# INFLUENCE OF MAN-MACHINE RATIO ON SYSTEM PERFORMANCE OF ONE-PERSON-MULTI-MACHINE SERIES PRODUCTION LINE 

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#### Abstract

The one-person-multi-machine assignment is a typical feature of lean production systems. The major disadvantage of this type of assignment is that it could cause system delay due to human failure. Therefore, it is important to analyze the degree of efficiency loss among machines caused by interference between operators and machines. In this paper, a methodology is developed based on the decomposition technique. The whole $U$-shape production line is modeled as several subsystems where the efficiency loss mentioned above can be treated as machine failure. Hence, each subsystem can be simplified as an unreliable workstation with a certain failure rate. With finite buffers between consecutive subsystems, the influence of human failure can be analyzed and verified with an industry-based case study. Data was collected from an automotive electronics plant as well as corresponding computational and simulation test results. Statistics show that the method developed in this paper will make contributions to solving industrial problems.


Keywords: production line, queueing theory, Markov process, system performance, decomposition
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## 1. INTRODUCTION

Nowadays, lean production is considered as a key strategy for improving the competitiveness of manufacturing operations. Multi-machine/process handling is a typical feature of lean production systems. Multi-Machine Handling means that one shop worker will move along a group of machines or pieces of equipment and operate them to perform multiple jobs by himself. Those machines and equipment are grouped together because of the similarity of processes involved or the similarity of the machines used.

Multi-process handling means that a shop worker will move down a row of machines or equipment arranged in the order of production processes and will perform all necessary jobs within the takt time. Operator-induced machine interference is a critical issue for multi-process handling work cell analysis. The accurate estimation of lean production system performance is challenging. In part, this challenge consists of estimating the dynamic and complex interactions that typically occur between the parts to be processed, machines, and operator(s) in lean production systems. However, to efficiently design and operate lean production systems, this estimation is crucial. Traditional discrete-event simulation, which permits accurate performance analysis within a wide array of systems (Law, 2007), is often time-consuming. An interesting alternative is to use analytical models based on queuing theory. Desruelle (1996) presented an analytical model that involved the resolution of two interacting queueing networks: an open network representing the interactions between the parts processed by the work cell and the machines, and a closed network representing the interactions between the machines and the workplace operator responsible for the setup and part loading/unloading operations. Yang (2005) used these models (Desruelle, 1996) to investigate the effect of external operations on the effectiveness of a one-man multi-machine production line. However, their

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methods were only used to evaluate the performance of a manufacturing workplace tended by a single operator when operatorinduced machine interference occurs, and as such, the results are limited to a small number of machines assigned to a single operator.


Figure 1. Lean production line composed of multi-process handling subsystems in series
Moreover, the abovementioned study did not solve the performance estimation problem for a lean production system containing multi-process handling subsystems in series, as shown in Figure 1. Although there exist many methods of modeling serial production lines, such as the stochastic-flow manufacturing network (Lin, 2013), queuing theory, open-closed loop queuing network method (Papadopoulos, 1993; Buzacott, 1993; Gershwin, 1987), and iterative method based on Markov chains (Lim, 1990; Jacobs, 1995; Kuo, 1996; Chiang, 2001; Li and Meerkov, 2001), most of these methods focus on random mechanical failure and repair events occurring at workstations represented by the exponential distributions for failure (MTBF) and repair (MTTR). Additionally, previous studies have made various assumptions regarding the conditions under which failure may occur, the time until failure occurs, failure duration, synchronicity, and so on. Most real systems are unsynchronized; that is, failure and repair times are uncertain and can lead to unsynchronized operation times. Other considerations include the buffer size, i.e., limited and unlimited storage space, configuration of the production line (serial assembly and identical or non-identical parallel machines), system performance analysis of issues related to capacity problems, blocking and starvation, and machine utilization. Miltenburg (2000) investigated the effect of a U-shaped line on system effectiveness. He found that effectiveness increases with a U-shaped line in comparison with a traditional straight line. Moreover, the author grouped tasks into stations, and the stations were positioned in a U-shaped line. He argued that there exists a unique straight line with the same effectiveness as the U -line and developed a procedure for constructing this equivalent straight line. However, he did not consider the effect of the operator on the performance of the production line. In terms of a one-man, one-machine, or automatic production line, the performance of the system is obviously determined by the machine performance and buffer size. At this time, the performance of the manufacturing system is mainly related to the reliability of the machine. However, for a lean production line composed of multi-process handling subsystems in series, as shown in Figure 1, the performance of the system is related not only to machine performance and buffer size but also to the man-machine configuration. At this time, the man-machine interference will greatly affect the performance of the system. Desruelle (1994) developed models to analyze the performance of workplaces with several operators with different skill levels based on queuing theory. However, his method focused on the evaluation problems of the performance of the multipleoperator cell.

