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DECISION MAKING APPROACH FOR BEST MOBILE PHONE SERVICE PROVIDER SELECTION USING LAPLACIAN ENERGY AND COSINE SIMILARITY MEASURES OF HESITANCY FUZZY GRAPH

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Telecommunication is one of the essential necessities of everyday life. In India, the telecommunications sector has seen a significant increase in the day-to-day. Telecommunications service companies hold data about their customers, and crisp graphs are used to depict these records. Examining and selecting the best mobile phone service providers (MPSPs) based on operational restrictions will help determine the best MPSPs. The analysis of MPSPs may be regarded as a difficult decision-making issue. The aim of this article is to provide an outline to examine the performance of MPSPs and the selection of the best MPSP for customers in India. The statistical data were obtained from the Telecom Regulatory Authority of India between April 2019 and March 2021. A novel approach for cosine similarity measures (CSM) among hesitancy fuzzy graphs (HFG) and estimating the certified repute scores of the experts by determining the ambiguous information of hesitancy fuzzy preference relations (HFPRs) and the regular cosine similarity grades from one separable HFPR to some others. And consider "objective" and "subjective" information given by experts. According to CSMs, we define the Laplacian energy of an HFG. This research provides a solution to a decision-making problem by applying the newly developed cosine similarity measure and the Laplacian energy of hesitancy fuzzy graphs. The ranking order of all alternatives and the best one is determined by calculating the cosine similarity between each alternative and the ideal alternative. Finally, an illustrated example is provided to show the applicability of the proposed approach to the decision-making problem as well as its effectiveness.

Keywords: Hesitancy Fuzzy Graph; Cosine Similarity Measures; Hesitancy Fuzzy Preference Relationships; Mobile Phone Service Provider; Laplacian Energy.

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1. INTRODUCTION

Communication is one of the most important aspects of human society and culture. Human society is developed with faster communication systems. Telecommunication is one of the most necessary and unavoidable systems in our daily lives. Some telecom companies preferred by people are Indian Mobile, Jio, Airtel, Cellone (BSNL), and VI.

As of January 2020, Jio is the largest mobile phone service provider in India, with about 426.2 million subscribers. Bharti Airtel Limited, commonly known as Airtel, is an Indian telecommunications company that operates in 20 countries across South Asia, Africa, and the Channel Islands. Airtel is the fifth-largest telecom service provider in the world, with over 327.3 million customers across 20 countries as of March 2012. Airtel is the second-largest mobile telecommunications company, measured by both subscribers and 2019 revenues, and had 439 million subscribers as of December 2019. At the end of 2019, Cellone Mobile had a pan-India presence in all 22 telecom circles in India. Cellone has 203 million subscribers, making it one of the largest mobile phone service providers in India. As of September 31, 2019, VI had a subscriber base of 119.8 million, making it the fourth-largest mobile telecommunications network in India and the 11th-largest mobile telecommunications network in the world.

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'Vodafone Idea Limited' was created on August 31, 2018 by the merger of Vodafone India and Idea Cellular. Customers' statistics are compiled by telecom service providers in order to discover stars and churners. It is extremely difficult and time-consuming to identify such individuals. Hadden *et al.* (2006) described complaint data-driven churn prediction. Nanavati *et al.* (2006) went on to explore the structural features of huge telecom call graphs. Dasgupta *et al.* (2008) then explored social links and their impact on churn in mobile telecom networks.

Zadeh (1965) modeled fuzzy notions by proposing the membership degree of each entity and introducing fuzzy sets. Atanassov (1986) generalized the fuzzy set and proposed the intuitionistic fuzzy set (IFS), which provides non-membership values with membership values. Torra V. (2010) presented the notion of a hesitant fuzzy set (HFS), which is another generalization of fuzzy sets in 2010. An HFS is denoted by a positive grade that takes the form of a finite subset of the closed interval [0, 1]. HFS and its applications have recently advanced significantly [Del Campo *et al.* (2017); Deepak, D., and Jhon, S.J. (2014); Faizi *et al.* (2017); Pei, Z., and Yi L. (2015); Qian *et al.* (2013); Thakur *et al.* (2014)]. Uncertainty is common in many real-world issues. Fuzzy sets [Rodriguez, R.M. Martinez, L., and Herrera, F] and their expansions have yielded promising solutions for dealing with uncertainty in a variety of applications. Rodriguez, R.M. *et al*. (2014) have focused on one of them, HFS, which addresses uncertain circumstances that frequently arise when the membership degree of an element in a set must be determined. Torra V. and Narukawa Y. (2009) defined several fundamental operations on HFS. Rodriguez, R.M. *et al.* (2011) developed the concept of hesitant fuzzy linguistic word sets. Furthermore, for linguistic fuzzy sets, a number of further expansions of the aforementioned distance metrics were devised by Xu, Z.S. (2005). HFS proves to be an effective technique for communicating uncertainty and ambiguity. Liao H.C. and Xu Z.S. (2014) suggested a group of hesitant fuzzy hybrid weighted aggregation operators to produce new hesitant fuzzy information, pointing out that these operators not only weigh the significance of the hesitant fuzzy arguments and their ordered roles simultaneously but also maintain the property of idempotency. Zhang (2013) introduced and explored a wide variety of hesitant fuzzy power aggregation operators. Liao and Xu (2014b) discovered that, while the HFHA and HFHG operators have some benefits over the HFWA, HFWG, HFOWA, HFOWG, HFHA, and HFHG operators in that they will always weigh all the provided assertions and about their arranged positions concurrently, they have a critical flaw in that they should not satisfy the fundamental property known as idempotency.

The preference relationship, being the most prevalent and dominant presentation of data, has piqued the interest of academics and has been extensively applied, particularly in multiple-criteria DM. Several other forms of preference relationships have been suggested up to this point, like the fuzzy preference relationship [Tanino, T. (1984)]. Atanassov K.T. (1999) introduced the ideas of intuitionistic fuzzy preference relations (IFPRs) and intuitionistic fuzzy graphs (IFGs). Karunambigai M.G. and Parvathi R. (2006) established IFGs and described operations among IFGs based on a specific instance of Atanassov's concept. Akram M. and Davvaz B. (2012) studied certain IFG features. Sahoo, S., and Pal, M. (2015) described certain IFG product operations and properties. Pathinathan T. *et al.* (2012) developed a novel graph structure termed HFG and explained various fundamental principles related to this structure. Despite the fact that Pathinathan *et al.* (2012) propose the notion of HFG, they do not allocate hesitant fuzzy elements (HFEs) to graph nodes and paths. They employ IF-values instead of HFEs, and these IF-values are represented as triples that include the membership, hesitancy, and non-membership degrees of nodes and paths. Given the fact that the HFS idea is close to the IFSs, there are some fundamental variations in their definitions and operations.

