_{rj}^t u_r^t,\qquad j=1,\dots,n,\quad r=1,\dots,s,\quad t=1,\dots,h\\
&p_j^{z^t}+q_{rj}^{z^t}\ge \hat{z}_{dj}^t w_d^t,\qquad j=1,\dots,n,\quad d=1,\dots,D,\quad t=1,\dots,h-1\\
&p_j^{y^t},q_{rj}^{y^t}\ge 0,\qquad j=1,\dots,n,\quad r=1,\dots,s,\quad t=1,\dots,h\\
&p_j^{z^t},q_{dj}^{z^t}\ge 0,\qquad j=1,\dots,n,\quad d=1,\dots,D,\quad t=1,\dots,h-1\\
&v_i^t,u_r^t,w_d^t\ge 0,\quad i=1,\dots,m,\quad r=1,\dots,s,\quad d=1,\dots,D,\quad t=1,\dots,h.
\end{aligned}\tag{16}$$

$$\begin{aligned}
&E_k^h=\max w\\
&s.t.
\end{aligned}\tag{17}$$

$$\begin{aligned}
&\textstyle\sum_{d=1}^{D} w_d^{h-1}z_{dk}^{h-1}+\sum_{i=1}^{M} v_i^h x_{ik}^h=1,\\
&w-\textstyle\sum_{r=1}^{S}u_r^h y_{rk}^h+p_k^{y^h}\Gamma_k^{y^h}+\sum_{r=1}^{S} q_{rk}^{y^h}\le 0,\\
&\left(\textstyle\sum_{d=1}^{D} w_d^1 z_{dj}^1+p_j^{z^1}\Gamma_j^{z^1}+\sum_{d=1}^{D} q_{dj}^{z^1}\right)+\left(\sum_{r=1}^{S}u_r^1 y_{rj}^1+p_j^{y^1}\Gamma_j^{y^1}+\sum q_{rj}^{y^1}\right)-\sum_{i=1}^{m} v_i x_{ij}\le 0, j=1,\dots,n\\
&\left(\textstyle\sum_{d=1}^{D} w_d^t z_{dj}^t+p_j^{z^t}\Gamma_j^{z^t}+\sum_{d=1}^{D} q_{dj}^{z^t}\right)+\left(\sum_{r=1}^{S}u_r^t y_{rj}^t+p_j^{y^t}\Gamma_j^{y^t}+\sum q_{rj}^{y^t}\right)\qquad-\left(\sum_{d=1}^{D} w_d^{t-1}z_{dj}^{t-1}-\right.\\
&\left.p_j^{z^{t-1}}\Gamma_j^{z^{t-1}}-\textstyle\sum_{d=1}^{D} q_{dj}^{z^{t-1}}\right)-\sum_{i=1}^{M} v_i^t x_{ij}^t\le 0, t=2,\dots,h-2,\quad j=1,\dots,n\\
&\left(\textstyle\sum_{d=1}^{D} w_d^{h-1}z_{dj}^{h-1}\right)+\left(\sum_{r=1}^{S}u_r^t y_{rj}^t+p_j^{y^t}\Gamma_j^{y^t}+\sum q_{rj}^{y^t}\right)-\left(\sum_{d=1}^{D} w_d^{h-2}z_{dj}^{h-2}-p_j^{z^{h-2}}\Gamma_j^{z^{h-2}}-\sum_{d=1}^{D} q_{dj}^{z^{h-2}}\right)-\\
&\textstyle\sum_{i=1}^{M} v_i^{h-1}x_{ij}^{h-1}\le 0,\qquad j=1,\dots,n\\
&\left(\textstyle\sum_{r=1}^{s}u_r^h y_{rj}^h+p_j^{y^h}\Gamma_j^{y^h}+\sum q_{rj}^{y^h}\right)-\sum_{d=1}^{D} w_d^{h-1}z_{dj}^{h-1}-\sum_{i=1}^{M} v_i^h x_{ij}^h\le 0,\ j=1,\dots,n
\end{aligned}$$

$$p_j^{y^t}+q_{rj}^{y^t}\ge \hat{y}_{rj}^t u_r^t,\qquad j=1,\dots,n,\quad r=1,\dots,s,\quad t=1,\dots,h$$
$$p_j^{z^t}+q_{rj}^{z^t}\ge \hat{z}_{dj}^t w_d^t,\qquad j=1,\dots,n,\quad d=1,\dots,D,\quad t=1,\dots,h-1$$

$p_j^{y^t}, q_{rj}^{y^t} \geq 0, \quad j = 1, \ldots, n, \quad r = 1, \ldots, s, \quad t = 1, \ldots, h$
$p_j^{z^t}, q_{dj}^{z^t} \geq 0, \quad j = 1, \ldots, n, \quad d = 1, \ldots, D, \quad t = 1, \ldots, h-1$
$v_i^t, u_r^t, w_d^t \geq 0, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s, \quad d = 1, \ldots, D, \quad t = 1, \ldots, h.$

3.2 Multi-Stage Robust Model Based on Dual of Kao and Hwang Model (model (6))

Applying the assumption of uncertainty (*) for intermediate and ultimate products, the model (6) is changed as follows.

$$
\begin{aligned}
& E_k = \min \theta \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq \theta x_{ik}^t, \quad i = 1, \ldots, m, \quad t = 1, \ldots, h \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) \left(z_{dj}^t + \eta_{dj}^{z^t} \hat{z}_{dj}^t\right) \geq 0, \quad \mathrm{d} = 1, \ldots, D, \quad t = 2, \ldots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^t \left(y_{rj} + \eta_{rj}^{y} \hat{y}_{rj}\right) \geq \left(y_{rk} + \eta_{rk}^{y} \hat{y}_{rk}\right), \quad \mathrm{r} = 1, \ldots, s, \quad t = 1, \ldots, h \\
& \lambda_j^t, \bar{z}_{dk}^t \geq 0, \quad j = 1, \ldots, n, \quad d = 1, \ldots, D, \quad t = 1, \ldots, h-1.
\end{aligned}
$$