In this study, our objective is to develop a computer-based approach that can answer the following question quickly: "How does the man-machine ratio affect the performance of a lean production system containing multi-process handling cells. Our proposed approach deals with the evaluation problems of the performance of a lean production line composed of the multi-process handling cell in series.

Our proposed approach partly adopts the solution infrastructure from Hu and Meerkov (2006) to model the lean production system. However, our approach is different from theirs in that: Hu and Meerkov did not consider the effect of the operator on the performance of the production line. We are not aware of any literature that explicitly models a lean production line composed of multi-process handling subsystems in series. In addition, the mathematical development in existing literature contributed to literature but failed to address its application to an industrial application adequately. Our research is based on an industrial application. Practical constraints are considered for model development and practice management insights for the proposed solution to an industrial problem.

The remainder of this paper describes the development of the proposed approach. In Section 2, we introduce the analytical model and present our methodology. In Section 3, we present the results for a set of discrete-event simulation experiments to assess the accuracy of the proposed analytical model. Finally, the conclusions drawn from this study are summarized in Section 4.

## 2. METHODOLOGY

The key to using this model for a lean production line is the ability to construct a virtual production line whose effectiveness is the same as the effectiveness of the lean production line. This is done as follows.

In the proposed method, the production line is first decomposed into several single-operator subsystems based on the number of operators. The machine interference problem in each subsystem is analyzed according to queuing theory, and the subsystem is considered as equivalent to an unreliable machine with the same effectiveness. Secondly, Markov chains are applied to establish equations between each simplified tandem queue subsystem and used to approximate the performance results for the entire system through an iterative process. In the joint man-machine operation, a machine is thought to interfere if the operator engages with another machine when it has completed its production. In other words, the machine is idle and waits for the attention of the operator. Because both the processing time and concurrence time are random, it is important to evaluate the percentage interference of machine $j=1,2, \ldots, n-1$ and the idleness percentage of the operator within a given time interval.


Figure 2. Queuing theory model of multi-process handling subsystem
To simplify the model, this study did not consider the reliability of the machine and assumed that the machine was $100 \%$ reliable. At this point, the efficiency loss of the subsystem will be caused entirely by the machine interference rather than by machine failure. Therefore, at any time interval, if a system does not have machine interference and a machine is not waiting for service, we assumed that the efficiency of the subsystem was $100 \%$.

By applying queuing theory, we considered each machine in the subsystem as a customer, and its interarrival times were assumed to follow Poisson distributions with the $\lambda$ parameter. Each operator was considered as a server, and the service times were assumed to be exponentially distributed with the $\mu$ parameter. Each subsystem was a single server queue with the first-come-first-served (FCFS) scheduling. Because $m$ customers were allowed into the system, at most, the $[M / M / 1 / m]$ model was applied to analyze the performance of the multi-process handling subsystem, as shown in Figure 2.

According to (Stewart, 2009), the equations for the mean number of customers in a system $L_{S}$, mean queue length $L_{q}$, average response time $W_{s}$ and average waiting time $W_{q}$, are expressed as follows:

$$
\begin{align*}
L_{s} & =\sum_{n=0}^{m} n P_{n}=P_{0} \rho \sum_{n=0}^{m} n \rho^{n-1}=P_{0} \rho \sum_{n=0}^{m} \frac{d}{d \rho}\left(\rho^{n}\right)=P_{0} \rho \frac{d}{d \rho}\left(\sum_{n=0}^{m} \rho^{n}\right)  \tag{1}\\
& =P_{0} \rho\left(\frac{d}{d \rho}\left[\frac{1-\rho^{m+1}}{1-\rho}\right]\right)=P_{0} \rho\left(\frac{1-(m+1) \rho^{m}+m \rho^{m+1}}{(1-\rho)^{2}}\right) \\
L_{q} & =L_{s}-\left(1-P_{0}\right)=L_{s}-\left(1-\frac{1-\rho}{1-\rho^{m+1}}\right)=L_{s}-\frac{\rho\left(1-\rho^{m}\right)}{1-\rho^{m+1}}  \tag{2}\\
W_{s} & =\frac{1}{\lambda^{\prime}} L_{s}  \tag{3}\\
W_{q} & =W_{s}-\frac{1}{\mu}  \tag{4}\\
P_{0} & =\frac{1-\rho}{1-\rho^{m+1}}, \tag{5}
\end{align*}
$$

where $\rho=\frac{\lambda}{\mu}, \lambda^{\prime}=\lambda\left(1-P_{m}\right)=\lambda\left(1-\rho^{m} P_{0}\right)=\lambda\left(1-\rho^{m} \frac{1-\rho}{1-\rho^{m+1}}\right)=\lambda \frac{1-\rho^{m}}{1-\rho^{m+1}}$ is the effective arrival rate, and $P_{n}$ denotes the probability of there being $n$ customers in the queuing system. Thus, $P_{0}$ represents the probability that a customer does not exist in the system. This probability of the system being idle is expressed in Eq. (5).

