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CONTRACT CHOICE FOR A BRAND-LED HYBRID COMPETING SUPPLY CHAIN CONSIDERING LOYAL CONSUMERS

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From the perspective of cooperation between brands and manufacturers, this paper constructs a hybrid channel supply chain with brands as the dominant players in the game, considering the different market structure factors of traditional and direct sales channels and introducing channel transfer coefficients and loyal consumers. Based on this model, we compare the optimal solutions of each variable and optimal returns under different decisions and study the wholesale price discount model and revenue compensation coordination mechanism for the hybrid channel. The effectiveness of the coordination mechanism is verified through numerical analysis of the wholesale price discount rate and the revenue compensation coefficient. Moreover, the effect of the parameters on the supply chain's profitability before and after coordination is also analyzed. The study shows that the smaller the channel transfer coefficient and the larger the demand of loyal consumers is, the more beneficial the increase in the supply chain system's profitability.

Keywords: Specific Markets; Brand Retailers-Led; Price Discount; Revenue Compensation.

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1. INTRODUCTION

With the rapid development of China's economy and the increasing popularity of the Internet, many international brand companies have entered the Chinese market. To pursue relatively low costs, most foreign brand companies cooperate with local manufacturers in China. They outsource low value-added processes (such as production) to manufacturers, while brand companies are responsible for R&D, i.e., research and development, sales, and other relatively high value-added processes. This mode of cooperation in which brand retailers entrust others to produce is Original Equipment Manufacturer (OEM) production, and the manufacturer company that undertakes the manufacturing task is called OEM (original equipment manufacturer), so a relatively large number of OEM enterprises have been formed in China. The manufacturers studied in this paper are mainly original equipment manufacturers, and this supply chain produced by manufacturers and sold by brand retailers is called the traditional channel supply chain.

At the same time, many manufacturers have begun to open up direct sales channels to expand their market share. This behavior may attract new consumers to generate additional revenue, but it also may conflict with traditional channels because the manufacturer's direct sales channels divert some of the brand's consumers with low prices and ease of operation. Poor coordination between the supplying parties may not only affect the partnership but also lead to a loss of supply chain system performance due to the asymmetry of information and power. For example, The Home Depot, the world's leading retailer of home building supplies, canceled partnerships with all manufacturers that have opened direct sales channels (Huang *et al.,* 2009), so coordination is one of the challenges in decision-making.

In the hybrid channel supply chain discussed in this paper, the brand retailer refers to mature enterprises in the external regional market, such as international companies, which have a certain brand effect and sell to both the local market and the external market. Therefore, this paper considers brand companies as the dominant player in the supply chain. The manufacturer is the OEM enterprise of the brand retailers, and the direct sales channel opened by the manufacturer is only for the local regional market because the manufacturer is in the early stage of development. Such a supply chain mainly occurs in

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the low-core technology industries, such as clothing, cosmetics, etc. For example, on the shopping app 'Necessity' came out in 2014, the merchants settled are the domestic OEMs of international beauty and clothing brands, such as PRADA and ARMANI, etc. These manufacturers sell their products to the brand retailers or directly online to local customers on the app. The platform connects users directly to the manufacturers, cutting out a range of unnecessary value-added activities such as the movement of goods and brand retailers' premiums, enabling users in the local regional market to purchase relatively low-priced but homogeneous products.

Such a situation also exists in the early stages of development of some home appliance industries. For example, Galanz, Changhong, and Aucma produced products for international brands such as Panasonic and Toshiba in the early years while also operating their products directly in the local regional market, thus forming a hybrid channel model. Although Galanz has now gone global through low-cost expansion and scale advantages, many manufacturers still sell to the local region market in the early stages of establishing direct sales channels.

The research questions of this paper are: How can a brand coordinate a hybrid channel supply chain after a manufacturer has established a direct sales channel serving a different market to achieve the effect of centralized decision-making? Can such coordination benefit both the manufacturer and the brand? To this end, we construct profit models for a brand-led hybrid channel supply chain, calculate the market demand function, and devise a mechanism for coordination with wholesale price discounts and revenue compensation contracts. Finally, the calculated wholesale price discount rate k_1 and income compensation coefficient k_2 are numerically analyzed to verify the effectiveness of the coordination mechanism.

2. LITERATURE REVIEW

There has been much research on dual-channel supply chains now. Park *et al.* (2003) studied the optimal channel choice of suppliers and found that adopting a multi-channel sales strategy by suppliers is conducive to improving their profitability and the overall performance of the supply chain but may reduce the profits of retailers. This is because manufacturers' new direct sales channels will cause a shift of some consumers from the traditional channels. Chen *et al.* (2017) studied the impact of the channel transfer coefficient on the supply chain performance under centralized and decentralized decisions, taking manufacturers as the core enterprises, and showed that single-channel supply chains can benefit from the introduction of new channels in some cases. In contrast to these works that investigate suppliers' options in choosing channel strategies, we focus on how brand retailers, as the dominant players in the supply chain, make their next choices after the supplier has entered the retail market because brand retailers have more resources and markets in industries such as apparel and FMCG (fast-moving consumer goods).

Also of interest is the large body of recent literature focused on retailer-led dual channels. Liu *et al.* (2022) showed that retailers charging manufacturers a small percentage of their retail price as a commission can benefit both parties, arguing that incentive contracts may have some impact on coordination strategies. Matsui (2017) studied the dynamic pricing problem in dual-channel supply chains and proposed to coordinate supply chains through contracts, such as two-part tariff or return policy contracts. Dumrongsiri *et al.* (2008) argued that when market demand is lower than a certain threshold, brand retailers could reasonably coordinate traditional channels and direct sales channels to increase the system's total profits. However, these studies didn't specifically explore how mechanisms coordinate dual-channel supply chains and only mentioned methods such as two-part tariffs and volume discount contracts as potential options for the goal of improving system performance. Channel conflict not only leads to price competition between direct sales channels and traditional channels but also leads to vertical competition between manufacturers and retailers, so channel coordination strategies are crucial. In contrast to these works, we have conducted an in-depth study of contract coordination mechanisms.

In response to the study of coordination contract selection, David *et al.* (2015) demonstrated that some known single-channel contracts couldn't coordinate dual-channel supply chains and proposed a linear volume discount contract that perfectly coordinated dual-channel supply chains. Shafiq *et al.* (2021) proposed a revenue-sharing and demand-redistribution model to coordinate and reduce risks. Chen *et al.* (2017) proposed a retailer profit contract that can coordinate a dual-channel supply chain which can determine the retailers' profit margin ranges. Channel members obtained the same profit under this agreement as under centralized decision-making. Kurata *et al.* (2017) studied based on Park's theory, arguing that an appropriate dynamic pricing strategy can benefit both parties. These papers mainly considered symmetric multiple retailers and didn't consider the impact of channel demand shifting, which is inconsistent with our research. According to Tsao *et al.* (2022), customers' consumption patterns and preferences may also lead to competition and conflict between upstream and downstream members of the supply chain, so it is necessary to consider consumer channel preference.

As for the wholesale price discount contract adopted in this study, some scholars have done a lot of research previously. Choi *et al.* (2022) discussed how to optimize the hybrid channel supply chains led by retailers through different contracts from a theoretical perspective, including simple wholesale price contracts, two-part tariff contracts, revenue sharing contracts, and volume discount contracts, and believed that all contracts are more effective for supply chain coordination but volume discount contracts because of risk. Chiang *et al. (*2003) analyzed the competition between direct sales prices and traditional prices and found that reducing wholesale prices by manufacturers could increase the total sales and profits of channel members. Zhang *et al. (*2017) showed that after manufacturers establish online channels, retailers may strategically offer to reduce wholesale prices, and if manufacturers didn't accept this offer, they may pay retailers an appropriate fee. Amrouche *et al. (*2022) investigated the role of various coordination mechanisms and found that neither promotional cost-sharing nor brand coordination strategies were optimal coordination solutions, and initiated a new mechanism, which is co-branding coordination. They also believed that wholesale price discounts could be used for coordination. Yao *et al. (*2005) also showed that proper adjustment of wholesale prices can reconcile the conflicts between traditional and direct channels. These studies all agree that the wholesale price discount contract can coordinate channels, but it is challenging to coordinate channels with wholesale price contracts alone. For this reason, this paper uses both wholesale price discounts and revenue compensation contracts.

However, Xu *et al. (*2014) argued that wholesale price contracts and revenue-sharing contracts couldn't achieve coordination in dual-channel supply chains because of the dual marginalization effect. Contrary to this work, we found that even under the decentralized decision-making that only considers their respective marginal benefits, revenue compensation based on wholesale price discount can be beneficial to both the system and members, and it is verified in Section 6 that the coordination contracts have effectiveness on the profit increase of hybrid channel supply chain, which is significantly different from the results of the above paper.