To obtain a robust solution, we must have:

$$
\begin{aligned}
& E_k = \min \theta \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij} \leq \theta x_{ik}, \quad i = 1, \ldots, m, \quad t = 1, \ldots, h \\
& \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) z_{dj}^t + \min_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \left(\lambda_j^t - \lambda_j^{t+1}\right) \eta_{dj}^{z^t} \hat{z}_{dj}^t \geq 0, \quad \mathrm{d} = 1, \ldots, D, \\
& t = 2, \ldots, h-1 \\
& \left(\sum_{j=1}^{n} \lambda_j^t y_{rj} + \min_{\eta_{rj}^{y} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{rj}^{y} \hat{y}_{rj}\right) \geq y_{rk} + \max_{\eta_{rk}^{y} \in [-1,1]} \eta_{rk}^{y} \hat{y}_{rk}, \quad r = 1, \ldots, s, \\
& t = 1, \ldots, h \\
& \lambda_j^t \geq 0, \quad j = 1, \ldots, n, \quad t = 1, \ldots, h.
\end{aligned}
$$

Or

$$
\begin{aligned}
& E_k = \min \theta \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq \theta x_{ik}^t, \quad i = 1, \ldots, m, \quad t = 1, \ldots, h \\
& \sum_{j=1}^{n} \lambda_j^t z_{dj}^t - \sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t + \min_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{dj}^{z^t} \hat{z}_{dj}^t - \max_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^{t+1} \eta_{dj}^{z^t} \hat{z}_{dj}^t \geq 0, \\
& \mathrm{d} = 1, \ldots, D, \quad t = 2, \ldots, h-1 \\
& \left(\sum_{j=1}^{n} \lambda_j^t y_{rj}^t + \min_{\eta_{rj}^{y^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{rj}^{y^t} \hat{y}_{rj}^t\right) \geq y_{rk}^t + \max_{\eta_{rj}^{y^t} \in [-1,1]} \eta_{rk}^{y^t} \hat{y}_{rk}^t, \\
& r = 1, \ldots, s, \quad t = 1, \ldots, h \\
& \lambda_j^t \geq 0, \quad j = 1, \ldots, n, \quad t = 1, \ldots, h.
\end{aligned}
$$

The first constraint won't change because it is dependent on input parameters that are presumed to be deterministic. With regard to the second constraint, for each $t = 1, \ldots, h$, the equation $\min_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{dj}^{z^t} \hat{z}_{dj}^t$ is equivalent to:

$$
\begin{aligned}
& -\min \left(\Gamma_{dj}^{z^t} p_j^{z^t} + \sum_{d=1}^{D} q_{dj}^{z^t}\right) \\
& \text{s.t.} \quad p_j^{z^t} + q_{dj}^{z^t} \geq \lambda_j^t \hat{z}_{dj}^t, \quad \mathrm{d} = 1, \ldots, D \\
& p_d^{z^t}, q_{dj}^{z^t} \geq 0, \quad \mathrm{d} = 1, \ldots, D
\end{aligned}
$$

In a similar vein, the robust counterpart of the expression $\max_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^{t+1} \eta_{dj}^{z^t} \hat{z}_{dj}^t$ is equal to:

$$
\begin{aligned}
& min\left(\Gamma_{dj}^{z^t} p_j^{z^t} + \sum_{d=1}^{D} q_{dj}^{z^t}\right) \\
& \text{s.t.} \quad p_j^{z^t} + q_{dj}^{z^t} \geq \lambda_j^{t+1} \hat{z}_{dj}^{t}, \quad \text{d} = 1, \dots, D \\
& p_d^{z^t}, q_{dj}^{z^t} \geq 0, \qquad \text{d} = 1, \dots, D
\end{aligned}
$$

and $\underset{\eta_{rj}^{y^t} \in [-1,1]}{min} \sum_{j=1}^{n} \lambda_j^t \eta_{rj}^{y^t} \hat{y}_{rj}^t$ is equal to

$$
\begin{aligned}
& -min\left(\Gamma_r^{y^t} p_r^{y^t} + \sum_{j=1}^{n} q_{rj}^{y^t}\right) \\
& s.t. \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \hat{y}_{rj}^t \lambda_j^t, \quad j = 1, \dots, n, \quad r = 1, \dots, s \\
& p_r^{y^t}, q_{rj}^{y^t} \geq 0, \qquad j = 1, \dots, n, \quad r = 1, \dots, s.
\end{aligned}
$$

As one can see, $max\, \eta_{rk}^{y^t} \hat{y}_{rk}^t$ for $\eta_{rk}^{y^t} \in [-1,1]$ equals to $\hat{y}_{rk}^t$. Therefore, the multi-stage robust counterpart DEA model based on model (6) is as follows:

$$
\begin{aligned}
& E_k = min\, \theta \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq \theta x_{ik}, \qquad i = 1, \dots, m \\
& \sum_{j=1}^{n} (\lambda_j^t - \lambda_j^{t+1}) z_{dj}^t - \left(\Gamma_d^{z^t} p_d^{z^t} + \sum_{j=1}^{n} q_{dj}^{z^t}\right) - \left(\Gamma_d^{z^t} p_d^{z^t} + \sum_{j=1}^{n} q_{dj}^{z^t}\right) \geq 0, \; d = 1, \dots, D \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t - \left(\Gamma_r^{y^t} p_r^{y^t} + \sum_{j \in J_r} q_{rj}^{y^t}\right) \geq y_{rk}^t + \hat{y}_{rk}^t, \qquad \text{r} = 1, \dots, s \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \lambda_j^t \hat{y}_{rj}^t \qquad j = 1, \dots, n, \qquad \text{r} = 1, \dots, s, \; t = 1, \dots, h \\
& p_d^{z^t} + q_{dj}^{z^t} \geq \hat{z}_{dj}^t \lambda_j^t, \quad j = 1, \dots, n, \; d = 1, \dots, D, \; t = 1, \dots, h-1 \\
& p_d^{z^t} + q_{dj}^{z^t} \geq \hat{z}_{dj}^t \lambda_j^{t+1}, \; j = 1, \dots, n, \; d = 1, \dots, D, \; t = 1, \dots, h-1 \\
& p_r^{y^t}, q_{rj}^{y^t} \geq 0, \qquad j = 1, \dots, n, \quad \text{r} = 1, \dots, s, \quad t = 1, \dots, h \\
& p_d^{z^t}, q_{dj}^{z^t} \geq 0, \qquad j = 1, \dots, n, \; d = 1, \dots, D, \; t = 1, \dots, h-1 \\
& \lambda_j^t \geq 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h.
\end{aligned} \tag{18}
$$