According to the imprecise information in human thinking, DM problems based on Zadeh's FSs were present in many aspects of organizations during the last century [Xu, Z.S. (2007); Braathen, S. *et al.* (2004); Xu, Z. *et al.* (2011)]. The DM offers its preferences on options involved in the group GDM approach using Zadeh's FSs. Preference relationships have been frequently employed in GDM [Chiclana, F *et al.*, (2001); Herrera-Viedma, E *et al.*, (2005); Fan, Z.P. *et al.*, (2010); Herrera-Viedma, E *et al.*, (2002)] as a simple technique for collecting and representing preferences. Rodriguez et al. created the hesitant fuzzy linguistic term set, explored its computational functions and characteristics, and used it for multi-criteria decision-making. In a DM issue, the HFS is more useful than the IFS in modeling reluctance in judgments about things. Likewise, the HFG is a generalization of the IFG and FG, but it is a more efficient approach than IFGs for modeling specific DM difficulties, such as DMs' reluctance in relation to nodes and paths. Xia, M., and Xu, Z. (2011) introduced the concept of hesitant fuzzy data aggregation in DM. The concept of similarity is important to human cognition. Similarity is important in recognition, taxonomy, and a variety of other domains. Numerous parts of the concept of similarity have remained unformalized. According to HFS, developing a substantial, generally relevant definition of similarity is a tough task. There is no acceptable, universally applicable definition of resemblance. There are a variety of specialized dentitions that have been successfully used in diagnostics, categorization, cluster analysis, and recognition. There are a variety of comparison metrics that are interpreted and used for various objectives [Xu, Z., and Xia, M. (2011)]. Farhadinia, B. (2014) used similarity and distance measurements for higher-order HFS. The idea of Laplacian energy of a fuzzy graph, proposed by Basha, S.S., and Kartheek, E. (2015), is extended to the Laplacian energy of an IFG. They also established an IFG's adjacency matrix, and an IFG's Laplacian energy is described in terms of its adjacency matrix. They also offered minimum and maximum values for an IFG's energy. Kartheek, E., and Sharief Basha, S. (2019) proposed that the idea of an IFG's

Laplacian energy be expanded to Laplacian energy in IFG operations. The Laplacian energy is also expanded in hesitancy fuzzy graphs from the Laplacian energy of intuitionistic fuzzy graphs on decision-making problems (Reddy NR; Basha S.S. (2023); Rajagopal Reddy N *et al.* (2023); Rajagopal Reddy N and Sharief Basha S (2023); Sharief Basha S *et al.* (2024)).

Han-Ying K. *et al.* (2021) presented an integrated approach to green supply chain network architecture in cloud computing platforms. Two phases are specified in this study: supply chain architecture and virtual machine allocation. In the first phase of the proposed two-stage approach, the supply chain network architecture choice is solved by considering three aims: reducing overall costs, decreasing carbon emissions, and optimizing service satisfaction levels. In the second step, based on the data requirements at the leader level, the placement of virtual machines on supply chain servers is determined by minimizing energy usage while maximizing physical machine efficacy. To validate the performance of the suggested methodologies, a case study and trials are carried out.

For the rest of this article, the structure is as follows: In Section 2o, preliminaries are presented. Section 3 describes the working procedure for decision-making problems (DMP) in HFPR. In Section 4, we discuss a real-time example using Laplacian energy of hesitancy fuzzy graphs based on cosine similarity measures in decision-making problems, and finally, Section 5 ends with the conclusion.

2. PRELIMINARIES

This section describes fundamental HGF ideas and terminologies, as well as a cosine SM for HFGs, which will be necessary for the systematic review.

Definition 1. Suppose $Y$is a finite nonempty set. An HFS on Ycanbe expressedas a function$h$, when implemented to Y gives a subset of [0,1] then the mathematical symbol is written as follows

$$E = \{\langle y, h_E(Y)\rangle | y\epsilon Y|\}$$

where $h_E(Y)$ is known as hesitant element and it is a collection of numbers in the range [0,1] indicating the membership degrees of the element $y\epsilon Y$to the set E.

Definition 2. Suppose $HG = (V, E, \mu, \gamma, \beta)$ is a HFG, where

(a) Consider $V = \{v_1, v_2, v_3 \dots v_n\}$ such that $\mu_1 : V \to [0,1]$,$\gamma_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$ are denotes the grade of membership, nonmembership and hesitant of the elements $t_i \in V$ and $\mu_1(t_i) + \gamma_1(t_i) + \beta_1(t_i) = 1$, where $\beta_1(t_i) = 1 - [\mu_1(t_i) + \gamma_1(t_i)]$ and $0 \le \mu_1(t_i) + \gamma_1(t_i) \le 1$

(b) Consider$E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$, $\gamma_2 : V \times V \to [0,1]$ and $\beta_2 : V \times V \to [0,1]$ are such that,
$\mu_2(t_i, t_j) \le \min[\mu_1(t_i), \mu_1(t_j)]$
$\gamma_2(t_i, t_j) \le \max[\gamma_1(t_i), \gamma_1(t_j)]$
$\beta_2(t_i, t_j) \le \min[\beta_1(t_i), \beta_1(t_j)]$
$and\ 0 \le \mu_2(t_i, t_j) + \gamma_2(t_i, t_j) + \beta_2(t_i, t_j) \le 1, \forall (t_i, t_j) \in E.$

Definition 3. Suppose $T = \{t_1, t_2, t_3 \dots t_n\}$ is a nonempty set, and then a HFPR H on T is obtainable by a matrix $H = (h_{ij})_{n\times n} \subset Y \times Y$, where $h_{ij} = \xi_{ij}{}^l$ for all $l = 1,2, \dots n$ is a HFE indicating the entire possible preference grade (s) of the objective t_i overt_j. Furthermore, h_{ij} must satisfy the following conditions:

$$\xi_{ij}{}^{\sigma(l)} + \xi_{ji}{}^{\sigma(n-l+1)} = 1, \xi_{ii} = 0, i = j = 1,2, \dots r$$

where $\xi_{ij}{}^{\sigma(l)}$is the l^{th} largest elements in h_{ij}.

Definition 4. Let $M = (r_{ij})_{n\times n}$ is the fuzzy preference relation (FPR), and then

$$r_{ij} = \frac{w_i}{w_i + w_j}, \qquad i, j \in N$$

where w_i is the weighting vector for the FPR $M = (r_{ij})_{n\times n}$ and $\sum_{i=1}^{n} w_i = 1,\ w_i > 0, i \in N$.

Definition 5. Consider a HFG $HG = (V, E, \mu, \gamma, \beta)$ and $(\alpha_i, \theta_i, \lambda_i)$ are the eigenroots of hesitancy fuzzy adjacency matrix $A(HG)$, then the Laplacian energy of HFG is indicated as follows as

$$LE(HG) = \left(LE\left(A_\mu(HG)\right), LE\left(A_\gamma(HG)\right), LE\left(A_\beta(HG)\right)\right)$$

where $A_\mu(HG), A_\gamma(HG)$, and $A_\beta(HG)$ is the membership matrix, nonmembership matrix, and hesitant element matrix of $A(HG)$ of HFG, α_i, θ_i and λ_i are the eigenroots of $A_\mu(HG)$, $A_\gamma(HG)$, and $A_\beta(HG)$ and also $LE\left(A_\mu(HG)\right), LE\left(A_\gamma(HG)\right)$, $LE\left(A_\beta(HG)\right)$ are Laplacian energy's of membership matrix $A_\mu(HG)$, nonmembership matrix $A_\gamma(HG)$, and hesitant element matrix $A_\beta(HG)$ of HFG. The Laplacian energy of $LE\left(A_\mu(HG)\right), LE\left(A_\gamma(HG)\right)$, $LE\left(A_\beta(HG)\right)$ of HFG is described as

$$LE\left(A_\mu(HG)\right) = \left|\alpha_i - \frac{2\sum_{1\le i\le j\le n}\mu(v_i, v_j)}{n}\right|$$
$$LE\left(A_\gamma(HG)\right) = \left|\theta_i - \frac{2\sum_{1\le i\le j\le n}\gamma(v_i, v_j)}{n}\right|$$
$$LE\left(A_\beta(HG)\right) = \left|\lambda_i - \frac{2\sum_{1\le i\le j\le n}\beta(v_i, v_j)}{n}\right|.$$