In general, many scholars have studied coordination strategies for hybrid channel supply chains, all emphasizing the importance and necessity of supply chain coordination for manufacturers and brands, arguing that a reasonably designed coordination strategy is conducive to resolving conflicts between members and proposing measures such as dynamic pricing, order payment transfer, revenue sharing, quantity discounts. Many scholars have also proposed wholesale price discount models, but most of them only adopt wholesale price discount contracts alone, which is challenging for coordination. As shown in subsection 5.1 of this paper, although the profits of brand retailers and systems after coordination are higher than those under decentralized decision-making, the profits of manufacturers are still damaged. To this end, we consider examining this issue through wholesale price discount contracts with revenue compensation, which is one of the key points of this research. In addition, in the context of the study, i.e., in the early stages of the development of direct sales channels, the market structure of manufacturers is much smaller than that of brand retailers, so the division of market structure in Lee *et al. (*2022) also provides a basis for the subsequent research in this paper.

Unlike most of the literature that discusses "how OEMs should establish their own direct sales channels without infringing on brand retailers," this paper focuses on the situation where manufacturers have established their own direct sales channels, and their market size is different from that of traditional supply chains at the initial stage of establishment. To solve the problems raised before, this paper mainly completes the following contents:

(1) Based on the disparity between the current market size of traditional channels and direct sales channels, the demand function and profit model of hybrid channel supply chains led by brand retailers are constructed considering the channel transfer coefficient and the fact that brand retailers have some loyal consumers in the local market (section 4);

(2) The channel coordination mechanism of wholesale price discount and income compensation is designed by comparing and analyzing the optimal results of centralized and decentralized decision-making (section 5);

(3) Explore the influence of channel transfer coefficient and brand loyalty consumers on the coordination strategy of hybrid channel supply chains (section 6).

3. MODEL BUILDING

This section introduces the relevant notations and parameters of the model and draws the hybrid channel supply chain structure diagram. Then, the assumptions of the model and the range of parameter variables are given, and the basic model is constructed.

3.1 Parameter Introduction

In such a hybrid supply chain, we use the superscripts D and T, respectively denote direct sales channels and traditional channels, and superscripts I, H, and C are the different decision types, centralized decision, decentralized decision, and decision coordination, respectively. At the same time, the subscripts M, R, and SC refer to manufacturers, brand retailers, and supply chain systems. The structural diagram of the channel model studied in this paper is shown in Figure 1.

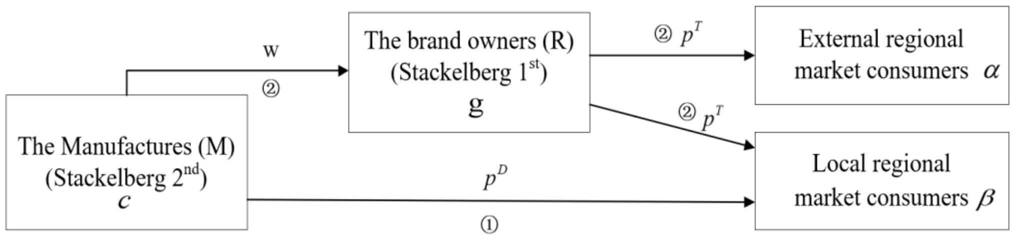

Figure 1. Channel Model Structural Diagram

According to Figure 1, original equipment manufacturers produce products at a unit cost of production c and sell them to the brand retailers at a unit wholesale price w; the brand retailers have their level of unit profit g and sell them at p^T to consumers in the external market sizable factor α and consumers in the local market sizable factor β. Meanwhile, the manufacturers have their direct sales channel and sell at p^D in the local market. For the elasticity coefficient of market demand concerning price, i.e., the price sensitivity factor, we use the symbol b.

In cases where OEMs can sell their products directly to the local regional market, the valuation function cannot distinguish between consumers with strong brand loyalty and those without due to the same product quality. To represent these consumers' preferences for branded products, we introduce the parameter δ and call them loyal consumers in the local regional market. These established and loyal consumers of these brand retailers in the local market buy products from the brand retailers as long as they can afford them, even if the quality of the products sold by the manufacturer and the brand retailer in the local market is the same. In addition, consumers generally shift demand between channels due to differences in perceptions of the price paid for the products, so we denote the extent of this demand diffusion by r and refer to this degree of demand diffusion as the channel shift coefficient, which is constantly greater than zero and only exists in the local regional markets.

Table 1 summarizes the notation and parameters in the hybrid channel supply chain model.

Table 1. Description of the Symbols in the Hybrid Channel Model

Symbols	Meanings	Symbols	Meanings
c	Unit production cost	T	Traditional channels
w	Unit wholesale price	D	Direct sales channels
g	Unit profit levels for brand retailers	M	Manufacturers
b	Price sensitivity factor	R	Retailers
r	Channel transfer coefficient	SC	Supply Chain Systems
δ	Demand of the brand's loyal consumers	α	External market sizable factor
p^D	Unit product price in the direct channel	β	Local market sizable factor
p^T	Unit product price in the traditional channel	H	Decentralized decision-making
Q	Demand	I	Centralized decision-making
π	Revenue	C	Coordination mechanisms

3.2 Model Assumptions

Based on the above parameters, we make some assumptions about the model.

(1) Assuming that the external regional market sizable factor α and the local regional market sizable factor β are both large enough, but the number of consumers in the external regional market is much larger than that in the local regional market.

(2) It is assumed that the unit product price of the brand retailer remains unchanged after the manufacturer establishes a direct sales channel, and the brand retailer sells at the same price in different markets (local regional market and external regional market) for following the principle of consistency in market pricing.

(3) This paper doesn't consider the product quality differences between channels, so it is assumed that the manufacturer and the brand retailer have the same production line and sell the same product quality.

3.3 Basic Model

According to the basic formula of market demand, in the traditional channel where there are only brands offer products at a price p^T, the market coverage in the external regional market is $\alpha - bp^T$, and the market coverage in the local region is $\beta - bp^T$. But when considering the demand of the brand's loyal consumers, we know that in external regional markets where OEMs do not sell their products, the market coverage of the brand is $\alpha - bp^T$. In the local market, the market coverage of the brand becomes $\min\{\delta, \beta - bp^T\}$, depending on how many loyal consumers can afford to buy the branded product, and the manufacturer's local market coverage becomes $\beta - bp^D - \min\{\delta, \beta - bp^T\}$.

The size of the existing local area market can generally be divided into three scenarios: (1)$\beta > \delta + bp^T$, (2)$\beta = \delta + bp^T$, (3)$\beta < \delta + bp^T$. The first and third scenarios mean that the local area market is large (small) enough that the number of people who can afford the branded product $\beta - bp^T$ is more (less) than the demand of loyal consumers δ. Based on the assumption that the local regional market is sufficiently large, the size of the market we study focuses on this scenario $\beta > \delta + bp^T$, i.e., that $\min\{\delta, \beta - bp^T\} = \delta$. In this case, although more people can afford to pay for the brand's products, only loyal consumers δ would only choose to buy from traditional channels, and the rest of this segment ($\beta - bp^D - \delta$) would all buy the same quality but cheaper products from the direct sales channels established by the manufacturer.

Therefore, the external regional market coverage of the brand retailer is $\alpha - bp^T$, the local regional market of the brand retailer is δ, while the manufacturer's coverage in the local regional market is $\beta - bp^D - \delta$.

Moreover, when local regional market consumers shift demand between channels due to differences in price perceptions, the demand of the direct sales channel in the local regional market is $\beta - bp^D - \delta + r(p^T - p^D)$, the demand for the traditional channel in the local regional market is $\delta + r(p^D - p^T)$, and the demand for the traditional channel in the external regional market is $\alpha - bp^T$.

In the following subsections, we consider the case where the brand retailer is the leader and the manufacturer is the follower and use this assumption to analyze the brand retailer's price strategy and associated profits.

4. DECISION MODELS FOR THE HYBRID CHANNEL SUPPLY CHAIN

In a hybrid channel supply chain, the market demand of direct sales channels Q^D and the market demand for traditional channels Q^T are, respectively: $Q^D = \beta - bp^D - \delta + r(p^T - p^D)$, and $Q^T = \alpha - bp^T + \delta + r(p^D - p^T)$. Then, the profits of manufacturers in direct sales channels and traditional channels are, respectively: $\pi_M^D = (p^D - c)Q^D$, and $\pi_M^T = (w - c)\,Q^T$. So, the total profits of the manufacturers and retailers are, respectively: $\pi_M = (p^D - c)Q^D + (w - c)Q^T$, and $\pi_R = \pi_R^T = (p^T - w)Q^T$. The profits across direct sales channels and traditional channels are, respectively: $\pi^T = (p^T - c)Q^T$, and $\pi^D = (p^D - c)Q^D$. The total profit of the supply chain systems is $\pi_{SC} = \pi_R + \pi_M = \pi^D + \pi^T$.

4.1 Centralized Decision-Making (I)

In the hybrid channel centralized decision, the manufacturer produces products at unit cost c and sells to direct channels and traditional channels. The brand retailer forms a strategic alliance with the manufacturer, i.e., first sets the unit wholesale price w^I to the unit cost c, then the manufacturer and the brand as a whole decide to optimize and jointly set the optimal price for the traditional and direct channels p^{IT} and p^{ID}.