As previously stated, in order to build the robust counterpart model, we assume that the stage's input values are certain and deterministic. We can calculate robust counterpart models for (7), (8), and (9) models using the same procedure.

$$
\begin{aligned}
& E_k^1 = min\, \theta^1 \\
& \text{s.t.} \quad \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq \theta^1 x_{ik}^1, \qquad i = 1, \dots, m, \\
& \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \leq 0, \qquad i = 1, \dots, m, \; t = 2, \dots, h \\
& \sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 - \sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 + \underset{\eta_{dj}^{z^1} \in [-1,1]}{min} \sum_{j=1}^{n} \lambda_j^1 \eta_{dj}^{z^1} \hat{z}_{dj}^1 - \underset{\eta_{dj}^{z^1} \in [-1,1]}{max} \sum_{j=1}^{n} \lambda_j^2 \eta_{dj}^{z^1} \hat{z}_{dj}^1 \geq z_{dk}^1 + \underset{\eta_{dj}^{z^1} \in [-1,1]}{max} \eta_{dj}^{z^1} \hat{z}_{dj}^1, \\
& \qquad d = 1, \dots, D, \\
& \sum_{j=1}^{n} \lambda_j^t z_{dj}^t - \sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t + \underset{\eta_{dj}^{z^t} \in [-1,1]}{min} \sum_{j=1}^{n} \lambda_j^t \eta_{dj}^{z^t} \hat{z}_{dj}^t - \underset{\eta_{dj}^{z^t} \in [-1,1]}{max} \sum_{j=1}^{n} \lambda_j^{t+1} \eta_{dj}^{z^t} \hat{z}_{dj}^t \geq 0, \quad d = 1, \dots, D, \\
& \qquad t = 2, \dots, h-1 \\
& \sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 + \underset{\eta_{rj}^{y^t} \in [-1,1]}{min} \sum_{j=1}^{n} \lambda_j^1 \eta_{rj}^{y^1} y_{rj}^1 \geq y_{rk}^1 + \underset{\eta_{rj}^{y^1} \in [-1,1]}{max} \eta_{rk}^{y^1} \hat{y}_{rk}^1, \qquad r = 1, \dots, s, \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t + \underset{\eta_{rj}^{y^t} \in [-1,1]}{min} \sum_{j=1}^{n} \lambda_j^t \eta_{rj}^{y^t} y_{rj}^t \geq 0, \qquad r = 1, \dots, s, \quad t = 2, \dots, h \\
& \lambda_j^t \geq 0, \eta_{rj}^{y^t} \geq 0, \eta_{dj}^{z^t} \geq 0 \qquad r = 1, \dots, s, \; d = 1, \dots, D, \; j = 1, \dots, n, \quad t = 1, \dots, h
\end{aligned} \tag{19}
$$

Or

$$E_k^1 = min\, \theta^1$$

$$
\begin{aligned}
&\text{s.t.} \quad \textstyle\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \le \theta^1 x_{ik}^1, \qquad i = 1, \dots, m, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le 0, \qquad i = 1, \dots, m, \quad t = 2, \dots, h \\
&\textstyle\sum_{j=1}^{n} \lambda_j^1 z_{dj}^1 - \sum_{j=1}^{n} \lambda_j^2 z_{dj}^1 - \Gamma_d^{z^1} p_d^{z^1} - \sum_{j=1}^{n} q_{dj}^{z^1} - \Gamma_d^{z^1} p_d^{z^1} - \sum_{j=1}^{n} q_{dj}^{z^1} \ge z_{dk}^1 + \hat{z}_{dk}^1, \\
&d = 1, \dots, D, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t z_{dj}^t - \sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^{n} q_{dj}^{z^t} - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^{n} q_{dj}^{z^t} \ge 0, \\
&d = 1, \dots, D, \ t = 2, \dots, h-1 \\
&\textstyle\sum_{j=1}^{n} \lambda_j^1 y_{rj}^1 - \Gamma_r^{y^1} p_r^{y^1} - \sum_{j=1}^{n} q_{rj}^{y^1} \ge y_{rk}^1 + \hat{y}_{rk}^1, \qquad r = 1, \dots, s, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t y_{rj}^t - \Gamma_r^{y^t} p_r^{y^t} - \sum_{j=1}^{n} q_{rj}^{y^t} \ge 0, \qquad r = 1, \dots, s, \quad t = 2, \dots, h \\
&p_d^{z^t} + q_{dj}^{z^t} \ge \lambda_j^t \hat{z}_{dj}^t, \qquad d = 1, \dots, D, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h-1 \\
&p_d^{z^t} + q_{dj}^{z^t} \ge \lambda_j^{t+1} \hat{z}_{dj}^t, \qquad d = 1, \dots, D, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h-1 \\
&p_r^{y^t} + q_{rj}^{y^t} \ge \lambda_j^t \hat{y}_{rj}^t, \qquad \mathrm{r} = 1, \dots, s, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h \\
&p_r^{y^t} + q_{rj}^{y^t} \ge \lambda_j^{t+1} \hat{y}_{rj}^t, \qquad \mathrm{r} = 1, \dots, s, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h \\
&p_d^{z^t}, q_{dj}^{z^t} \ge 0, \qquad d = 1, \dots, D, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h \\
&p_r^{y^t}, q_{rj}^{y^t} \ge 0, \qquad \mathrm{r} = 1, \dots, s, \quad \mathrm{j} = 1, \dots, n, \quad \mathrm{t} = 1, \dots, h \\
&\lambda_j^t \ge 0, \qquad j = 1, \dots, n, \quad t = 1, \dots, h
\end{aligned}
$$