Definition 6. If P and R are HFGs, then the SMs from P to Rbe denoted as $S(P, R)$, it has the following characteristics:

i) $0 \le S(P, R) \le 1$; $(S1)$
ii) $S(P, R) = 1,\ iff A = B$; $(S2)$
iii) $S(P, R) = S(R, P)$; $(S3)$
iv) If $P \subseteq R \subseteq B$, then $S(P, B) \le S(P, R)$and$S(P, B) \le S(R, B)$. $(S4)$

Definition 7. Suppose that P and R are two HFGs in $T = \{t_1, t_2, \ldots., t_n\}$. Based on the extension of the CSMs for HFGs, then the weighted CSM between the HFGs P and R are defined as follows:

$$CS(P, R) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_P(t_i)\mu_R(t_i) + \gamma_P(t_i)\gamma_R(t_i) + \beta_P(t_i)\beta_R(t_i)}{\sqrt{\mu_P{}^2(t_i) + \gamma_P{}^2(t_i) + \beta_P{}^2(t_i)}\sqrt{\mu_R{}^2(t_i) + \gamma_R{}^2(t_i) + \beta_R{}^2(t_i)}}$$

where $\mu_P(t_i)$ is the degree of the membership element, $\gamma_P(t_i)$ be the degree of the nonmembership element and $\beta_P(t_i)$ be the degree of the hesitant element. Thus, the CSM between HFGs P and R are satisfies the following conditions:

i) $0 \le CS(P, R) \le 1$; $(CS1)$
ii) $CS(P, R) = CS(R, P)$; $(CS2)$
iii) $CS(P, R) = 1,\ iff A = B$; then
$\mu_P(t_i) = \mu_R(t_i),\ \gamma_P(t_i) = \gamma_R(t_i)$, and $\beta_P(t_i) = \beta_R(t_i), i = 1,2,3, \ldots n$ $(CS3)$

Note that we put $n = 1$, then the CSM between HFGs P and R becomes the correlation coefficient between HFGs P and R is $CS(P, R) = k(P, R)$.

3. THE DECISION-MAKING PROBLEMS BY HESITANCY FUZZY PREFERENCE RELATIONS

Suppose that $T = \{t_1, t_2, \ldots., t_n\}$ be the replacement set, and $Y = \{y_1, y_2, \ldots., y_n\}$ be the expert set. The expert y_l deals the evidence of optimal to all replacements and forms HFPRs

$$M^{(l)} = \left(a_{ij}{}^{(l)}\right)_{m\times m}$$

where $a_{ij}{}^{(l)} = \left(\mu_{ij}{}^{(l)}, \gamma_{ij}{}^{(l)}, \beta_{ij}{}^{(l)}\right)$, $0 \leq \mu_{ij}{}^{(l)} + \gamma_{ij}{}^{(l)} + \beta_{ij}{}^{(l)} \leq 1$ and $\mu_{ij}{}^{(l)} = \gamma_{ij}{}^{(l)} = \beta_{ij}{}^{(l)} = 0,\ \forall\, i, j = 1, 2, 3, \dots, n.$

3.1 Working Procedure

In this sector, working procedure is constructed for DMP concentrated on HFPRs.

We define an impartial scoring vector as $C = c_1, c_2, \dots, c_m$ of experts for DMP based on HFPRs, where $C_b > 0, b = 1,2,3,..,l$ and the entire scoring values of the experts is equal to one is denoted as $\sum_{i=1}^{l} C_i = 1$.

Stage I. Evaluate the Laplacian energy $LE\left(M^{(b)}\right)$ of $M^{(b)}$:

$$E\left(M^{(k)}\right) = \left|\sum_{i=1}^{n} k_i\right| \tag{1}$$

Stage II. Evaluate the scores ${C_b}^1$, determined by $E\left(M^{(k)}\right)$, of the expert e_b:

$${C_b}^1 = \left(\left(C_\mu\right)_i, \left(C_\gamma\right)_i, \left(C_\beta\right)_i\right) = \left[\frac{LE\left(\left(D_\mu\right)_i\right)}{\sum_{r=1}^{l} LE\left(\left(D_\mu\right)_r\right)}, \frac{LE\left(\left(D_\gamma\right)_i\right)}{\sum_{r=1}^{l} LE\left(\left(D_\gamma\right)_r\right)}, \frac{LE\left(\left(D_\beta\right)_i\right)}{\sum_{r=1}^{l} LE\left(\left(D_\beta\right)_r\right)}\right] \tag{2}$$

Stage III. Evaluate the CSM $CS\left(M^{(b)}, M^{(d)}\right)$ between $M^{(b)}$ and $M^{(d)}$ for every $b \neq d$

$$CS\left(M^{(b)}, M^{(d)}\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_{M^{(b)}}(t_i)\,\mu_{M^{(d)}}(t_i) + \gamma_{M^{(b)}}(t_i)\,\gamma_{M^{(d)}}(t_i) + \beta_{M^{(b)}}(t_i)\beta_{M^{(d)}}(t_i)}{\sqrt{{\mu_{M^{(b)}}}^2(t_i) + {\gamma_{M^{(b)}}}^2(t_i) + {\beta_{M^{(b)}}}^2(t_i)}\sqrt{{\mu_{M^{(d)}}}^2(t_i) + {\gamma_{M^{(d)}}}^2(t_i) + {\beta_{M^{(d)}}}^2(t_i)}} \tag{3}$$

The mean cosine similarity degree $CS\left(M^{(b)}\right)$ of $M^{(b)}$ to the others is calculated by

$$CS\left(M^{(b)}\right) = \frac{1}{m-1}\sum_{i=1, b\neq d}^{n} CS\left(M^{(b)}, M^{(d)}\right), b = 1,2,3,..,l \tag{4}$$

Stage IV. Evaluate the scores ${C_b}^a$, determined by $CS\left(M^{(b)}\right)$ of the expert e_b:

$${C_b}^a = \frac{CS\left(M^{(b)}\right)}{\sum_{i=1}^{l} CS(M^{(i)})}, b = 1,2,3,..,l \tag{5}$$

Stage V. Evaluate the "objective" scores ${C_b}^2$ of the expert e_b

$${C_b}^2 = \eta {C_b}^1 + (1-\eta){C_b}^a, \qquad \forall\, \eta \epsilon [0,1],\ \ b = 1,2,3,..,l \tag{6}$$

Stage VI. Evaluate the subjective and objective scores ${C_b}^1$ and ${C_b}^2$ of the expert e_b

$$C_b = \gamma {C_b}^1 + (1-\gamma){C_b}^2, \qquad \forall\ \gamma \epsilon [0,1], \qquad b = 1,2,3,..,l \tag{7}$$

3.2. Working Procedure-I

Stage I. Evaluate the mean hesitancy fuzzy values (HFVs) ${r_i}^{(k)}$ of replacements t_i to the other replacements

$${r_i}^{(k)} = \frac{1}{n}\sum_{j=1}^{n} {r_{ij}}^{(k)},\ \ j = 1,2,3,..,n. \tag{8}$$

Stage II. Calculate the values of $r_i^{(k)}$ equivalent to m experts in to a collection of HFVs of the replacements t_i to other replacements.

$$r_i^{(k)} = \sum_{b=1}^{l} (C_b)\left(r_{ij}^{(k)}\right) \tag{9}$$

Stage III. Calculate the score function of r_i

$$CS(r_i) = \frac{\mu_i - \gamma_i + \beta_i}{\sqrt{\mu_i^2 + \gamma_i^2 + \beta_i^2}} \tag{10}$$

where the highest value of the score function is the greater of the replacement t_i and then build a ranking order of the replacements.