The sequence of decisions is as follows: (1) the manufacturer produces the product at c and sells the product to the brand retailer at $w^I = c$; (2) the manufacturer and the brand retailer decide both the selling price in the direct sales channel p^{ID} and the selling price in the traditional channel p^{IT}.

At this point, the system optimization objective is shown as '**Equation 1**'. The specific expansion formula is placed in Appendix A, and the following equations are similar.

Lemma 1.

(1) In the hybrid channel centralized decision, the optimal unit product price for the direct and traditional channels, respectively as: $p^{ID*} = \frac{b^2c + r(\alpha+\beta) + b(2cr + \beta - \delta)}{2b(b+2r)}$, $p^{IT*} = \frac{b^2c + r(\alpha+\beta) + b(2cr + \alpha + \delta)}{2b(b+2r)}$.

(2) The optimal demand for the direct and traditional channels is yielded respectively as: $Q^{ID*} = \frac{\beta - \delta - bc}{2}$, $Q^{IT*} =$

$\frac{\delta-bc+\alpha}{2}$.

(3) The optimal profit for the direct and traditional channels is yielded as follows: $\pi^{ID^*} = \frac{(bc-\beta+\delta)(b^2c+2bcr-r\alpha-b\beta-r\beta+b\delta)}{4b(b+2r)}$, $\pi^{IT^*} = \frac{(\delta-bc+\alpha)(r\alpha+r\beta-b^2c-2bcr+b\alpha+b\delta)}{4b(b+2r)}$.

The detailed reasoning process is described in Appendix B. Through the above, the total profit of the supply chain is shown as '**Equation 2**'.

4.2 Decentralized Decision-Making (H)

Assume that the brand retailer is the dominant player in the Stackelberg game, and the manufacturer is the follower. In the hybrid channel decentralization decision, the decision sequence is as follows: (1) the brand retailer first aims to maximize its revenue and makes a decision on the unit product profit level g^H; (2) based on the unit production cost c and the optimal profit level g^H. The brand has decided that the manufacturer then makes decisions on the optimal wholesale price w^H and the optimal direct channel's price p^{HD}; (3) finally, the brand retailer determines the market price of the product in the traditional channel in the hybrid channel p^{HT}, where $p^{HT} = w^H + g^H$.

At this point, the decision objectives of the system are:

$$\max_{g^H} \pi_R^H = g^H Q^{HT} \tag{1}$$

$$\max_{p^{HD}、w^H} \pi_M^H = (p^{HD} - c)\, Q^{HD} + (w - c) Q^{HT} \tag{2}$$

Lemma 2.

(1) In the hybrid channel decentralized decision, the optimal unit product price of the direct and traditional channels, respectively as follows: $p^{HD^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\beta-\delta)}{2b(b+2r)}$, $p^{HT^*} = \frac{b^3c+2r^2(\alpha+\beta)+2br(2cr+3\alpha+\beta+2\delta)+b^2[4cr+3(\alpha+\delta)]}{4b(b+r)(b+2r)}$.

(2) The optimal wholesale price is obtained as follows: $w^{H^*} = \frac{3b^3c+2r^2(\alpha+\beta)+2br(2cr+\alpha+\beta)+b^2(8cr+\alpha+\delta)}{4b(b+r)(b+2r)}$.

(3) The optimal demand for the direct and traditional channels is yielded respectively as: $Q^{HD^*} = \frac{b(2\beta-3cr-2\delta)-2b^2c+r(\alpha+2\beta-\delta)}{4(b+r)}$, $Q^{HT^*} = \frac{1}{4}(\alpha - bc + \delta)$.

(4) The optimal profit of the direct sales channel is: $\pi^{HD^*} = \pi_M^{HD^*} = \frac{[b^2c-r(\alpha+\beta)+b(2cr-\beta+\delta)][2b^2c+r(\delta-\alpha-2\beta)+b(3cr-2\beta+2\delta)]}{8b(b+r)(b+2r)}$, the profit of the traditional channel is: $\pi^{HT^*} = \frac{(bc-\alpha-\delta)\{3b^3c-2r^2(\alpha+\beta)+2br(2cr-3\alpha-\beta-2\delta)+b^2[8cr-3(\alpha+\delta)]\}}{16b(b+r)(b+2r)}$.

The subsequent detailed reasoning process is also described in Appendix B. Appendix B will also show the profits of different members on different channels. Through the above, the total profit of the supply chain is obtained as '**Equation 3**'.

4.3 Comparison and Optimization Analysis Under Different Decisions

According to the basic model of the brand-led hybrid channel supply chain under the centralized and decentralized decision, we obtain the optimal unit product price, demand, and profit of the direct sales channel and the traditional channel, and we can give the following propositions by comparing the results of this game, where $\Delta\,(I^* - H^*)$ denotes the difference in price, demand volume, or profit of each channel and supply chain member under centralized and decentralized decisions.

4.3.1 Comparison of Hybrid Channel Supply Chains Under Different Decisions

- **Proposition 1.** Unit product price in the direct sales channel is equal under centralized decision-making I^* and decentralized decision-making H^*, i.e., $p^{ID^*} - p^{HD^*} = 0$.
- **Proposition 2.** The demand for the direct sales channel under decentralized decision-making H^* is larger than the demand for the direct sales channel under centralized decision-making I^*, so the profit of the direct sales channel under decentralized decision-making H^* is also larger than the profit of the direct sales channel under centralized decision-making I^*, i.e., $Q^{ID^*} - Q^{HD^*} < 0$ (**Equation 4**), $\pi^{ID^*} - \pi^{HD^*} < 0$ (**Equation 5**).
- **Proposition 3.** The wholesale price of the traditional channel under decentralized decision-making H^* is greater than that under centralized decision-making I^*, i.e., $w^{I^*} - w^{H^*} < 0$ (**Equation 6**), and the retail price of the traditional channel under decentralized decision-making H^* is greater than that under the centralized decision-making I^*, i.e. $p^{IT^*} - p^{HT^*} < 0$ (**Equation 7**). This deviation in wholesale and retail prices in the traditional channel under different decisions leads to greater sales in the traditional channel under the centralized decision-making I^* than that under the decentralized decision-making H^*, i.e., $Q^{IT^*} - Q^{HT^*} > 0$ (**Equation 8**).
- **Proposition 4.** While the manufacturers are more profitable in the case of the decentralized decision H^* in the traditional channel, i.e., $\pi_M^{HT^*} - \pi_M^{IT^*} > 0$ (**Equation 9**), the profit of the brand in traditional channels under the centralized decision I^* is greater than that under the decentralized decision H^*, i.e., $\pi_R^{IT^*} - \pi_R^{HT^*} > 0$ (**Equation 10**), and the total profit of traditional channels under centralized decision-making I^* is also greater than that under decentralized decision-making H^*, i.e., $\pi^{IT^*} - \pi^{HT^*} > 0$ (**Equation 11**). The total profit of the hybrid channel supply chain under centralized decision-making is much larger than that under decentralized decision-making, i.e., $\pi_{SC}^{I^*} - \pi_{SC}^{H^*} > 0$ (**Equation 12**).

4.3.2 Optimization Analysis of Hybrid Channel Supply Chains Under Different Decisions

As shown in the above propositions, in hybrid channel supply chains, a dual-channel supply chain can be coordinated if the optimal decisions under decentralized and centralized can be close to consistent.

When the demand of brand loyal customers δ is larger, the more significant the total profit increment of the supply chain will be. The comparison reveals that the optimal retail price of the traditional channel is larger under decentralized decision-making than under centralized decision-making. The total profit of the supply chain system under centralized decision-making increases by $(\alpha - bc + \delta)^2/[16(b + r)]$ than decentralized decision-making. This increase is a decreasing function of the manufacturer's production cost c and an increasing function of the brand loyal consumers' demand δ in the local regional market, which shows that there is room to improve the efficiency of the hybrid channel supply chain. In a brand-led hybrid channel market, the key to Pareto improvement lies in how the brand can design a coordination mechanism with the manufacturer to coordinate the direct sales channel and the traditional channel, improve the overall profitability of the supply chain, and achieve each member's revenue increment.

5. HYBRID CHANNEL SUPPLY CHAIN COORDINATION

Corollary 1. Compared to the decentralized decision, prices of the direct channel remain unchanged under the centralized decision ($p^{ID^*} = p^{HD^*}$), while the retail price of the traditional channel decreases. It is this price decrease that triggers a channel shift in market demand: the demand in the traditional channel, mainly in the external regional markets, increases significantly, leading to increased profits in the traditional channel at $\frac{[b^3c+2br(2cr-\alpha-\beta)-2r^2(\alpha+\beta)+b^2(4cr-\alpha-\delta)](bc-\alpha-\delta)}{16b(b+r)(b+2r)}$; the decline in the direct channel's market demand leads to a reduction of profits in the direct sales channel at $\frac{r(\alpha-bc+\delta)[b^2c-r(\alpha+\beta)+b(2cr-\beta+\delta)]}{8b(b+r)(b+2r)}$.