and for each T, $T = 2, \dots, h-1$, we must have

$$
\begin{aligned}
&E_k^T = \min \theta^T \\
&\text{s.t.} \quad \textstyle\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le \theta^T x_{ik}^T, \quad i = 1, \dots, m, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le 0, \quad i = 1, \dots, m, \quad t = 1, \dots, h, t \ne T, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^{T-1} z_{dj}^{T-1} - \sum_{j=1}^{n} \lambda_j^T z_{dj}^{T-1} + \theta^T z_{dj}^{T-1} \ge 0, \quad d = 1, \dots, D, \\
&\sum_{j=1}^{n} \lambda_j^T z_{dj}^T + \min_{\eta_{dj}^{z^T} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^T \hat{z}_{dj}^T - \sum_{j=1}^{n} \lambda_j^{T+1} z_{dj}^T - \max_{\eta_d^{z^T} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^{T+1} \eta_{dj}^{z^T} \hat{z}_{dj}^T \ge z_{dj}^T + \max_{\eta_{dj}^{z^T} \in [-1,1]} \eta_{dj}^{z^T} \hat{z}_{dj}^T, \qquad d = 1, \dots, D, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t z_{dj}^t + \min\limits_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{dj}^{z^t} \hat{z}_{dj}^t - \sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t - \max\limits_{\eta_{dj}^{z^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^{t+1} \eta_{dj}^{z^t} \hat{z}_{dj}^t \ge 0, \\
&d = 1, \dots, D, \quad t = 1, \dots, h-1, t \ne T-1, T, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^T y_{rj}^T + \min\limits_{\eta_{rj}^{y^T} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^T \eta_{rj}^{y^T} \hat{y}_{rj}^T \ge y_{ik}^T + \max\limits_{\eta_{rj}^{y^T} \in [-1,1]} \eta_{rk}^{y^T} \hat{y}_{rk}^T, \qquad r = 1, \dots, s, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t y_{rj}^t + \min\limits_{\eta_{rj}^{y^t} \in [-1,1]} \sum_{j=1}^{n} \lambda_j^t \eta_{rj}^{y^t} \hat{y}_{rj}^t \ge 0, \qquad r = 1, \dots, s, \quad t = 1, \dots, h, t \ne T, \\
&\lambda_j^t \ge 0, \qquad r = 1, \dots, s, \quad t = 1, \dots, h
\end{aligned}
$$

Or

$$
\begin{aligned}
&E_k^T = \min \theta^T \\
&\text{s.t.} \quad \textstyle\sum_{j=1}^{n} \lambda_j^T x_{ij}^T \le \theta^T x_{ik}^T, \qquad i = 1, \dots, m, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le 0, \qquad i = 1, \dots, m, \quad t = 2, \dots, h, t \ne T, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^{T-1} z_{dj}^{T-1} - \sum_{j=1}^{n} \lambda_j^T z_{dj}^{T-1} + \theta^T z_{dk}^{T-1} \ge 0, \ d = 1, \dots, D, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^T z_{dj}^T - \sum_{j=1}^{n} \lambda_j^{T+1} z_{dj}^T - \Gamma_d^{z^T} p_d^{z^T} - \sum_{j=1}^{n} q_{dj}^{z^T} - \Gamma_d^{z^T} p_d^{z^T} - \sum_{j=1}^{n} q_{dj}^{z^T} \ge z_{dk}^T + \hat{z}_{dk}^T, d = 1, \dots, D, \\
&\textstyle\sum_{j=1}^{n} \lambda_j^t z_{dj}^t - \sum_{j=1}^{n} \lambda_j^{t+1} z_{dj}^t - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^{n} q_{dj}^{z^t} - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^{n} q_{dj}^{z^t} \ge 0, d = 1, \dots, D, \ t = 1, \dots, h-1, \\
&t \ne T-1, T \\
&\textstyle\sum_{j=1}^{n} \lambda_j^T y_{rj}^T - \Gamma_r^{y^T} p_r^{y^T} - \sum_{j=1}^{n} q_{rj}^{y^T} \ge y_{rk}^T + \hat{y}_{rk}^T, \qquad r = 1, \dots, s, \\
&\sum_{j=1}^{n} \lambda_j^t y_{rj}^t - \Gamma_r^{y^t} p_r^{y^t} - \sum_{j=1}^{n} q_{rj}^{y^t} \ge 0, \qquad r = 1, \dots, s, \quad t = 1, \dots, h, t \ne T
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
& p_d^{z^t} + q_{dj}^{z^t} \geq \lambda_j^t \hat{z}_{dj}^t, && d = 1,\ldots,D, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h-1 \\
& p_d^{z^t} + q_{dj}^{z^t} \geq \lambda_j^{t+1} \hat{z}_{dj}^t, && d = 1,\ldots,D, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h-1 \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \lambda_j^t \hat{y}_{rj}^t, && r = 1,\ldots,s, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \lambda_j^{t+1} \hat{y}_{rj}^t, && r = 1,\ldots,s, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h \\
& p_d^{z^t}, q_{dj}^{z^t} \geq 0, && d = 1,\ldots,D, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h \\
& p_r^{y^t}, q_{rj}^{y^t} \geq 0, && r = 1,\ldots,s, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h \\
& \lambda_j^t \geq 0, && j = 1,\ldots,n, \quad t = 1,\ldots,h
\end{aligned}
$$

Also, for the last stage, we have

$$
\begin{aligned}
& E_k^h = min\, \theta^h \\
& \text{s.t.} \quad \textstyle\sum_{j=1}^n \lambda_j^t x_{ij}^t \leq 0, \quad i = 1,\ldots,m, \quad t = 1,\ldots,h-1, \\
& \textstyle\sum_{j=1}^n \lambda_j^h x_{ij}^h \leq \theta^h x_{ik}^h, \quad i = 1,\ldots,m, \\
& \textstyle\sum_{j=1}^n \lambda_j^t z_{dj}^t - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^n q_{dj}^{z^t} - \sum_{j=1}^n \lambda_j^{t+1} z_{dj}^t - \Gamma_d^{z^t} p_d^{z^t} - \sum_{j=1}^n q_{dj}^{z^t} \geq 0, \\
& d = 1,\ldots,D, \quad t = 1,\ldots,h-2, \\
& \textstyle\sum_{j=1}^n \lambda_j^{h-1} z_{dj}^{h-1} - \sum_{j=1}^n \lambda_j^h z_{dj}^{h-1} \geq -\theta^h z_{ik}^{h-1}, \quad d = 1,\ldots,D, \\
& \textstyle\sum_{j=1}^n \lambda_j^t y_{rj}^t - \Gamma_r^{y^t} p_r^{y^t} - \sum_{j=1}^n q_{rj}^{y^t} \geq 0, \quad r = 1,\ldots,s, \quad t = 1,\ldots,h-1, \\
& \textstyle\sum_{j=1}^n \lambda_j^h y_{rj}^h - \Gamma_r^{y^h} p_r^{y^h} - \sum_{j=1}^n q_{rj}^{y^h} \geq y_{rk}^h + \hat{y}_{rk}^h, \quad r = 1,\ldots,s, \\
& p_d^{z^t} + q_{dj}^{z^t} \geq \lambda_j^t \hat{z}_{dj}^t, \quad j = 1,\ldots,n, \quad d = 1,\ldots,D, \quad t = 1,\ldots,h-1 \\
& p_d^{z^t} + q_{dj}^{z^t} \geq \lambda_j^{t+1} \hat{z}_{dj}^t, \quad j = 1,\ldots,n, \quad d = 1,\ldots,D, \quad t = 1,\ldots,h-1 \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \lambda_j^t \hat{y}_{rj}^t, \quad j = 1,\ldots,n, \quad r = 1,\ldots,s, \quad t = 1,\ldots,h-1 \\
& p_r^{y^t} + q_{rj}^{y^t} \geq \lambda_j^{t+1} \hat{y}_{rj}^t, \quad j = 1,\ldots,n, \quad r = 1,\ldots,s, \quad t = 1,\ldots,h-1 \\
& p_d^{z^t}, q_{dj}^{z^t} \geq 0, \quad j = 1,\ldots,n, \quad d = 1,\ldots,D, \quad t = 1,\ldots,h-1 \\
& p_r^{y^t}, q_{rj}^{y^t} \geq 0, \quad j = 1,\ldots,n, \quad r = 1,\ldots,s, \quad t = 1,\ldots,h-1 \\
& \lambda_j^t \geq 0, \quad j = 1,\ldots,n, \quad t = 1,\ldots,h.
\end{aligned}
\tag{21}
$$