3.3. Working Procedure-II

Stage I. Evaluate the cooperative HFPR $M = \left(r_{ij}\right)_{n\times n}$ by

$$r_{ij} = \left(\sum_{b=1}^{l} C_b\mu_{ij}^{(b)}, \sum_{b=1}^{l} C_b\gamma_{ij}^{(b)}, \sum_{b=1}^{l} C_b\beta_{ij}^{(b)}\right), \quad \forall \quad i,j = 1,2,3,\dots,n \tag{11}$$

Stage II. Calculate the CSMs $CS(M^i, M^+)$ between M^i and M^+ for every replacement t_i

$$CS\left(M^{(i)}, M^{(+)}\right) = \frac{1}{n}\sum_{j=i}^{n}\left|\frac{\mu_{ij}(1) - \gamma_{ij}(0) + \beta_{ij}(1)}{\sqrt{\mu_{ij}^2 + \gamma_{ij}^2 + \beta_{ij}^2}}\right| = \frac{1}{n}\sum_{j=i}^{n}\left|\frac{\mu_{ij} + \beta_{ij}}{\sqrt{\mu_{ij}^2 + \gamma_{ij}^2 + \beta_{ij}^2}}\right| \tag{12}$$

Stage III. Calculate the CSMs $CS(M^i, M^-)$ between M^i and M^- for every replacement t_i

$$CS\left(M^{(i)}, M^{(+)}\right) = \frac{1}{n}\sum_{j=i}^{n}\left|\frac{\mu_{ij}(0) - \gamma_{ij}(1) + \beta_{ij}(0)}{\sqrt{\mu_{ij}^2 + \gamma_{ij}^2 + \beta_{ij}^2}}\right| = \frac{1}{n}\sum_{j=i}^{n}\left|\frac{-\gamma_{ij}}{\sqrt{\mu_{ij}^2 + \gamma_{ij}^2 + \beta_{ij}^2}}\right| \tag{13}$$

Stage IV. Evaluate the values of $g(t_i)$, for every replacement t_i

$$g(t_i) = \frac{CS\left(M^{(i)}, M^{(+)}\right)}{CS(M^{(i)}, M^{(+)}) + CS(M^{(i)}, M^{(-)})} \tag{14}$$

The highest value of $g(t_i)$ is greater to the replacements t_i. And we estimate the rank of the replacements. Procedures *I* and *II* are given to illustrate how to achieve absorbed scores to classify replacements in the two following instances. Now, the order of ranking of the replacements is conformed.

A brief overview of the framework of assessment ranking order for the alternatives utilizing the working methods can be seen in Figure 1 below.

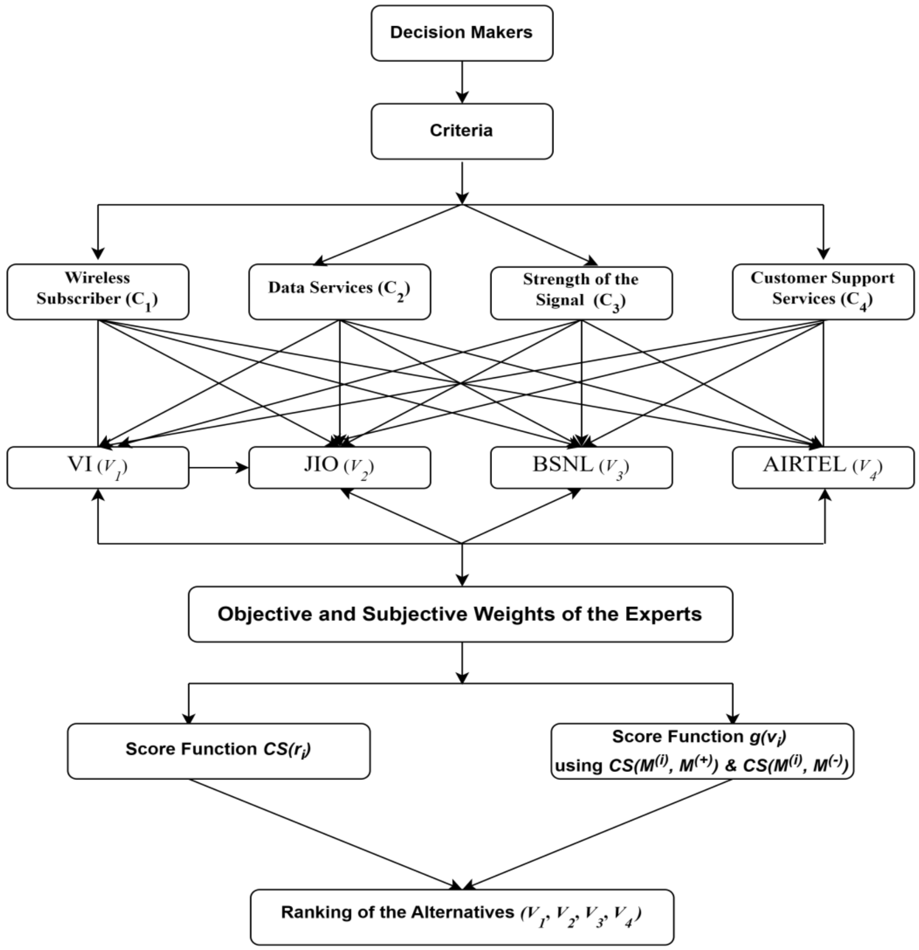

Figure 1. The framework of evaluation ranking order for the alternatives.

4. APPLICATION: SELECTION OF FINEST CELLULAR MOBILE SERVICE PROVIDER

This section provides a quick overview of telecommunications and illustrates relevant examples.

4.1. Importance of Telecommunications

The Government of India recognizes the importance of providing world-class telecommunication services and information for the rapid economic and social growth of the country. It is crucial not only for the growth of the information technology sector but also has far-reaching implications for the country's overall economy. This industry is likewise expected to contribute a significant portion of the country's GDP in the future. As a result, it is critical for the government to have a comprehensive and forward-thinking telecommunications strategy that offers an enabling environment for the development of this sector.

4.2. Cellular Mobile Service Providers

Cellular Mobile Service Providers (CMSP) will be able to provide mobile phone services, including the ability to transport their own long-distance traffic inside their service area, without the need for a separate permit. Simultaneous interconnection among permitted CMSPs and other kinds of service providers (including another CMSP) in their region of operation is authorized, as is infrastructure sharing with other sorts of service suppliers. It is suggested to examine spectrum utilization on a regular basis, bearing in mind the developing situation of spectrum availability, optimal spectrum usage, market demands, competition, and other public interests.