From the increase of system profitability under centralized decision-making, the focus of optimizing the hybrid channel supply chain by centralized decision-making is the traditional channel, not the direct channel.

Therefore, via analyzing the optimal unit product price and profitability under different decisions, the brand retailers should make their retail price as close as possible to that under the centralized decision by reducing the wholesale price of traditional channels. Simultaneously, the role of channel transfer should be considered to coordinate the brand-led hybrid channel supply chain.

However, manufacturers are mainly unlikely to accept such a wholesale price discount coordination mechanism from brands, believing that it would be detrimental to their profits. For manufacturers to allow lower wholesale prices, brands need to compensate manufacturers with a percentage of their revenue so that both parties can achieve Pareto improvements in their profits, thus achieving harmonization in the hybrid channel supply chain.

5.1 Wholesale price discount coordination mechanism

The sequence of decisions in this subsection is roughly the same as in the decentralized-making: (1) First, the brand retailer decides on its profit level g^{HC} to maximize its revenue; (2) Based on the unit production cost c and the brand's optimal profit level g^{HC}, the manufacturer then decides on the optimal price of the direct channel p^{HDC}, and the optimal wholesale price w^{HC}, where $w^{HC} = k_1(p^{HDC} - c) + c$, $k_1(0 < k_1 < 1)$ is the wholesale price discount rate that satisfies $c < w^{HC} < p^{HDC}$; (3) Finally, the brand decides the traditional channel's unit product price p^{HTC}, where $p^{HTC} = w^{HC} + g^{HC}$.

At this point, by combining equations and substituting, we can obtain the profit function of manufacturers after wholesale price discount coordination under decentralized decision (**Equation 13**).

Lemma 3.

(1) In the hybrid channel supply chain coordination, the optimal unit product price of the direct sales channel is obtained as '**Equation 14'**, and the optimal unit product price of the traditional channel is obtained as '**Equation 15**'.

(2) The optimal wholesale price of the traditional channel is shown in '**Equation 16**'.

To achieve hybrid channel supply chain coordination, the optimal price decision of direct and traditional channels under the price discount mechanism should be equal to the optimal decision under centralized decision-making, respectively, i.e., such that $p^{HDC^*} = p^{ID^*}$ and $p^{HTC^*} = p^{IT^*}$.

Substituting the equations of p^{HDC^*}, p^{ID^*}, p^{HTC^*}, and p^{IT^*}, the wholesale price discount rate is solved: $k_1 = \frac{r}{b+r}$.

Lemma 4.

In the hybrid channel supply chain, after the wholesale price discount coordination mechanism, when $k_1 = \frac{r}{b+r}$, the optimal unit product price of the direct and the traditional channels is: $p^{HDC^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\beta-\delta)}{2b(b+2r)}$, $p^{HTC^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\alpha+\delta)}{2b(b+2r)}$.

(1) The profits of the manufacturer and the brand retailer are: $\pi_M^{HC1^*} = \frac{\left[b^2c-r\,(\alpha+\beta)+b\,(2cr-\beta+\delta)\right]^2}{4b(b+r)(b+2r)}$, $\pi_R^{HC1^*} = \frac{(\alpha-bc+\delta)^2}{4(b+r)}$.

Therefore, the total profit of the supply chain is obtained as '**Equation 17**'.

Corollary 2. By making the price of direct channels and traditional channels under the wholesale price discount mechanism equal to the optimal unit product price under the centralized decision, respectively, we can obtain the wholesale price discount rate $k_1 = r/(b+r)$, at which point the supply chain system can achieve a total profit of no less than the centralized decision.

However, as both parties are rational individuals, effective coordination of a hybrid channel supply chain can only be achieved by ensuring a win-win situation. The following is an analysis of the change in the profitability of the supply chain system and its members before and after coordination.

Comparing the profit under decentralized decision-making after and before the wholesale price discount coordination mechanism, the profit differential of the supply chain system is: $\Delta\pi_{SC} = \pi_{SC}^{HC^*} - \pi_{SC}^{H^*} = (\alpha - bc + \delta)^2/[16(b+r)] > 0$, the profit differential of the manufacturer is: $\Delta\pi_M = \pi_M^{HC1^*} - \left(\pi_M^{HD^*} + \pi_M^{HT^*}\right) = -(\alpha - bc + \delta)^2/[16(b+r)] < 0$, the profit differential of the brand retailer is: $\Delta\pi_R = \pi_R^{HC1^*} - \pi_R^{HT^*} = (\alpha - bc + \delta)^2/[8(b+r)] > 0$.

Corollary 3. An increase in brand retailer profits and a decrease in manufacturer profits after wholesale price discount coordination led to an increase in total profits in the hybrid channel supply chain, and the increase in supply chain system profits equals the decrease in manufacturer profits.

5.2 Earnings Compensation Mechanism

Since the wholesale price discount coordination hurts the manufacturer's profit, the manufacturer will not easily agree to lower wholesale prices. In this case, the brand needs to compensate the manufacturer's profit based on the wholesale price discount to ensure that both parties achieve a Pareto improvement in their profits. The strategy used in this paper is the brand retailer compensates the manufacturer for a percentage k_2 of its profits after the wholesale price discount.

At this point, the profits of the manufacturer and the brand are, respectively:

$$\pi_M^{HC2^*} = \pi_M^{HC1^*} + k_2\pi_R^{HC1^*} \tag{3}$$

$$\pi_R^{HC2^*} = \pi_R^{HC1^*} - k_2\pi_R^{HC1^*} \tag{4}$$

The revenue compensation strategy is effective only if the profits of both the manufacturer and the brand retailer after compensation are greater than the profits of each party in the hybrid channel decentralized decision, i.e., let $\pi_M^{HC2^*} \geq \pi_M^{H^*}$ and $\pi_R^{HC2^*} \geq \pi_R^{H^*}$, and then solve it by substituting as $1/4 < k_2 < 1/2$.

Corollary 4. Under the price discount coordination mechanism, brands are willing to compensate manufacturers because their profits and the total supply chain profits are significantly increased, but manufacturers' profits are reduced. The study shows that brand retailers compensate manufacturers for their profits by a revenue compensation factor of $k_2(1/4 < k_2 < 1/2)$, which can lead to a win-win situation for both members and realize Pareto's improvement in the profitability of the hybrid channel supply chain.

6. EXAMPLE ANALYSIS

To further demonstrate the coordination mechanism's effectiveness, we analyze an arithmetic example.

Assuming that the external regional market size for the traditional channel is $\alpha = 280$, the local regional market size for the direct and traditional channels is $\beta = 120$, the manufacturer's unit production cost is $c = 1.8$, and the price sensitivity factor is $b = 3$. According to Jiang et al. (2016), we can get $0 < r < 9.332b$, which leads to $r \in (0, 27.996)$, the channel shift factor is assumed to be $r = 27$ in this paper; the proportion of loyal brand consumers' demand in the local regional market is 5%. Therefore $\delta = 6$.

A comparison of the optimal decision results for the hybrid channel supply chain under centralized versus decentralized decision-making is shown in Table 2.

Table 2. The Optimal Decision for Supply Chains with Centralized versus Decentralized Decision-making

Type	p^D	p^T	Q^D	Q^T	π^D	π^T	π_M	π_R	π_{SC}
Centralized decision-making	33.5	35.0	54.3	140.3	1720.2	4656.2	1720.2	4656.2	6376.4
Decentralized decision-making	33.5	37.3	117.4	67.2	3720.2	2492.2	5884.3	328.1	6212.4

As can be seen from Table 2, the optimal supply chain profit under centralized decision-making is 6376.4, while the optimal supply chain profit under decentralized decision-making is 6212.4, with a loss of 164 in system profit. At the same time, the retail price of traditional channels under the centralized decision mode is lower than that under the decentralized decision, while the total demand of the dual channels under the centralized decision is more significant than that under the decentralized decision.

6.1 About Wholesale Price Discount Rate k_1

The parameters are assumed to be consistent with the previous subsection. The wholesale price discount rate k_1's impact on supply chain members and system revenue is detailed in Table 3.