4. NUMERICAL EXAMPLE

Two examples are provided in this part to demonstrate the applicability of the produced models. The first is a case study of data from Iran's top 40 dairy supply chains, which Khalili-Damghani and Taghavi-Fard (2012) used.

The proposed robust models were written in LINGO 17.0 software and run on a Pentium V laptop with a Core i7, 2 GHz CPU and Windows 8.1 running on 8 GB of RAM.

4.1 Application of the RNDEA Model in a Dairy Supply Chain: a Three-Stage Example

Food industries with a continuous production process are typically more complex than other sectors due to issues such as product perishability, a large number of completed goods, a wide range of manufacturing routes, specialized storage equipment, common and by-products, and changeable recipes.

Figure 2. Three stage network

Each dairy company is viewed as its own supply chain, with three-stage sub-processes linked in a chain of JIT methods, prospective agility indicators, contingency agility indicators, and performance goals (Khalili-Damghani and Taghavi-Fard, 2012).

Khalili-Damghani and Taghavi-Fard (2012) gathered supply chain managers' opinions about the values of indicators in various sectors of a specific supply chain and, used triangular fuzzy numbers to represent each criterion, and then used a three-stage fuzzy DEA model for performance evaluation of dairy supply chains. (For a full discussion regarding these examples, see Khalili-Damghani and Taghavi-Fard (2012)). They solved the problem and determined the lower and upper bounds of its efficiency score. The findings of the models are displayed in Table 1's LB-TSFDEA and UP-TSFDEA columns, respectively.

We use the conceptual model (Figure 2) and data used in that research and convert the fuzzy data to robust data for run the models suggested in our research. For this, we Consider $x_i = \frac{(x_i^L + x_i^U)}{2}and\hat{x}_i = \frac{(x_i^U - x_i^L)}{2}$.

First, a three-stage DEA is used to calculate the specific total efficiency of DMUs. The efficiency value is determined using model (2) for this purpose and is shown in the second column of Table 1.

Table 1. Total efficiency values of the three-stage model with Bertsimas *et al*. technique

	model (14)			model (2)	model (18)		
	20%	10%	5%		5%	10%	20%
DMU 1	0.300	0.398	0.455	**0.511**	0.747	0.861	0.981
DMU 2	0.163	0.202	0.227	**0.261**	0.496	0.561	0.641
DMU 3	0.123	0.157	0.178	**0.204**	0.349	0.395	0.452
DMU 4	0.180	0.227	0.260	**0.296**	0.528	0.605	0.686
DMU 5	0.329	0.416	0.475	**0.567**	1.093	1.252	1.471
DMU 6	0.264	0.325	0.360	**0.412**	0.639	0.706	0.804
DMU 7	0.169	0.221	0.259	**0.301**	0.482	0.553	0.641
DMU 8	0.335	0.411	0.455	**0.527**	0.906	1.026	1.173
DMU 9	0.139	0.181	0.206	**0.236**	0.386	0.446	0.505
DMU 10	0.241	0.312	0.353	**0.397**	0.594	0.690	0.779
DMU 11	0.174	0.229	0.271	**0.320**	0.761	0.894	1.028
DMU 12	0.234	0.293	0.336	**0.382**	0.536	0.640	0.727
DMU 13	0.302	0.383	0.437	**0.500**	0.791	0.892	1.028
DMU 14	0.294	0.357	0.394	**0.444**	0.668	0.741	0.854
DMU 15	0.306	0.382	0.434	**0.494**	0.750	0.833	0.964
DMU 16	0.242	0.313	0.360	**0.411**	0.686	0.793	0.861
DMU 17	0.240	0.292	0.326	**0.396**	0.615	0.678	0.762
DMU 18	0.261	0.292	0.389	**0.436**	0.640	0.741	0.840
DMU 19	0.301	0.380	0.436	**0.507**	0.837	0.968	1.144
DMU 20	0.177	0.232	0.236	**0.296**	0.543	0.622	0.702
DMU 21	0.268	0.356	0.415	**0.467**	0.670	0.767	0.883
DMU 22	0.184	0.240	0.263	**0.286**	0.559	0.650	0.712
DMU 23	0.224	0.297	0.338	**0.397**	0.625	0.691	0.785
DMU 24	0.226	0.291	0.322	**0.361**	0.575	0.667	0.732
DMU 25	0.381	0.482	0.545	**0.638**	1.056	1.218	1.406
DMU 26	0.238	0.309	0.356	**0.405**	0.596	0.674	0.760
DMU 27	0.269	0.370	0.432	**0.496**	0.758	0.893	1.032
DMU 28	0.301	0.390	0.449	**0.523**	0.785	0.883	0.990
DMU 29	0.326	0.415	0.467	**0.531**	0.931	1.053	1.223
DMU 30	0.172	0.220	0.246	**0.277**	0.430	0.496	0.565
DMU 31	0.221	0.282	0.324	**0.374**	0.643	0.750	0.834
DMU 32	0.327	0.324	0.376	**0.420**	0.606	0.670	0.785
DMU 33	0.299	0.363	0.408	**0.456**	0.687	0.774	0.887
DMU 34	0.274	0.339	0.378	**0.417**	0.628	0.705	0.793
DMU 35	0.447	0.562	0.630	**0.717**	1.049	1.153	1.339
DMU 36	0.269	0.349	0.398	**0.452**	0.742	0.844	0.966
DMU 37	0.243	0.301	0.336	**0.395**	0.747	0.855	0.963