4.3. Cellular Networks Report and Other Service Providers

A ceasefire in India's cellular industry between four big operators in 2019 substantially altered the dynamics. According to OpenSignal, even as the price difference in India reduces, customers' experience with mobile networks remains a crucial predictor of switching mobile providers. According to our most recent assessment of Indian mobile network quality, the business is very competitive, with providers attempting to differentiate it by providing the best service to their customers. Open Signal is an impartial worldwide standard for assessing the mobile experience of consumers. Our industry studies are the definitive reference for understanding the genuine wireless communication experience that customers have.

Analysis performed on Opensignal smartphone users. Leavers includes our smartphone users who changed their mobiles services provider during the 30-day period days starting on December 1, 2020 and represents their mobile experience during the 30-days before they changed. All network users represent the typical experience on each mobile network during the 30-days period days starting on December 1, 2020. For example, according to Open Signal information, all Indian smart phone users on the four largest telecommunication services who transferred their mobile service provider had a poorer experience prior to switching than the normal mobile experience on their original network, as shown in Figure 2.

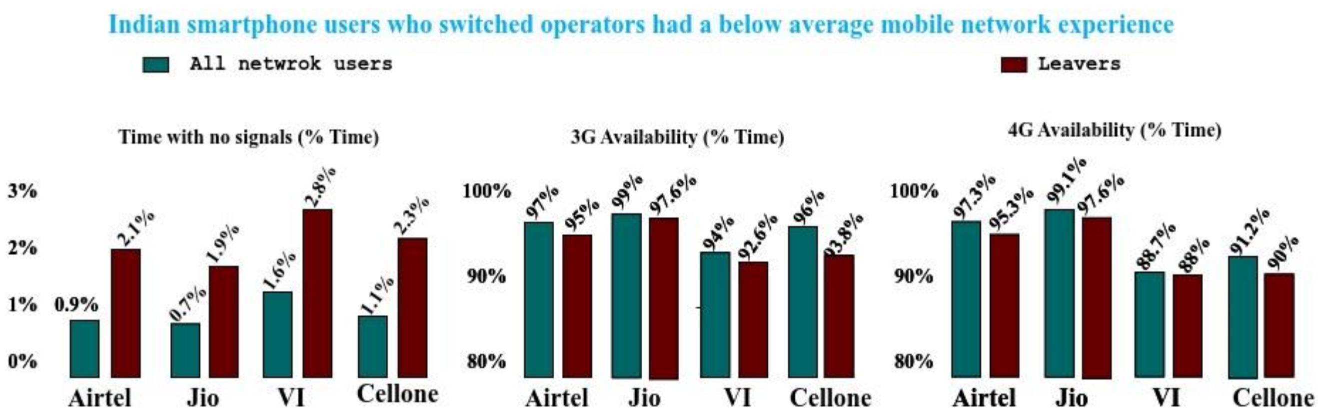

Figure 2. An average mobile network experience on the largest telecommunication services

In accordance with OpenSignal information, all Indian smart phone users on the telecommunication services who transferred their mobile service provider had a poorer experience prior to switching than the normal mobile experience on their original network, as shown in the results in the following Table 1.

Table 1. Indian smartphone users who switched operators had an average mobile network experience

Service Providers	Time with no signal (% Time)		3G Availability (% Time)		4G Availability (% Time)	
	All Network Users	Leavers	All Network Users	Leavers	All Network Users	Leavers
Airtel	0.9	2.1	97	95.2	97.3	95.3
Jio	0.7	1.9	99	97.6	99.1	97.6
VI	1.6	2.8	94	92.6	88.7	88.0
BSNL	1.1	2.3	96	93.8	91.2	90.0

Other service providers will be able to operate by utilising facilities supplied through different access operators for applications such as tele-banking, tele-medicine, tele-education, tele-trading, and e-commerce. There will be no permit cost, but enrollment in certain services will be necessary. These service providers will not violate the jurisdiction of other access providers and will not offer switched telephony. The government of India has granted a new permit to the Global Mobile Personal Communication Services (GMPCS) sector.

Example: We Assume $A = \{v_1, v_2, v_3, v_4\}$ is the set of cellular mobile service providers (alternatives $v_1 = VI, v_2 = JIO, v_3 = BSNL$ and $v_4 = AIRTEL$), and $C = \{c_1, c_2, c_3, c_4\}$ be the set of five criteria for specifying quality service providers in telecommunication relation to Wireless Subscriber, Data Services, Strength of the Signal, Prices for Prepaid Plans, and Customer Support Services with preference information provided in the form of HFPR $R = (r_{ij})$, where $R = (\mu_{ij}, \gamma_{ij}, \beta_{ij})$ for the specified constraints appears in the matrices below, respectively. We consider that in the GDM issue, we have four replacements t_i and three experts e_i, for all $i = 1, 2, 3$. Consider every expert scores are $0, 0.1, 0.3, 0.5, 0.7,$ and 0.9. The four replacement units are composed of the following every expert $e_i, (i = 1, 2, 3)$ and the HFPRs $M^{(b)} = r_{ij}{}^{(b)}, (b = 1, 2, 3)$ are constructed individually. The steps for selecting a process are listed below. First and foremost, the DMs offered language ratings for the criteria using a weighted scale of significance (see $Table$ 1 and 2). Similarly, the DMs assign ratings to options using the proper scales, as shown in the Table 2 and Table 3.

Table 2. The standards for selecting a mobile service provider.

Standards	**Definition**	**Unit Type**
Wireless Subscriber (C_1)	Number linked to prepaid wireless telecommunication service	Benefit
Data Services (C_2)	A data service is a type of telecommunications service that sends data instead of speech.	Cost
Strength of the Signal (C_3)	The received signal frequency as determined at the receiver's antenna	Benefit
Customer Support Services (C_4)	A group of individuals that assist consumers who are having problems with a company's products or services	Benefit

Table 3. Linguistic variables for the importance weights of each standard.

Linguistic term	**Weight**
Very Very high (VVH)	(0.5, 0.1, 0.3)
Very high (VH)	(0.4, 0.2, 0.2)
High (H)	(0.3, 0, 0.4)
Medium (M)	(0.2, 0.3, 0)
Low (H)	(0.1, 0.1, 0.2)
Very low (VL)	(0, 0.2, 0.1)
Very Very low (VVL)	(0, 0, 0.1)

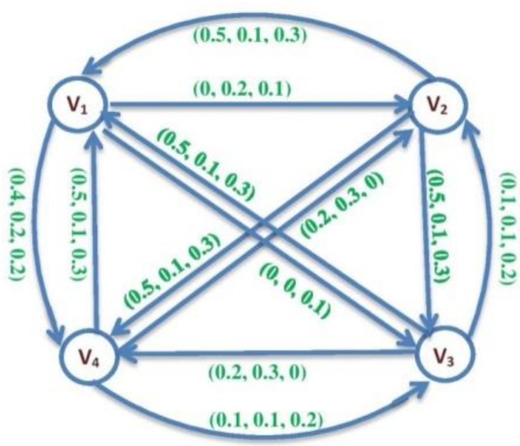

Figure 3. Wireless subscribers of HFPR of HFG

We develop the matrix from Figure 3, we get

$$M^{(1)} = M(HG) = \begin{bmatrix} (0,0,0) & (0,0.2,0.1) & (0,0,0.1) & (0.4,0.2,0.2) \\ (0.5,0.1,0.3) & (0,0,0) & (0.5,0.1,0.3) & (0.5,0.1,0.3) \\ (0.5,0.1,0.3) & (0.1,0.1,0.2) & (0,0,0) & (0.2,0.3,0) \\ (0.5,0.1,0.3) & (0.2,0.3,0) & (0.1,0.1,0.2) & (0,0,0) \end{bmatrix}$$