Table 3. The Wholesale Price Discount Rates k_1's Impact on Supply Chain Members and System

r	1	3	6	9	12	15	18	21	24	27
k_1	0.25	0.5	0.6667	0.75	0.8	0.8333	0.8571	0.875	0.8889	0.9
π_R^{HC1}	4921.02	3280.68	2187.12	1640.34	1312.27	1093.56	937.338	820.17	729.04	656.136
$\pi_R^{H^*}$	2460.51	1640.34	1093.56	820.17	656.136	546.78	468.669	410.085	364.52	328.068
$\Delta\pi_R^1$	2460.51	1640.34	1093.56	820.17	656.134	546.78	468.669	410.085	364.52	328.068
π_M^{HC1}	2130.1	3441.73	4370.94	4847.28	5136.22	5330.03	5469.01	5573.53	5655	5720.27
$\pi_M^{H^*}$	3360.36	4261.9	4917.72	5257.37	5464.29	5603.42	5703.34	5778.58	5837.26	5884.3
$\Delta\pi_M^1$	-1230.26	-820.17	-546.78	-410.09	-328.07	-273.39	-234.33	-205.05	-182.26	-164.03
π_{SC}^{HC}	7051.12	6722.41	6558.06	6487.62	6448.49	6423.59	6406.35	6393.7	6384.04	6376.41
$\pi_{SC}^{H^*}$	5820.87	5902.25	6011.28	6077.54	6120.42	6150.2	6172.01	6188.66	6201.78	6212.37
$\Delta\pi_{SC}$	1230.25	820.16	546.78	410.08	328.07	273.39	234.34	205.04	182.26	164.04

The relationship between the parameters is shown in Figure 2 and Figure 3.

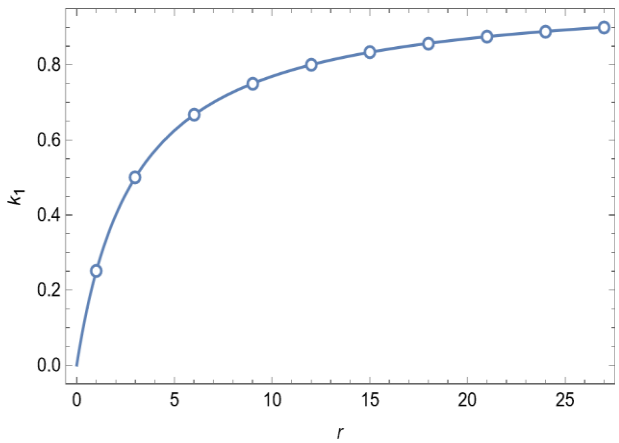

Figure 2. The Trend of k_1 Concerning r

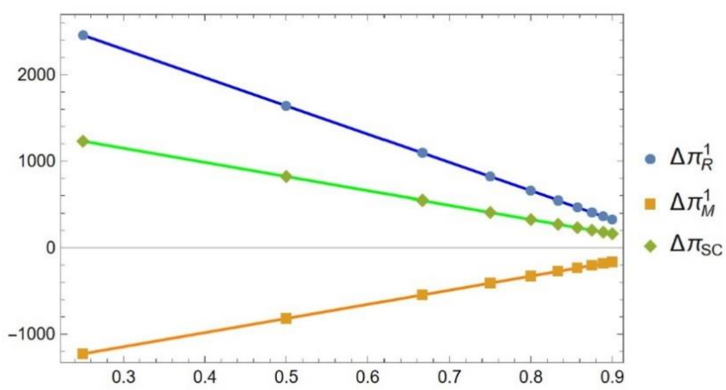

Figure 3. The Trend of $\Delta\pi_R^1$, $\Delta\pi_M^1$ and $\Delta\pi_{SC}$ with Respect to k_1

As shown in Figure 2, as r increases, k_1 also increases, i.e., the wholesale price discount rate $k_1 = r/(b+r)$ increases monotonically with the channel shift coefficient r's increment.

From Table 3, the profit of the brand π_R^{HC1} decreases as the channel transfer coefficient r increases under wholesale price discount coordination, and the manufacturer's profit π_M^{HC1} increases as the channel transfer coefficient r increases.

According to the value range of $\Delta\pi_R^1$, $\Delta\pi_M^1$, and $\Delta\pi_S$. In Figure 3, we know that the profit of the brand retailer and supply chain system is larger than before after the wholesale price discount coordination, and the profit of manufacturers is smaller than before; that is, the revenue increment of the brand retailer and supply chain system has been greater than zero, and the revenue reduction of the manufacturer has been greater than zero too. However, with the increase of the k_1's value, the incremental profit of the brand retailer decreases, and the profit reduction of the manufacturer also decreases than before, the total incremental profit of the hybrid channel supply chain system decreases. Therefore, the smaller the channel transfer coefficient r is, the closer the k_1's value is to the lower limit, which is more beneficial to the total profit increase of the hybrid channel supply chain system.

6.2 About The Revenue Compensation Factor k_2

Because the value of k_1 changes as r changes, to further analyze the role of the revenue compensation factor k_2 under the wholesale price discount rate, it is helpful to take $r = 6$ from Table 3, where $k_1 = 0.6667$ and corresponding $\pi_R^{HC1} = 2187.12$, $\pi_M^{HC1} = 4370.94$, yielding the effect of the revenue compensation coefficient k_2 on the returns of members of the hybrid channel supply chain, as shown in Table 4 and Figure 4.

Table 4. The Impact of Revenue Compensation Factors k_2 on Supply Chain Members and System Revenue

k_2	0.25	0.3	0.35	0.4	0.45	0.5
π_R^{HC2}	1640.34	1530.98	1421.63	1312.27	1202.92	1093.56
$\pi_R^{H^*}$	1093.56					
$\Delta\pi_R^2$	546.78	437.42	328.07	218.71	109.36	0
π_M^{HC2}	4917.72	5027.08	5136.43	5245.79	5355.14	5464.5
$\pi_M^{H^*}$	4917.72					
$\Delta\pi_M^2$	0	109.36	218.71	328.07	437.42	546.78
$\Delta\pi_{SC}$	546.78					

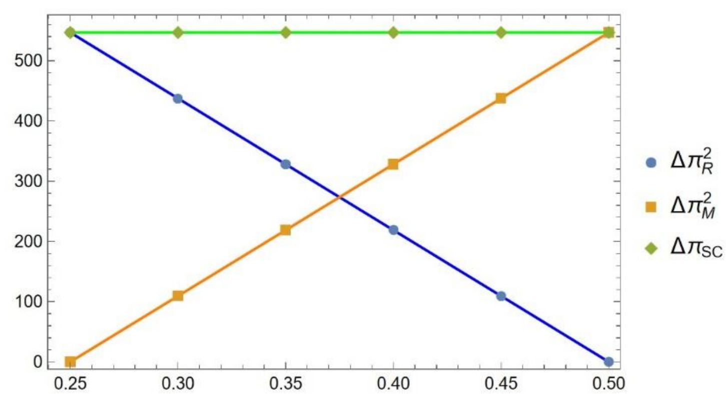

Figure 4. The Trend of $\Delta\pi_R^2$, $\Delta\pi_M^2$ and $\Delta\pi_{SC}$ with Respect to k_2

As shown in Figure 4, when $k_2 = 0.25$, the incremental profit of the brand is precisely equal to the incremental profit of the supply chain system, while the profit of the manufacturer is precisely equal to its profit under the decentralization decision before coordination. As the value of the revenue compensation coefficient k_2 increases, the decrease in the incremental profit of the brand is precisely equal to the increase in the incremental profit of the manufacturer; when $k_2 = 0.5$, the brand's profit is precisely equal to its profit in the pre-coordination decentralization decision, and the manufacturer's incremental profit is precisely equal to the incremental profit of the total supply chain system.

7. CONCLUSIONS AND FURTHER RESEARCH

This paper constructs demand functions for direct sales channels and traditional channels in a hybrid channel. Then, assuming that the retailer is the dominant player in the game, this study compares and analyses the optimal price, market demand, and profit of supply chain members under different decisions and finds that the competition between direct sales channels and traditional channels under decentralized decision will lead to inter-channel conflict, which directly affects the efficiency of the supply chain. To mitigate this conflict and enable channel coordination in hybrid channel supply chains, the brand retailer requires the manufacturer to give themselves a discount on the wholesale price $k_1\left(\frac{r}{b+r}\right)$ while keeping the price of the direct sales channel unchanged. The brand retailer sells products at the optimal price under the centralized decision and compensates the manufacturer for a proportional share of the proceeds $k_2(0.25 < k_2 < 0.5)$.

From the study of wholesale price discounts and revenue compensation mechanisms, it can be shown that the brand retailer can maximize the total profit of the hybrid channel supply chain by designing the contract correctly and effectively incentivizing the manufacturer's cooperation to achieve a win-win situation for the supply chain members.

Although the effectiveness of the designed coordination mechanism is validated by numerical analysis, further research is still needed. For example, since the brand retailer owns the core technology and reprocesses the products produced by the original equipment manufacturer, it is possible to consider the case where there are quality differences in the products sold in hybrid channel supply chains, then embed consumer quality utility model for decision making and coordination, thus benefiting the brand retailer, the manufacturer, and the consumer at the same time. In addition, the external regional market of this paper can be specifically referred to as the international market, and the local internal regional market is specifically referred to as the domestic market. When brand retailers sell their products to the international market, the tariffs and export rebates can be considered in the model, which provides a new direction for subsequent research.

REFERENCES

Abdulla, H., Ketzenberg, M. E., and Abbey, J. D. (2019). Taking Stock of Consumer Returns: A Review and Classification of The Literature. *Journal of Operations Management*, 65(6): 560-605.