	model (14)			model (2)	model (18)		
	20%	**10%**	**5%**		**5%**	**10%**	**20%**
DMU 38	0.172	0.217	0.244	**0.273**	0.459	0.513	0.560
DMU 39	0.202	0.264	0.306	**0.355**	0.588	0.674	0.773
DMU 40	0.175	0.231	0.270	**0.309**	0.481	0.561	0.611
Average	0.2498	0.315125	0.35875	0.411175	0.66655	0.7597	0.86605
Max	0.447	0.562	0.63	0.717	1.093	1.252	1.471
Min	0.123	0.157	0.178	0.204	0.349	0.395	0.452
Correlation	0.97015	0.99059	0.99609		0.91739	0.90290	0.90298

Accompanying the model (14) and the model (18) are calculated and shown in columns 3 through 8 of the accompanying table, assuming a deviation of 5, 10, and 20% of the nominal data values.

The total efficiency values presented in Table 1 are also plotted in the following diagram. As can be seen from the efficiency values shown in Table 1 as well as in Figure 3, the robust efficiency scores obtained from model (14) give lower bounds to the efficiency scores of the nominal three-stage model, and model (18) provide upper bounds for them. Therefore, one can provide an efficiency interval for each DMU using the robust efficiency scores obtained from Model (14) and Model (18). Furthermore, as shown in the last line of Table 1, the results obtained for the upper and lower bounds of the DMU's efficiency score have a high correlation with the definite efficiency values of the DMUs, with the lower limit values being more accurate. Furthermore, as the conservatism coefficient in the Bertsimas technique decreases, the accuracy of model calculations increases, and the results of the proposed models converge to the efficiency values obtained from definitive data.

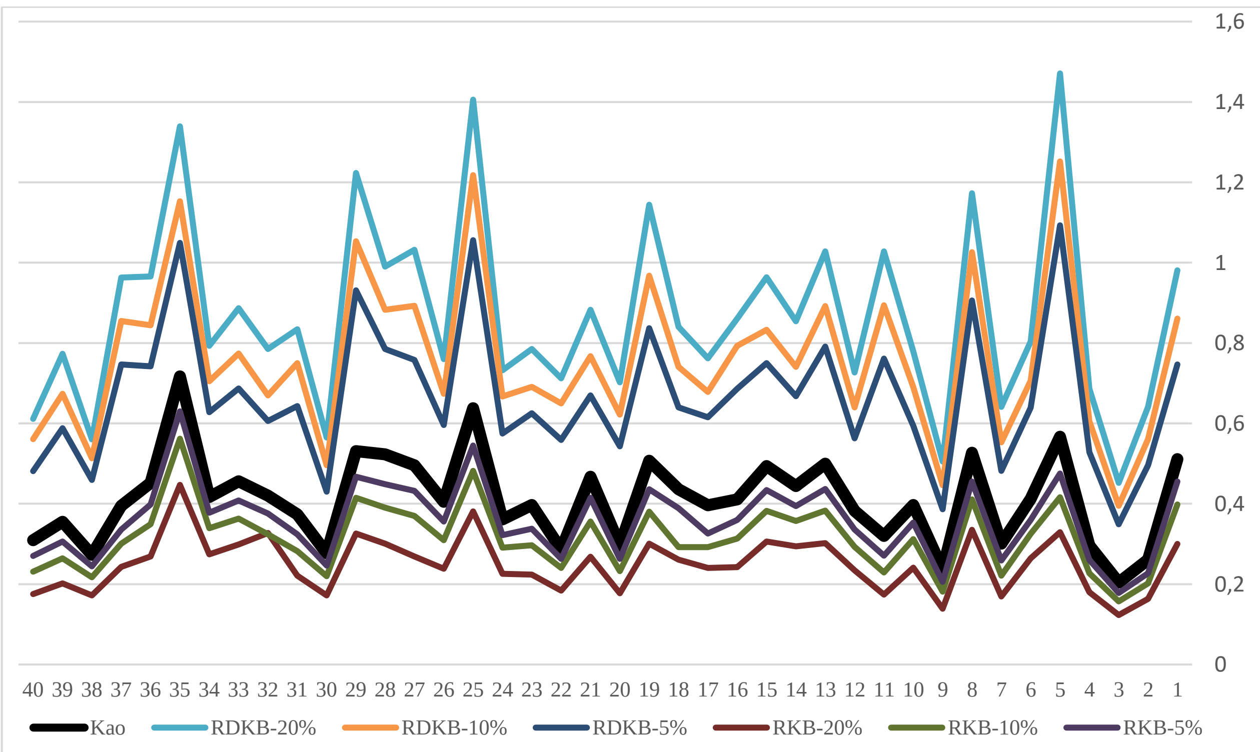

Figure 3. Diagram of nominal and robust efficiency scores

The relative efficiency of sub-DMUs was calculated using (26) to (31) and (38) to (41). The results have been represented in Table 7. It is notable that the experts' order of preference was first, second, and third sub-DMU in our case.