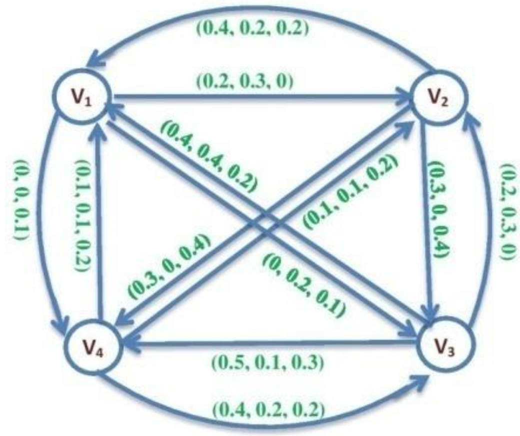

Figure 4. Data services of HFPR of HFG

We develop the matrix from Figure 4, we get

$$M^{(2)} = M(HG) = \begin{bmatrix} (0,0,0) & (0.2,0.3,0) & (0,0.2,0.1) & (0,0,0.1) \\ (0.4,0.2,0.2) & (0,0,0) & (0.3,0,0.4) & (0.3,0,0.4) \\ (0.4,0.2,0.2) & (0.2,0.3,0) & (0,0,0) & (0.5,0.1,0.3) \\ (0.1,0.1,0.2) & (0.1,0.1,0.2) & (0.4,0.2,0.2) & (0,0,0) \end{bmatrix}$$

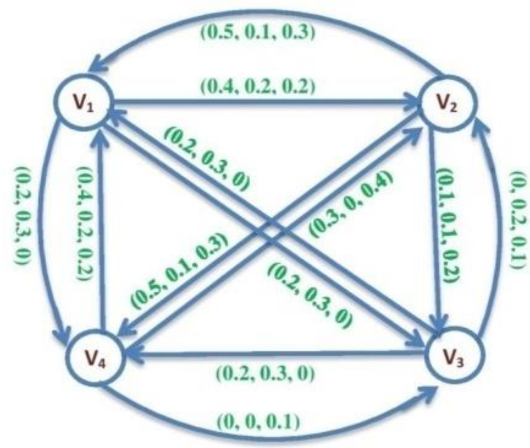

Figure 5. Strength of the Signal of HFPR of HFG

We develop the matrix from Figure 5, we get

$$M^{(3)} = M(HG) = \begin{bmatrix} (0,0,0) & (0.4,0.2,0.2) & (0.2,0.3,0) & (0.2,0.3,0) \\ (0.5,0.1,0.3) & (0,0,0) & (0.1,0.1,0.2) & (0.5,0.1,0.3) \\ (0.2,0.3,0) & (0,0.2,0.1) & (0,0,0) & (0.2,0.3,0) \\ (0.4,0.2,0.2) & (0.3,0,0.4) & (0,0,0.1) & (0,0,0) \end{bmatrix}$$

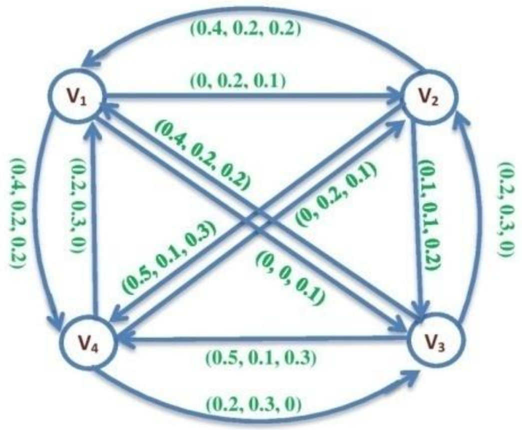

Figure 6. Customer Support Services of HFPR of HFG

We develop the matrix from Figure 6, we get

$$M^{(4)} = M(HG) = \begin{bmatrix} (0,0,0) & (0,0.2,0.1) & (0,0,0.1) & (0.4,0.2,0.2) \\ (0.4,0.2,0.2) & (0,0,0) & (0.1,0.1,0.2) & (0.5,0.1,0.3) \\ (0.4,0.2,0.2) & (0.2,0.3,0) & (0,0,0) & (0.5,0.1,0.3) \\ (0.2,0.3,0) & (0,0.2,0.1) & (0.2,0.3,0) & (0,0,0) \end{bmatrix}$$

Stage I. From equation (1), determine the Laplacian energy of an adjacency matrices $M^{(1)}, M^{(2)}$, $M^{(3)}$ and $M^{(4)}$ of HFG are

$$LE\left(M^{(1)}\right) = (1.7500, 0.8500, 1.1140)$$
$$LE\left(M^{(2)}\right) = (2.1334, 1.0214, 1.5000)$$
$$LE\left(M^{(3)}\right) = (2.0746, 1.5000, 1.5336)$$
$$LE\left(M^{(4)}\right) = (1.5000, 1.1000, 1.0464)$$

Stage II. From equation (3), determine the scores ${C_i}^1$of all the experts e_i is determined by using (3) we get

$${w_1}^1 = [0.4712, 0.2289, 0.3000]$$
$${w_2}^1 = [0.4583, 0.2194, 0.3223]$$
$${w_3}^1 = [0.4061, 0.2937, 0.3002]$$
$${w_4}^1 = [0.4114, 0.3017, 0.2870]$$

Stage III. From equation (5), determine the CSMs $CS\left(M^{(b)}, M^{(d)}\right)$ between $M^{(b)}$and $M^{(d)}$we get

$$CS\left(M^{(1)}, M^{(2)}\right) = 2.4974,$$
$$CS\left(M^{(1)}, M^{(3)}\right) = 2.1559,$$
$$CS\left(M^{(1)}, M^{(4)}\right) = 2.4639,$$
$$CS\left(M^{(3)}, M^{(4)}\right) = 1.9007$$
$$CS\left(M^{(2)}, M^{(3)}\right) = 2.1500$$
$$CS\left(M^{(2)}, M^{(4)}\right) = 2.3751$$

From equation (6), the average CS degree (CSD) $CS\left(M^{(b)}\right)$of $M^{(b)}$ is

$$CS\left(M^{(1)}\right) = 2.3724,$$
$$CS\left(M^{(2)}\right) = 2.3408,$$

$CS(M^{(3)}) = 2.0689,$
$CS(M^{(4)}) = 2.2466.$

Stage IV. From equation (7), determine the values of the scores ${C_b}^a$ for every expert e_i is

$C^b = (0.2628, 0.2291, 0.2594, 0.2488)$

Stage V. From equation (8) and $\eta = 0.5$, determine the objective scores ${C_b}^2$ for every expert e_i is

${C_1}^2 = [0.3670, 0.2459, 0.2814]$
${C_2}^2 = [0.3437, 0.2243, 0.2757]$
${C_3}^2 = [0.3328, 0.2766, 0.2798]$
${C_4}^2 = [0.3301, 0.2753, 0.2679]$

Stage VI. From equation (8) and $\gamma = 0.5$, determine the scores of an objective and subjective of every expert e_i, we have

$C_1 = [0.4191, 0.2374, 0.2907]$
$C_2 = [0.4010, 0.2219, 0.2990]$
$C_3 = [0.3695, 0.2852, 0.2900]$
$C_4 = [0.3708, 0.2885, 0.2775]$