Amrouche, N., Pei, Z., and Yan, R. L. (2022). Mail-In-Rebate and Coordination Strategies for Brand Competition. *International Journal of Production Economics*, 247(13): 147-162.

Chen, J., Zhang, H., and Sun, Y. (2012). Implementing Coordination Contracts in A Manufacturer Stackelberg Dual-Channel Supply Chain. *Omega*, 40(5): 571-583.

Chen, J. X., Liang, L., Yao, D. Q., and Sun, S.N. (2017). Price and Quality Decisions in Dual-Channel Supply Chains. *European Journal of Operational Research*, 259(21): 935-948.

Chen, X., Zhang, H., Zhang, M., and Chen, J. (2017). Optimal Decisions in A Retailer Stackelberg Supply Chain. *International Journal of Production Economics*, 187(6): 260-270.

Chiang, W. K., Chhajed, D., and Hess, J. D. (2003). Direct Marketing, Indirect Profits: A Strategic Analysis of Dual-Channel Supply-Chain Design. *Management Science*, 49(1): 1-20.

Choi, T. M., Wang, Y. J., and Shen, B. (2022). Selling to Profit-Target-Oriented Retailers: Optimal Supply Chain Contracting with Bargaining. *International Journal of Production Economics*, 250(26): 216-232.

David, A. and Adida, E. (2015). Competition and Coordination in A Two-Channel Supply Chain. *Production & Operations Management*, 24(8): 1358-1370.

Dumrongsiri, A., Fan, M., Jain, A., and Moinzadeh, K. (2008). A Supply Chain Model with Direct and Retail Channels. *European Journal of Operational Research*, 187(3): 691-718.

Huang, J., Xiao, T. J., and Sheng, Z. H. (2009). A Research Review of Multi-Channel Supply Chain Management. *Scientific Research Management*, 30(5): 25-32.

Jiang, J. D., Li, B. Y., and He, Y. (2016). A Brand-Led Collaborative Study on Price Discounts and Revenue Compensation in The OEM Supply Chain of Mixed Channels. *Journal of Management Engineering*, 30(4): 145-152.

Kurata, H., Yao, D. Q., and Liu, J. J. (2007). Pricing Policies Under Direct Vs. Indirect Channel Competition and National Vs. Store Brand Competition. *European Journal of Operational Research*, 180(1): 262-281.

Lee, H. H., Chang, T. K, Jean, K., and Kuo, C. W. (2022). Channel Design and OEM Growth in A Multi-Market Setup. *European Journal of Operational Research*, 296(3): 862-872.

Li, B., Chen, P., Li, Q. H., and Wang, W. G. (2014). Dual-Channel Supply Chain Pricing Decisions with A Risk-Averse Retailer. *International Journal of Production Research*, 52(23): 7132-7147.

Liu, W. H., Liang, Y. J., Tang, O., and Ma, X. (2022). Channel Competition and Collaboration in The Presence of Hybrid Retailing. *Transportation Research Part E: Logistics and Transportation Review*, 160(13): 35-49.

Matsui, K. (2017). When Should A Manufacturer Set Its Direct Price and Wholesale Price in Dual-Channel Supply Chains? *European Journal of Operational Research*, 258(27): 501-511.

Park, S. Y. and Keh, H. T. (2003). Modeling Hybrid Distribution Channels: A Game-Theoretic Analysis. *Journal of Retailing and Consumer Services*, 10(3): 155-167.

Pu, X. J., Gong, L., and Han, X. H. (2017). Consumer Free Riding: Coordinating Sales Efforts in A Dual-Channel Supply Chain. *Electronic Commerce Research and Applications*, 22(4): 1-12.

Shafiq, M. and Luong, H. T. (2021). Optimal Quantity Allocation Decision in Revenue Sharing Contract. *International Journal of Industrial Engineering: Theory, Applications and Practice*, 28(6): 644-664.

Shang, W. F. and Yang, L. (2015). Contract Negotiation and Risk Preferences in Dual-Channel Supply Chain Coordination. *International Journal of Production Research*, 53(16): 4837-4856.

Tsao, Y.C., Linh, V.T., and Chen, T.H. (2022). Duopoly Market Competition of Pricing and Service Policies Under A Dual-Sale Channel. *International Journal of Industrial Engineering: Theory, Applications and Practice*, 29(2): 267-282.

Xiao, T. J., Choi, T. M., and Cheng, T. C. E. (2014). Product Variety and Channel Structure Strategy for A Retailer-Stackelberg Supply Chain. *European Journal of Operational Research*, 233(1): 114-124.

Xu, G. Y., Dan, B., Zhang, X. M., and Liu, C. (2014). Coordinating A Dual-Channel Supply Chain with Risk-Averse Under A Two-Way Revenue-Sharing Contract. *International Journal of Production Economics*, 147(7): 171-179.

Yao, D. Q. and Liu, J. J. (2005). Competitive Pricing of Mixed Retail and E-Tail Distribution Channels. *Omega*, 33(3): 235-247.

Yu, M. C. and Goh, M. (2014). A Multi-Objective Approach to Supply Chain Visibility and Risk. *European Journal of Operational Research*, 233(1): 125-130.

Zhang, P., He, Y., and Shi, C. V. (2017). Retailer's Channel Structure Choice: Online, Offline or Dual Channels. *International Journal of Production Economics*, 191(6): 37-50.

Zhang, P., Xiong, Y., and Xiong, Z. K. (2015). Coordination of A Dual-Channel Supply Chain After Demand or Production Cost Disruptions. *International Journal of Production Research*, 53(10): 3141-3160.

APPENDIX A - THE SPECIFIC EXPANSION EQUATIONS

Equation 1.

$$\max_{p^{ID},p^{IT}} \pi_{SC}^{I} = (p^{ID}-c)Q^{ID} + (p^{IT}-c)Q^{IT} = -(b+r){p^{ID}}^2 - (b+r){p^{IT}}^2 + (\beta-\delta+bc)p^{ID} + (\alpha+\delta+bc)p^{IT} + 2rp^{ID}p^{IT} - (\alpha+\beta)c.$$

Equation 2.

$$\pi_{SC}^{I^*} = \frac{2b^3c^2 + 2b^2c(2cr-\alpha-\beta) + r(\alpha+\beta)^2 + b[\alpha^2+\beta^2-4cr(\alpha+\beta)+2\alpha\delta+2\delta(\delta-\beta)]}{4b(b+2r)}$$

Equation 3.

$$\pi_{SC}^{H^*} = \frac{7b^4c^2+4r^2(\alpha+\beta)^2+2b^3c(11cr-3\alpha-4\beta+\delta)+2br[3\alpha^2+4\alpha\beta+4\beta^2-8cr(\alpha+\beta)+2\alpha\delta-4\beta\delta+3\delta^2]+b^2[16c^2r^2+3\alpha^2+4\beta^2+6\alpha\delta-8\beta\delta+7\delta^2+4cr(\delta-5\alpha-6\beta)]}{16b(b+r)(b+2r)}.$$

Equation 4.

$$Q^{ID^*} - Q^{HD^*} = \frac{r(bc-\alpha-\delta)}{4(b+r)}$$

Equation 5.

$$\pi^{ID^*} - \pi^{HD^*} = \frac{r(\alpha-bc+\delta)[b^2c-r(\alpha+\beta)+b(2cr-\beta+\delta)]}{8b(b+r)(b+2r)}$$

Equation 6.

$$w^{I^*} - w^{H^*} = \frac{b^3c+2br(2cr-\alpha-\beta)-2r^2(\alpha+\beta)+b^2(4cr-\alpha-\delta)}{4b(b+r)(b+2r)}$$

Equation 7.

$$p^{IT^*} - p^{HT^*} = \frac{bc-\alpha-\delta}{4(b+r)}$$

Equation 8.

$$Q^{IT^*} - Q^{HT^*} = \frac{1}{4}(\alpha-bc+\delta)$$

Equation 9.

$$\pi_M^{HT^*} - \pi_M^{IT^*} = \frac{(\alpha-bc+\delta)[2r^2(\alpha+\beta)-b^3c+2br(\alpha+\beta-2cr)+b^2(\alpha+\delta-4cr)]}{16b(b+r)(b+2r)}$$

Equation 10.

$$\pi_R^{IT^*} - \pi_R^{HT^*} = \frac{(\alpha-bc+\delta)[2r^2(\alpha+\beta)-b^3c+2br(\alpha+\beta-2cr)+b^2(\alpha-4cr+\delta)]}{8b(b+r)(b+2r)}$$

Equation 11.

$$\pi^{IT^*} - \pi^{HT^*} = \frac{[b^3c + 2br(2cr - \alpha - \beta) - 2r^2(\alpha + \beta) + b^2(4cr - \alpha - \delta)](bc - \alpha - \delta)}{16b(b + r)(b + 2r)}$$

Equation 12.

$$\pi_{SC}^{I^*} - \pi_{SC}^{H^*} = \frac{(\alpha - bc + \delta)^2}{16(b + r)}$$

Equation 13.