Table 2. Sub-DMUs efficiency values of the three-stage model with the Bertsimas *et al*. technique

	Stage 1			Stage 2			Stage 3		
	R-kao	**Kao**	**R-Dkao**	**R-kao**	**Kao**	**R-Dkao**	**R-kao**	**Kao**	**R-Dkao**
DMU1	0.82097	0.95985	1.30272	0.94110	1.00000	1.56044	0.93075	1.00000	1.64860
DMU2	0.75670	0.84810	1.35227	0.91304	1.00000	1.96538	0.65013	0.75651	0.97762
DMU3	0.87769	1.00000	1.73087	0.94301	1.00000	1.68668	0.30760	0.37508	0.36528
DMU4	0.78833	0.99605	1.43794	0.90738	1.00000	2.35086	0.67378	1.00000	1.22337
DMU5	0.94544	1.00000	1.91883	0.52301	0.67389	1.26612	0.85822	1.00000	1.72163
DMU6	0.65716	0.82042	1.23681	0.88451	1.00000	1.54794	0.94118	1.00000	1.49995
DMU7	0.55265	0.71797	1.07077	0.93333	1.00000	1.55556	0.40020	0.52545	0.56994
DMU8	0.93548	1.00000	1.75398	0.38429	0.58921	1.27091	0.92885	1.00000	1.85396
DMU9	0.58676	0.83656	1.56368	0.62636	0.81904	1.29992	0.58244	0.78618	0.72380
DMU10	0.74573	0.93072	1.38781	0.84970	1.00000	2.07097	0.92000	1.00000	0.89895
DMU11	0.93231	1.00000	2.57224	0.47970	0.59308	0.92505	0.44772	0.64411	0.79281
DMU12	0.59484	0.76305	1.26428	0.92079	1.00000	1.91111	0.69736	0.84811	0.70627
DMU13	0.94737	1.00000	1.66702	0.44211	0.64634	1.09607	0.90880	1.00000	1.09227
DMU14	0.70529	1.00000	2.15572	0.93548	1.00000	1.63587	0.79266	0.91577	0.64253
DMU15	0.61565	0.75888	1.17380	0.93346	1.00000	2.07815	0.72349	0.85017	0.69003
DMU16	0.82607	1.00000	1.62182	0.91639	1.00000	1.53852	0.67457	0.85514	0.97945
DMU17	0.95108	1.00000	1.61446	0.68696	0.88712	1.39135	0.57985	0.67471	0.56641
DMU18	0.47999	0.66364	1.12719	0.87936	1.00000	2.10998	0.70318	0.81162	0.95206
DMU19	0.71555	0.86087	1.16652	0.86387	1.00000	1.98218	0.90448	1.00000	1.55424
DMU20	0.74090	0.88516	1.30939	0.84474	1.00000	1.49963	0.67395	0.85460	1.20039
DMU21	0.77664	1.00000	1.76587	0.68860	0.95563	1.69301	0.77242	0.98094	1.69994
DMU22	0.74690	0.84810	1.32109	0.91304	1.00000	1.97129	0.82230	0.98030	1.42846
DMU23	0.87769	1.00000	1.73087	0.93464	1.00000	1.67484	0.67823	0.81684	0.67464
DMU24	0.78833	0.99605	1.45909	0.88884	1.00000	2.30497	0.79070	1.00000	1.26967
DMU25	0.94495	1.00000	1.88053	0.51240	0.68252	1.13298	0.92035	1.00000	2.20979
DMU26	0.65354	0.96985	1.66739	0.88451	1.00000	1.54794	0.86574	1.00000	1.41671
DMU27	0.58089	0.73605	1.13962	0.89583	1.00000	1.50000	0.67287	0.82044	0.87611
DMU28	0.93384	1.00000	1.72803	0.37180	0.52923	1.08616	0.82209	1.00000	1.14503
DMU29	0.82652	1.00000	1.99401	0.59994	0.74559	1.10607	0.90065	1.00000	1.25193
DMU30	0.72112	0.88644	1.30140	0.90411	1.00000	1.95604	0.83537	0.99540	0.96723
DMU31	0.92857	1.00000	1.72650	0.88886	1.00000	1.62188	0.56532	0.67508	0.72720
DMU32	0.58877	0.73488	1.33686	0.88888	0.99928	1.68889	0.85110	1.00000	0.82855
DMU33	0.89163	0.96147	1.52525	0.40603	0.48837	0.73303	0.92671	1.00000	1.33541
DMU34	0.77346	1.00000	2.15572	0.91398	1.00000	1.54727	0.82461	0.96820	0.63587
DMU35	0.74855	0.95087	1.87114	0.92007	1.00000	1.54131	0.91411	1.00000	1.13001
DMU36	0.84005	1.00000	1.62268	0.94872	1.00000	1.53852	0.77150	0.95505	1.03429
DMU37	0.92495	0.99642	1.53860	0.69053	0.89254	1.40342	0.65645	0.77332	0.88253
DMU38	0.44453	0.69807	1.18117	0.69053	1.00000	2.20992	0.44291	0.62794	0.67320
DMU39	0.71420	0.87640	1.20527	0.85415	1.00000	1.85540	0.68749	0.87639	1.06672
DMU40	0.73290	0.87809	1.28184	0.84207	1.00000	1.52724	0.68150	0.85269	0.97094
Average	0.76535	0.914349	1.546526	0.786153	0.912546	1.609572	0.742541	0.880501	1.072095
Min	0.44453	0.66364	1.07077	0.3718	0.48837	0.73303	0.3076	0.37508	0.36528
Max	0.95108	1	2.57224	0.94872	1	2.35086	0.94118	1	2.20979
Correlation	0.877869		0.733266	0.952559		0.773214	0.942328		0.67356

As shown in Table 2, the proposed models' findings provide appropriate constraints for the efficiency of each stage. The results obtained show that the third step has a reduced efficiency. However, the lower bounds of the efficiency score obtained have higher correlation and accuracy.

4.2 Comparison of TSBRNDEA with TSFDEA

Khalili-Damghani and Taghavifard (2012) used a three-stage fuzzy DEA model (TSFDEA) to tackle this problem and estimated the lower and upper bound values of efficiency scores. The table below displays the upper and lower bound values generated by them, as well as the efficiency scores calculated by our proposed models.