According to working procedure-*I*
Stage I. From equation (10), the average HFVs ${r_i}^{(b)}$ of the alternative t_i to the other alternatives is calculated below

${r_1}^{(1)} = (0.1333, 0.1333, 0.1333)$ ${r_2}^{(1)} = (0.0667, 0.1667, 0.0333)$
${r_3}^{(1)} = (0.2667, 0.2667, 0.0667)$ ${r_4}^{(1)} = (0.1333, 0.1333, 0.1333)$

${r_1}^{(2)} = (0.5000, 0.1000, 0.3000)$ ${r_2}^{(2)} = (0.3333, 0.0667, 0.3333)$
${r_3}^{(2)} = (0.3667, 0.1000, 0.0667)$ ${r_4}^{(2)} = (0.3333, 0.1333, 0.1333)$

${r_1}^{(3)} = (0.2667, 0.1667, 0.1667)$ ${r_2}^{(3)} = (0.3667, 0.2000, 0.1667)$
${r_3}^{(3)} = (0.1333, 0.2667, 0.0333)$ ${r_4}^{(3)} = (0.3667, 0.2000, 0.1667)$

${r_1}^{(4)} = (0.2667, 0.1667, 0.1667)$ ${r_2}^{(4)} = (0.2000, 0.1333, 0.2000)$
${r_3}^{(4)} = (0.2333, 0.0667, 0.2000)$ ${r_4}^{(4)} = (0.1333, 0.2667, 0.0333)$

Stage II. From equation (11), determine the values of r_i, we get

$r_1 = (0.2306, 0.1832, 0.1050),$ $r_2 = (0.6023, 0.1055, 0.3290)$
$r_3 = (0.4440, 0.2177, 0.1542),$ $r_4 = (0.3276, 0.1651, 0.1755)$

Stage III. From equation (12), the values of the score function $CS(r_i)$ of r_i is determined as below

$CS(r_1) = 0.4874,$
$CS(r_2) = 1.1893,$
$CS(r_3) = 0.7346,$
$CS(r_4) = 0.8311$

Therefore $CS(r_2) > CS(r_4) > CS(r_3) > CS(r_1)$
Hence

$v_2 > v_4 > v_3 > v_1$

As a result, v_2 is in the leading rank, v_1 is in the lowest rank, and v_3 and v_4 are in the middle position orders.

According to working procedure-II

In this part, we offer the ranking conclusions ability by our relative CS technique.
Stage I. The HFPR's cooperative $M = \left(r_{ij}\right)_{n\times n}$ are calculated by using (13), we get

$$M = \begin{bmatrix} (0,0,0) & (0.2280,0.2468,0.1148) & (0.0739,0.1299,0.0867) & (0.3899,0.1907,0.1435) \\ (0.7030,0.1543,0.2895) & (0,0,0) & (0.4039,0.0811,0.3203) & (0.5892,0.1382,0.2901) \\ (0.5922,0.2114,0.2025) & (0.1963,0.2339,0.0871) & (0,0,0) & (0.5436,0.2078,0.1730) \\ (0.4716,0.1895,0.2050) & (0.2348,0.1511,0.2036) & (0.2765,0.1547,0.1469) & (0,0,0) \end{bmatrix}$$

Stage II. From equation (14), the CSMs $CS(M^i, M^+)$ between M^i and M^+ for every alternative t_i is calculated as below

$$CS(M^1, M^+) = 0.7645,$$
$$CS(M^2, M^+) = 0.9943,$$
$$CS(M^3, M^+) = 0.8190,$$
$$CS(M^4, M^+) = 0.9289$$

Stage III. From equation (15), the CSMs $CS(M^i, M^-)$ between M^i and M^- for every alternative t_i is calculated as below

$$CS(M^1, M^-) = 0.4660,$$
$$CS(M^2, M^-) = 0.1401,$$
$$CS(M^3, M^-) = 0.3497,$$
$$CS(M^4, M^-) = 0.3065$$

Stage IV. From equation (16), the values of $g(v_i)$, for every alternative t_i is determined as below

$$g(v_1) = 0.6216,$$
$$g(v_2) = 0.7008,$$
$$g(v_3) = 0.8765,$$
$$g(v_4) = 0.7519.$$

Hence $g(v_2) > g(v_4) > g(v_3) > g(v_1)$

Therefore,

$$v_2 > v_4 > v_3 > v_1$$

As a result, v_2 is in the leading rank, v_1 is in the lowest rank, and v_3 and v_4 are in the middle position orders.

Correspondingly, we calculate the place position outcomes of the values $\gamma = 0, 0.1, 0.3, 0.5, 0.7,$ and 0.9 when $\eta = 0, 0.1, 0.3, 0.5, 0.7,$ and 0.9 by using working procedures *I* and *II* in the following Table 4 and Table 5.

Table 4.The ranking order of the alternatives for various values of η and γ using working procedure-I

$\boldsymbol{\eta}$	${C_i}^2(i = 1, 2, 3, 4)$	$\boldsymbol{\gamma}$	$\boldsymbol{C_i}(i = 1, 2, 3, 4)$	$\boldsymbol{r_i}(i = 1, 2, 3, 4)$
0	(0.2628, 0.2628, 0.2628)	***0***	(0.2628, 0.2628, 0.2628)	(0.1527, 0.1756, 0.0931)
	(0.2291, 0.2291, 0.2291)		(0.2291, 0.2291, 0.2291)	(0.3858, 0.1007, 0.2824)
	(0.2594, 0.2594, 0.2594)		(0.2594, 0.2594, 0.2594)	(0.2799, 0.2086, 0.1321)
	(0.2488, 0.2488, 0.2488)		(0.2488, 0.2488, 0.2488)	(0.2096, 0.1580, 0.1498)
0.1	(0.2836, 0.2594, 0.2665)	***0.1***	(0.3024, 0.2873, 0.2699)	(0.1724, 0.1843, 0.0961)
	(0.2520, 0.2281, 0.2384)		(0.2726, 0.2541, 0.2526)	(0.4406, 0.1060, 0.2942)
	(0.2741, 0.2628, 0.2635)		(0.2873, 0.2761, 0.2672)	(0.3215, 0.2187, 0.1377)
	(0.2651, 0.2541, 0.2526)		(0.2797, 0.2589, 0.2560)	(0.2395, 0.1656, 0.1563)
	(0.3258, 0.2526, 0.2740)		(0.3694, 0.2455, 0.2818)	(0.2056, 0.1807, 0.1007)