$$\pi_M^{HC} = (p^{HDC} - c)\{\beta - bp^{HDC} + [c + g^{HC} - ck_1 + (k_1 - 1)p^{HDC}]r - \delta + k_1[\alpha + \delta - b(c + g^{HC} - ck_1 + k_1p^{HDC}) - (c + g^{HC} - ck_1 + k_1p^{HDC} - p^{HDC})r]\}$$

Equation 14.

$$p^{HDC^*} = \frac{\begin{array}{c} b^3c\{4+k_1[9k_1+k_1{}^2(4k_1-1)-2]\}+(k_1-1)^2r^2[4c(k_1-1)^2r+(2+k_1)\alpha+3\beta+(k_1-1)\delta]+b^2\{2c[7+3k_1{}^2(5+2k_1{}^2-3k_1)-11k_1]r+4(\beta-\delta) \\ +3k_1{}^2(\beta-\delta)+2k_1(\alpha+\delta)+k_1{}^3(\alpha+\delta)\}+br\{3c(k_1-1)^2[5+k_1(4k_1-3)]r+2[(1+2k_1+k_1{}^3)\alpha+4\beta+3\beta(k_1-1)k_1+(k_1-1)(3+k_1{}^2-2k_1)\delta]\} \end{array}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k_1]r+(k_1-1)^2r^2\}}.$$

Equation 15.

$$p^{HTC^*} = \frac{\begin{array}{c} b^3c[2+(k_1-1)k_1](2+k_1{}^2)+(k_1-1)^2r^2\{4c(k_1-1)^2r+2(2\alpha+\beta+\delta)+k_1[(3k_1-4)\alpha+\beta-5\delta+3k_1\delta]\}+b^2\{2c[7+k_1(10k_1+3k_1{}^3-4k_1{}^2-8)]r+(2+k_1{}^2)[2\alpha+ \\ 3k_1{}^2\alpha+k_1\beta+(2+3k_1{}^2-k_1)\delta]\}+br\{c(k_1-1)^2[14+k_1(9k_1-5)]r+2[4\alpha+(k_1-1)\alpha k_1(6+3k_1{}^2-2k_1)+\beta+k_1(2+k_1{}^2)\beta+3\delta+k_1(8k_1+3k_1{}^3-6k_1{}^2-8)\delta]\} \end{array}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k]r+(k_1-1)^2r^2\}}.$$

Equation 16.

$$w^{HC^*} = \frac{\begin{array}{c} b^3c[2+(k_1-1)k_1](4+3k_1{}^2)+(k_1-1)^2r^2\{4c(k_1-1)^2r+k_1[(2+k_1)\alpha+3\beta+(k_1-1)\delta]\}+br\{c(k_1-1)^2[20+k_1(11k_1-13)]r+2k_1[(1+2k_1+k_1{}^3) \\ \alpha+4\beta+3\beta(k_1-1)k_1+(k_1-1)(3+k_1{}^2-2k_1)\delta]\}+b^2\{2c[12+k_1(19k_1+5k_1{}^3-11k_1{}^2-17)]r+k_1[4(\beta-\delta)+3k_1{}^2(\beta-\delta)+2k_1(\alpha+\delta)+k_1{}^3(\alpha+\delta)]\} \end{array}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k]r+(k_1-1)^2r^2\}}.$$

Equation 17.

$$\pi_{SC}^{HC^*} = \frac{b(b + 2r)(\alpha - bc + \delta)^2 + [b^2c - r(\alpha + \beta) + b(2cr - \beta + \delta)]^2}{4b(b + r)(b + 2r)}$$

APPENDIX B - PROCESS OF PROOF

Lemma 1. (1) The specific solution process is as follows: based on the total profit function of the supply chain system, the derivation shows that the objective function is a joint concave function of p^{ID} and p^{IT}, so there exists an optimal retail price to maximize the objective function. We solve the system of first-order partial derivative equations $\frac{\partial \pi_{SC}^{I}}{\partial p^{ID}} = 0$ and $\frac{\partial \pi_{SC}^{I}}{\partial p^{IT}} = 0$, and find the optimal unit product price for the direct and traditional channels, respectively as: $p^{ID^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\beta-\delta)}{2b(b+2r)}$, $p^{IT^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\alpha+\delta)}{2b(b+2r)}$.

Lemma 1. (2) Substituting the optimal price of the direct and traditional channels under the centralized decision-making into the market demand equations, the optimal demand for the direct and traditional channels is yielded respectively as: $Q^{ID^*} = \frac{\beta-\delta-bc}{2}$, $Q^{IT^*} = \frac{\delta-bc+\alpha}{2}$.

Lemma 1. (3) Substituting the optimal price and the optimal demand of the direct sales channel under centralized decision into the profit equation, we obtain the optimal profit of the direct channel: $\pi^{ID^*} = \frac{(bc-\beta+\delta)(b^2c+2bcr-r\alpha-b\beta-r\beta+b\delta)}{4b(b+2r)}$, and similarly, the optimal profit of the traditional channel can be obtained as: $\pi^{IT^*} = \frac{(\delta-bc+\alpha)(r\alpha+r\beta-b^2c-2bcr+b\alpha+b\delta)}{4b(b+2r)}$.

Lemma 2. (1) (2) The respective profit decision objective functions of the retailer and the manufacturer under decentralized decision-making can be solved by using the inverse-order solution method. Firstly, combining the demand equations of the direct and traditional channels under decentralized decision-making, and $p^{HT} = w^H + g^H$, then substituting them into the manufacturer's profit function, we obtain:

$$\pi_M^H = (p^{HD} - c)[\beta - bp^{HD} - \delta + r(p^{HT} - p^{HD})] + (w - c)[\alpha - bp^{HT} + \delta + r(p^{HD} - p^{HT})] = (p^{HD} - c)[\beta - bp^{HD} - \delta + r(w^H + g^H - p^{HD})] + (w - c)[\alpha - b(w^H + g^H) + \delta + r(p^{HD} - w^H - g^H)].$$

The derivation shows that the manufacturer's profit objective function is a joint concave function of p^{HD} and p^{HT}, so there exists an optimal retail price to maximize the objective function. We solve the system of first-order partial derivative equations $\frac{\partial \pi_M^H}{\partial p^{HD}} = 0$, and, $\frac{\partial \pi_M^H}{\partial p^{HT}} = 0$, and find the optimal price of the direct channel and the optimal wholesale price as: $p^{HD^*} = \frac{b^2c+r(\alpha+\beta)+b(2cr+\beta-\delta)}{2b(b+2r)}$, $w^{H^*} = \frac{b^2(c-g^H)+r(\alpha+\beta)+b(2cr-2g^Hr+\alpha+\delta)}{2b(b+2r)}$. Taking the optimal price of the direct sales channel p^{HD^*}, the optimal wholesale price w^{H^*}, and $p^{HT} = w^H + g^H$ into the brand retailer's profit under decentralized decision, we obtain: $\pi_R^H = g^H[\alpha - bp^{HT} + \delta + r(p^{HD} - p^{HT})] = \frac{1}{2}g^H[\alpha + \delta - g^Hr - b(c + g^H)]$.

Let $\frac{\partial \pi_R^H}{\partial g^H} = 0$, and the optimal profit level per product unit is solved as: $g^{H^*} = \frac{\alpha-bc+\delta}{2(b+r)}$. Put g^{H^*} into the wholesale price just found $w^{H^*} = \frac{b^2(c-g^H)+r(\alpha+\beta)+b(2cr-2g^Hr+\alpha+\delta)}{2b(b+2r)}$, the optimal wholesale price is obtained as: $w^{H^*} = \frac{3b^3c+2r^2(\alpha+\beta)+2br(2cr+\alpha+\beta)+b^2(8cr+\alpha+\delta)}{4b(b+r)(b+2r)}$.

From the optimal profit level per product unit g^{H^*}, and the optimal wholesale price w^{H^*}, the optimal unit product price of the traditional channel is obtained as follows: $p^{HT^*}=\frac{b^3c+2r^2(\alpha+\beta)+2br(2cr+3\alpha+\beta+2\delta)+b^2[4cr+3(\alpha+\delta)]}{4b(b+r)(b+2r)}$.

Lemma 2. (3) Substituting the optimal price of the direct and traditional channels under the decentralized decision-making into the market demand equations, the optimal demand for the direct and traditional channels is yielded respectively as: $Q^{HD^*}=\frac{b(2\beta-3cr-2\delta)-2b^2c+r(\alpha+2\beta-\delta)}{4(b+r)}$, $Q^{HT^*}=\frac{1}{4}(\alpha-bc+\delta)$.