Table 3. Comparison values of the TSBRNDEA model and FTSDEA model

	Model (14) 20%	LB TSFDEA	Model (14) 10%	Model (5)	Model (18) 10%	UB TSFDEA	Model (18) 20%
DMU1	0.300	0.348	0.398	0.511	0.861	0.958	0.981
DMU2	0.163	0.240	0.202	0.261	0.561	0.723	0.641
DMU3	0.123	0.130	0.157	0.205	0.395	0.533	0.452
DMU4	0.180	0.220	0.227	0.296	0.605	0.785	0.686
DMU5	0.329	0.434	0.416	0.567	1.252	0.776	1.471
DMU6	0.264	0.321	0.325	0.412	0.706	0.906	0.804
DMU7	0.169	0.289	0.221	0.301	0.553	0.705	0.641
DMU8	0.335	0.466	0.411	0.527	1.026	0.576	1.173
DMU9	0.139	0.249	0.181	0.236	0.446	0.746	0.505
DMU10	0.241	0.282	0.312	0.397	0.690	0.807	0.779
DMU11	0.174	0.227	0.229	0.320	0.894	0.644	1.028
DMU12	0.234	0.328	0.293	0.382	0.640	0.965	0.727
DMU13	0.302	0.347	0.383	0.500	0.892	0.645	1.028
DMU14	0.294	0.353	0.357	0.444	0.741	0.932	0.854
DMU15	0.306	0.380	0.382	0.493	0.833	0.847	0.964
DMU16	0.242	0.364	0.313	0.411	0.793	0.887	0.861
DMU17	0.240	0.263	0.292	0.369	0.678	0.693	0.762
DMU18	0.261	0.362	0.292	0.436	0.741	0.677	0.840
DMU19	0.301	0.453	0.380	0.507	0.968	0.900	1.144
DMU20	0.177	0.235	0.232	0.296	0.622	0.671	0.702
DMU21	0.268	0.339	0.356	0.467	0.767	0.946	0.883
DMU22	0.184	0.274	0.240	0.286	0.650	0.810	0.712
DMU23	0.224	0.288	0.297	0.397	0.691	0.870	0.785
DMU24	0.226	0.308	0.291	0.361	0.667	0.920	0.732
DMU25	0.381	0.552	0.482	0.638	1.128	0.788	1.406
DMU26	0.238	0.307	0.309	0.405	0.674	0.868	0.760
DMU27	0.269	0.453	0.370	0.496	0.893	1.000	1.032
DMU28	0.301	0.453	0.390	0.523	0.883	0.576	0.990
DMU29	0.326	0.455	0.415	0.531	1.053	0.888	1.223
DMU30	0.172	0.192	0.220	0.277	0.496	0.718	0.565
DMU31	0.221	0.327	0.282	0.374	0.750	0.810	0.834
DMU32	0.327	0.347	0.324	0.420	0.670	0.969	0.785
DMU33	0.299	0.408	0.363	0.456	0.774	0.659	0.887
DMU34	0.274	0.335	0.339	0.417	0.705	0.892	0.793
DMU35	0.447	0.849	0.562	0.716	1.153	0.865	1.339
DMU36	0.269	0.399	0.349	0.452	0.844	0.914	0.966
DMU37	0.243	0.294	0.301	0.394	0.855	0.676	0.963
DMU38	0.172	0.213	0.317	0.272	0.513	0.614	0.560
DMU39	0.202	0.320	0.264	0.355	0.674	0.860	0.773
DMU40	0.175	0.244	0.231	0.309	0.561	0.804	0.611
Average	0.2498	0.3412	0.317625	0.410425	0.75745	0.795575	0.86605
Max	0.447	0.849	0.562	0.716	1.252	1	1.471
Min	0.123	0.13	0.157	0.205	0.395	0.533	0.452
Correlation	0.969388	0.911342	0.969944	1	0.902426	0.293912	0.904376

It can be seen from Table 3 that the efficiency interval of our proposed model with a 10% deviation from the nominal data is more accurate than the TSFDEA model, and its efficiency interval with a 20% deviation is significantly accurate. When the data in Table 3 are compared, it is clear that the suggested technique's findings, while having good accuracy, have a larger correlation coefficient than the fuzzy method.

5. CONCLUSION

The efficiency of n-stage series systems was evaluated in this article by taking into account the structure and internal processes, as well as under uncertain conditions, using the network data envelopment analysis model. Initially, models for evaluating the efficiency of an n-stage network with deterministic data were presented. When only the upper and lower bounds of the data are known, the output data and non-deterministic intermediate products are introduced. The Bertsimas and Sim (2004) approach was then used, and robust optimization models for n-stage networks with non-deterministic outputs and intermediate products were demonstrated. In an n-stage network model with non-deterministic outputs and intermediate products, the models presented in this study calculate the upper and lower bounds of efficiency. The results show that the proposed approach is effective for evaluating dairy companies' supply chain performance and internal activities in the face of data uncertainty.

According to the performance evaluation analysis for these companies, the third stage is the most inefficient; therefore, it is recommended that different companies pay more attention to the processes of performance goals in order to increase the efficiency of their performance.

In addition, the results for the efficiency intervals of the decision-making units were compared to the results from Khalili *et al*. (2012)'s three-stage network fuzzy envelope analysis model. It was obtained using the fuzzy method; additionally, the proposed method yielded higher correlation coefficients. According to Table 1, we find that the bounds of the total efficiency calculated by the proposed models almost maintain the ranking of the units, and on the other hand, the lower the deviation of the data, the higher the accuracy of the obtained values. By reducing the amount of deviation in the data from 20% to 10% and then 5% of the data-deterministic value, the average lower efficiency bound increases from 0.2498 to 0.315125 and then 0.358750, and the average upper-efficiency bound increases from 0.86605 to 0.7597. and then it decreases to 0.6655, while the definitive efficiency value is 0.411175. Also, the correlation coefficient of the lower bounds increased from 0.97015 to 0.99059 and then 0.99609, and the correlation coefficient of the upper bounds increased from 0.90298 to 0.90290 and then 0.91739, which shows that the proposed model has better accuracy and performance for calculating the lower bound. Also, paying attention to the results of Table 2 for calculating the upper and lower bounds of the efficiency of the intermediate steps confirms the above results. The proposed models for calculating the lower bound have higher accuracy and correlation coefficients than the models proposed for calculating the upper bound.

In Table 3, the results of the proposed models with the results of the TSFDEA model presented by Khalili *et al*, it can be seen that the proposed model has comparable accuracy, although it has a higher correlation coefficient. Also has Especially in the upper limit, the accuracy of the values of the proposed model is higher, and it also has a higher correlation coefficient.

In this paper, a novel approach to evaluating the performance of envelopment analysis of network data with non-deterministic data is presented. The models presented here are for series multiphase systems with non-deterministic outputs and intermediate products. This method is applicable to all types of network systems, including parallel network systems with performance evaluation indicators such as undesirable inputs and outputs used in contracts and new models. Because most industries' data contains uncertainty, the model presented in this article can be applied to a variety of industries.

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