η	$C_i^2(i=1,2,3,4)$	γ	$C_i(i=1,2,3,4)$	$r_i\,(i=1,2,3,4)$
0.3	$(0.2979, 0.2262, 0.257s1)$	***0.3***	$(0.3460, 0.2242, 0.2767)$	$(0.5330, 0.1040, 0.1916)$
	$(0.3034, 0.2697, 0.2716)$		$(0.3342, 0.2769, 0.2716)$	$(0.3915, 0.2418, 0.1469)$
	$(0.2976, 0.2{,}647, 0.2603)$		$(0.3317, 0.2758, 0.2638)$	$(0.2898, 0.1628, 0.1656)$
0.5	$(0.3670, 0.2459, 0.2814)$	***0.5***	$(0.4191, 0.2374, 0.2907)$	$(0.2306, 0.1832, 0.1050)$
	$(0.3437, 0.2243, 0.2757)$		$(0.4010, 0.2219, 0.2990)$	$(0.6023, 0.1055, 0.3290)$
	$(0.3328, 0.2766, 0.2798)$		$(0.3695, 0.2852, 0.2900)$	$(0.4440, 0.2177, 0.1542)$
	$(0.3301, 0.2753, 0.2679)$		$(0.3708, 0.2885, 0.2775)$	$(0.3276, 0.1651, 0.1755)$
0.7	$(0.4087, 0.2391, 0.2888)$	***0.7***	$(0.4525, 0.2320, 0.2966)$	$(0.2476, 0.1847, 0.1076)$
	$(0.3895, 0.2223, 0.2934)$		$(0.4377, 0.2203, 0.3136)$	$(0.6485, 0.1065, 0.3968)$
	$(0.3621, 0.2834, 0.2880)$		$(0.3929, 0.2906, 0.2965)$	$(0.4791, 0.2196, 0.1589)$
	$(0.3626, 0.2858, 0.2755)$		$(0.3968, 0.2969, 0.2836)$	$(0.3528, 0.1667, 0.1809)$
0.9	$(0.4504, 0.2323, 0.2963)$	***0.9***	$(0.4961, 0.2292, 0.2996)$	$(0.2555, 0.1855, 0.1088)$
	$(0.4354, 0.2204, 0.3130)$		$(0.4560, 0.2195, 0.3214)$	$(0.6715, 0.1071, 0.3438)$
	$(0.3914, 0.2903, 0.2961)$		$(0.4046, 0.2934, 0.2998)$	$(0.4965, 0.2206, 0.1613)$
	$(0.3951, 0.2964, 0.2832)$		$(0.4098, 0.3012, 0.2866)$	$(0.3653, 0.1674, 0.1837)$

Table 5. The ranking order of the alternatives by using working procedure

γ	$CS(r_1)$	$CS(r_2)$	$CS(r_3)$	$CS(r_4)$	***Ranking***
0	0.2801	1.1615	0.5450	0.6664	$v_2 > v_4 > v_3 > v_1$
0.1	0.3118	1.1638	0.5830	0.6966	$v_2 > v_4 > v_3 > v_1$
0.3	0.4306	1.1830	0.6844	0.7879	$v_2 > v_4 > v_3 > v_1$
0.5	0.4874	1.1893	0.7346	0.8311	$v_2 > v_4 > v_3 > v_1$
0.7	0.5212	1.2229	0.7601	0.8533	$v_2 > v_4 > v_3 > v_1$
0.9	0.5354	1.1919	0.7714	0.8213	$v_2 > v_4 > v_3 > v_1$

According to the working procedure, by replace the values of $\gamma = 0, 0.1, 0.3$ $0.5, 0.7$ and 0.9, we get the same conclusions for all of the choices.

Therefore,

$$v_2 > v_4 > v_3 > v_1$$

Hence t_2 place the highest position, while t_1 place the last position, finally t_3 and t_4 places the center position orders, and which are mentioned in the above tables.

Procedure-II

Table 6. The ranking order of the alternatives for various values of γ using working procedure-II

γ	$C_i(i=1,2,3,4)$	$CS(M^i, M^+)$	$CS(M^i, M^-)$
0	$(0.2628, 0.2628, 0.2628)$	$(0.7075, 1.0184, 0.7746, 0.8894)$	$(0.5294, 0.1534, 0.4390, 0.3918)$
	$(0.2291, 0.2291, 0.2291)$		
	$(0.2594, 0.2594, 0.2594)$		
	$(0.2488, 0.2488, 0.2488)$		
0.1	$(0.3024, 0.2873, 0.2699)$	$(0.7193, 1.0049, 0.7837, 0.8962)$	$(0.5185, 0.1801, 0.4239, 0.3777)$
	$(0.2726, 0.2541, 0.2526)$		
	$(0.2873, 0.2761, 0.2672)$		
	$(0.2797, 0.2589, 0.2560)$		
0.3	$(0.3694, 0.2455, 0.2818)$	$(0.7562, 0.9815, 0.8163, 0.9192)$	$(0.4623, 0.1548, 0.3722, 0.3306)$
	$(0.3460, 0.2242, 0.2767)$		
	$(0.3342, 0.2769, 0.2716)$		
	$(0.3317, 0.2758, 0.2638)$		
	$(0.4191, 0.2374, 0.2907)$	$(0.7654, 0.9943, 0.8190, 0.9289)$	$(0.4660, 0.1401, 0.3497, 0.3065)$

γ	$C_i (i = 1, 2, 3, 4)$	$CS(M^i, M^+)$	$CS(M^i, M^-)$
0.5	(0.4010, 0.2219, 0.2990)		
	(0.3695, 0.2852, 0.2900)		
	(0.3708, 0.2885, 0.2775)		
0.7	(0.4525, 0.2320, 0.2966)	(0.7845, 0.8241, 0.9911, 0.9298)	(0.4456, 0.1330, 0.5505, 0.2676)
	(0.4377, 0.2203, 0.3136)		
	(0.3929, 0.2906, 0.2965)		
	(0.3968, 0.2969,0.2836)		
0.9	(0.4525, 0.2320, 0.2966)	(0.7884, 1.0076, 0.8264, 0.9345s)	(0.4406, 0.1055, 0.3289, 0.2857)
	(0.4377, 0.2203, 0.3136)		
	(0.3929, 0.2906, 0.2965)		
	(0.3968, 0.2969,0.2836)		

Table 7. The ranking order of the alternatives by using working procedure

γ	$g(t_1)$	$g(t_2)$	$g(t_3)$	$g(t_4)$	***Ranking***
0	0.5720	0.8691	0.6383	0.6942	$v_2 > v_4 > v_3 > v_1$
0.1	0.5811	0.6490	0.8480	0.7035	$v_2 > v_4 > v_3 > v_1$
0.3	0.6206	0.8638	0.6868	0.7355	$v_2 > v_4 > v_3 > v_1$
0.5	0.6216	0.7008	0.8765	0.7519	$v_2 > v_4 > v_3 > v_1$
0.7	0.6380	0.8817	0.7028	0.7765	$v_2 > v_4 > v_3 > v_1$
0.9	0.6415	0.9052	0.7153	0.7659	$v_2 > v_4 > v_3 > v_1$

According to the working procedure, by replace the values of $\gamma = 0, 0.1, 0.3\ 0.5, 0.7$ and 0.9, we get the same conclusions for all of the choices.

Therefore,

$$v_2 > v_4 > v_3 > v_1$$

Hence t_2 place the highest position, while t_1 place the last position, finally t_3 and t_4 places the center position orders,and which are mentioned in the above Table 6 and Table 7.

5. CONCLUSION

This research introduced an innovative process for evaluating the relative reputational scores of an expert by calculating the unclear information of HFPRs and the average grade of a cosine similarity measure of a particular HFPR compared to all the others. Also, this research established the CSMs and Laplacian energy on the undetermined signs of HFPRs. We constructed a tool for evaluating the score values of experts that takes both the subjective and objective scores of the experts into consideration. The scored CSMs were applied to decision-making issues, and the outcomes are explained in more detail. This study illustrated a real-time numerical example to find the finest cellular mobile service provider. After applying working procedures I and II, we obtained the best cellular mobile service provider (JIO) in both cases.

In the future, we will implement this technique based on the correlation coefficient using HFG information and its application to decision-making issues. Xu and Xia investigated the distance and correlation measures of the hesitant fuzzy set and applied them to medical diagnosis and decision-making.

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