Lemma 2. (4) From the optimal price p^{HD^*} and the optimal demand Q^{HD^*} of the direct sales channel under decentralized decision, it is obtained that the optimal profit of the direct sales channel is:

$$\pi^{HD^*}=\pi_M^{HD^*}=\frac{[b^2c-r(\alpha+\beta)+b(2cr-\beta+\delta)][2b^2c+r(\delta-\alpha-2\beta)+b(3cr-2\beta+2\delta)]}{8b(b+r)(b+2r)}.$$

From the optimal unit product profit level for brand retailer g^{H^*} and the optimal demand of the traditional channel Q^{HT^*}, it follows that the manufacturer's profit in the traditional channel is:

$$\pi_M^{HT^*}=\frac{(\alpha-bc+\delta)[2r^2(\alpha+\beta)-b^3c+2br(\alpha+\beta-2cr)+b^2(\alpha+\delta-4cr)]}{16b(b+r)(b+2r)}.$$

From the optimal profit of the manufacturer in the direct sales channel $\pi_M^{HD^*}$ and the manufacturer's optimal profit in the traditional channel $\pi_M^{HT^*}$, it follows that the total profit of the manufacturer in the hybrid channel is:

$$\pi_M^{H^*}=\frac{5b^4c^2+4r^2(\alpha+\beta)^2+2br[(\alpha+2\beta-\delta)^2-8cr(\alpha+\beta)]+2b^3c(9cr-\alpha-4\beta+3\delta)+b^2[16c^2r^2+\alpha^2+4\beta^2-12cr(\alpha+2\beta-\delta)+2\alpha\delta-8\beta\delta+5\delta^2]}{16b(b+r)(b+2r)}.$$

From the optimal unit product profit level for brand retailer g^{H^*} and the optimal demand of the traditional channel Q^{HT^*}, it follows that the profit of the brand retailer in the traditional/hybrid channel is: $\pi_R^{H^*}=\pi_R^{HT^*}=\frac{(\alpha-bc+\delta)^2}{8\,(b+r)}$.

The total profit of the traditional channel can be obtained by the profit of the manufacturer and the brand retailer in the traditional channel $\pi_M^{HT^*}$, and, $\pi_R^{HT^*}$: $\pi^{HT^*}=\frac{(bc-\alpha-\delta)\,\{3b^3c-2r^2(\alpha+\beta)+2br(2cr-3\alpha-\beta-2\delta)+b^2\,[8cr-3(\alpha+\delta)]\}}{16b(b+r)(b+2r)}$.

Proposition 1. Since the primary function for ordinary market demand is $\alpha/\beta-bp$ and $p>c$, therefore $\alpha>bc$, $\beta>bc$; and because $\beta\geq\delta+bp^T$, $p^T>c$, so $\beta\geq\delta+bc$, therefore $b\beta-b\delta-b^2c>0$, the next propositions prove similarly.

Lemma 3. Based on the profit function of the manufacturer under decentralized decision-making after coordination, the derivation shows that the objective function is a concave function of p^{HDC}, so there exists an optimal retail price to maximize the objective function. Let $\frac{\partial\pi_M^{HC}}{\partial p^{HDC}}=0$, and find the optimal unit product price for the direct sales channel as: $p^{HDC^*}=\frac{b[c-(c+g^{HC})k_1+2c{k_1}^2]+g^{HC}r+2c(k_1-1)^2r-g^{HC}k_1r+k_1\alpha+\beta+(k_1-1)\delta}{2b(1+{k_1}^2)+2(k_1-1)^2r}$. Combining the market demand of the traditional channel after coordination Q^{HTC}, p^{HDC^*}, $p^{HTC}=w^{HC}+g^{HC}$, and, $w^{HC}=k_1(p^{HDC}-c)+c$, then substituting them into the profit of the retailer π_R^{HC}, we can obtain: $\pi_R^{HC}=g^{HC}*$

$$\frac{b[2(\alpha+\delta)-c(k_1-3)(k_1-1)r-2g^{HC}(2+{k_1}^2-k_1)r+k_1(\delta-\beta+\alpha k_1+\delta k_1)]-b^2[2c+(k_1-1)ck_1+g^{HC}(2+{k_1}^2)]-(k_1-1)r[g^{HC}(k_1-1)r+2\alpha+\beta+\delta-k_1(\alpha+\delta)]}{2b(1+{k_1}^2)+2(k_1-1)^2r}.$$

The derivation shows that the profit of the retailer π_R^{HC} is a concave function of g^{HC}, so let $\frac{\partial\pi_R^{HC}}{\partial g^{HC}}=0$, the optimal profit level per unit of product is found as:

$$g^{HC^*}=\frac{b^2c(k_1-2-k_1{}^2)+(k_1-1)r[(k_1-2)\alpha-\beta+(k_1-1)\delta]+b\{2(\alpha+\delta)-c(k_1-3)(k_1-1)r+k_1[\delta-\beta+k_1(\alpha+\delta)]\}}{2b^2(2+k_1{}^2)+4b[2+(k_1-1)k_1]r+2(k_1-1)^2r^2}.$$

Put g^{HC^*} into p^{HDC^*}, the optimal unit product price for the direct sales channel is obtained as:

$$p^{HDC^*}=\frac{\begin{gathered}b^3c(4+9k_1{}^2+4k_1{}^4-k_1{}^3-2k_1)+(k_1-1)^2r^2[4c(k_1-1)^2r+2\alpha+\alpha k_1+3\beta+11k_1]r+4(\beta-\delta)+3k_1{}^2(\beta-\delta)+2k_1(\alpha+\\ \delta)+k_1{}^3(\alpha+\delta)\}+br\{3c(k_1-1)^2(5+4k_1{}^2-3k_1)r+2[(1+2k_1+k_1{}^3)\alpha+4\beta+3\beta(k_1-1)k_1+(k_1-1)(3+k_1{}^2-2k_1)\delta]\}\end{gathered}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k_1]r+(k_1-1)^2r^2\}}.$$

Then, we can solve the optimal wholesale price for the traditional channel through the optimal unit product price of the direct sales channel p^{HDC^*}:

$$w^{HC^*}=\frac{\begin{gathered}b^3c[2+(k_1-1)k_1](4+3k_1{}^2)+(k_1-1)^2r^2\{4c(k_1-1)^2r+k_1[(2+k_1)\alpha+3\beta+(k_1-1)\delta]\}+br\{c(k_1-1)^2[20+k_1\\(11k_1-13)]r+2k_1[(1+2k_1+k_1{}^3)\alpha+4\beta+3\beta(k_1-1)k_1+(k_1-1)(3+k_1{}^2-2k_1)\delta]\}+b^2\{2c[12+k_1\\(19k_1+5k_1{}^3-11k_1{}^2-17)]r+k_1[4(\beta-\delta)+3k_1{}^2(\beta-\delta)+2k_1(\alpha+\delta)+k_1{}^3(\alpha+\delta)]\}\end{gathered}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k]r+(k_1-1)^2r^2\}}.$$

From the optimal profit level per product unit g^{HC^*} and the optimal wholesale price of the traditional channel w^{HC^*}, the optimal price for the traditional channel is obtained as follows:

$$p^{HTC^*}=\frac{\begin{gathered}b^3c[2+(k_1-1)k_1](2+k_1{}^2)+(k_1-1)^2r^2\{4c(k_1-1)^2r+2(2\alpha+\beta+\delta)+k_1[(3k_1-4)\alpha+\beta-5\delta+3k_1\delta]\}+b^2\{2c\\ [7+k_1(10k_1+3k_1{}^3-4k_1{}^2-8)]r+(2+k_1{}^2)[2\alpha+3k_1{}^2\alpha+k_1\beta+(2+3k_1{}^2-k_1)\delta]\}+br\{c(k_1-1)^2\\ [14+k_1(9k_1-5)]r+2[4\alpha+(k_1-1)\alpha k_1(6+3k_1{}^2-2k_1)+\beta+k_1(2+k_1{}^2)\beta+3\delta+k_1(8k_1+3k_1{}^3-6k_1{}^2-8)\delta]\}\end{gathered}}{4[b(1+k_1{}^2)+(k_1-1)^2r]\{b^2(2+k_1{}^2)+2b[2+(k_1-1)k]r+(k_1-1)^2r^2\}}.$$

Lemma 4. (2) Substituting the wholesale price discount rate k_1 and the optimal price of direct channel p^{HDC} into the equation $w^{HC^*}=k_1(p^{HDC}-c)+c$, the wholesale price that the manufacturer gets from the branded retailer can be obtained as: $w^{HC^*}=\frac{2b^3c+5b^2cr+r^2(\alpha+\beta)+br(2cr+\beta-\delta)}{2b(b+r)(b+2r)}$. Then, we can find that the profit per unit of product for brand retailers is: $g^{HC^*}=p^{HTC}-w^{HC}=\frac{\alpha-bc+\delta}{2(b+r)}$. Therefore, under the wholesale price discount mechanism, substituting the optimal results into equations π_M^{HC} and π_R^{HC}, we can get the profits of the manufacturer and the brand as: $\pi_M^{HC1^*}=\frac{[b^2c-r\,(\alpha+\beta)+b\,(2cr-\beta+\delta)]^2}{4b(b+r)(b+2r)}$, $\pi_R^{HC1^*}=\frac{(\alpha-bc+\delta)^2}{4(b+r)}$.