We observed that the actual production time of each subsystem increased, contrary to a control group for which each machine did not need service from an operator, owing to man-machine interference. In other words, the actual efficiency of
a subsystem declined if a machine was waiting for service. In such cases, assuming that the efficiency of the control group is $100 \%$, the actual efficiency of each subsystem without considering machine failure can be defined as follows:

$$
\begin{align*}
e & =\frac{\text { processing time (including loading and unloading) }}{\text { processing time (including loading and unloading) }+ \text { waiting time }}  \tag{6}\\
& =\frac{m(1+\rho)}{m(1+\rho)+\lambda W_{q}}=\frac{1}{1+\frac{\lambda W_{q}}{m(1+\rho)}}
\end{align*}
$$



Figure 3. Simplified system method
The simplification of the multi-process handling subsystem into a workstation with efficiency $e$ forms the basis for the next iterative merger, as shown in Figure 3.

The analytical performance method of the simplified production system was based on a performance analysis method for serial lines with Bernoulli machines developed by Hu and Meerkov (2006). To make this paper self-contained, a brief review of this method is presented below.

Let us consider the simplified serial production line shown in Figure 3, where the rectangles representing the workstations and circles are the buffers. Let us assume that the line operates according to the following assumptions:
The system consists of $M$ workstations made up of machines and people that do not fail. $M$ workstations are arranged serially, and M-1 buffers separate each consecutive pair of workstations.

The balanced workstations have identical cycle times, Tc. The time axis is slotted with the slot duration Tc. The workstations begin operating at the beginning of each time slot.

Each buffer is characterized by its capacity, i.e., $N_{i}<+\infty, 1 \leq i \leq M-1$.
Workstations can be blocked and starved owing to man-machine interference. Workstation $i$ is starved during a time slot if the buffer $\mathrm{i}-1$ is empty at the beginning of the time slot. Workstation 1 is never starved for parts.

Workstation i is blocked during a time slot if buffer i has Ni parts at the beginning of the time slot, and Workstation $i+1$ fails to participate during the time slot. Workstation $M$ is never blocked by the ready goods buffer.

Workstations that define whether operator/machine interference occurs at random obey the Bernoulli reliability model, i.e., workstation $i=1, \ldots, M$ is neither locked nor starved during a time slot, produces a part with probability $e_{i}$, while it fails to do so with a probability of $1-e_{i}$. Parameter $e_{i}$ is the efficiency of workstation $i$.

For $M=2$, the production rate (PR) of a serial line is expressed by Eq. (7) using Markov chain analysis.

$$
\begin{align*}
P R & =e_{1}\left[1-Q\left(e_{2}, e_{1}, N\right)\right] \\
& =e_{2}\left[1-Q\left(e_{1}, e_{2}, N\right)\right], \tag{7}
\end{align*}
$$

where

$$
Q(x, y, N)=\left\{\begin{array}{l}
\frac{(1-x)(1-\alpha)}{1-\frac{x}{y} \alpha^{N}}, \text { if } \quad x \neq y  \tag{8}\\
\frac{1-x}{N+1-x}, \text { if } \quad x=y
\end{array} \quad \text { and } \quad \alpha=\frac{x(1-y)}{y(1-x)}\right.
$$

or $M>2$, and closed formulas for PR cannot be derived. Therefore, a recursive aggregation procedure was developed and led to a PR estimate with an accuracy that was typically within $1 \%$. This procedure consisted of backward and forward
aggregations. With backward aggregation, the last two workstations were aggregated into a single workstation by using Eq. (9). Then, this aggregated workstation was aggregated again with the $(M-2) t h$ workstation, and so on, until all workstations were aggregated into a single workstation. The efficiencies of the aggregated workstations are denoted as $e_{i}^{b}, i=1, \ldots, M-$ 1 , where $b$ means backward aggregation. In forward aggregation, the first workstation was aggregated with the aggregated workstation representing the last $M-1$ workstations. Then, this workstation was aggregated again with the aggregation of the last M-2 workstations, and so on, until all workstations were aggregated into a single workstation. The efficiencies of these aggregated workstations are denoted as $e_{i}^{f}, i=2, \ldots, M$, where $f$ means forward aggregation. Then, the procedure was repeated by alternating between the backward and forward aggregations. Formally, this process is represented in Eq. (9). Figure 4, shows an analytical model flow diagram of obtaining the performance estimates with regard to lean production line composed of multi-process handling subsystems in series.

$$
\begin{align*}
P R & =e_{1}^{b}=e_{M}^{f} \\
& =e_{i+1}^{b}\left[1-Q\left(e_{i}^{f}, e_{i+1}^{b}, N_{i}\right)\right]  \tag{9}\\
& =e_{i}^{f}\left[1-Q\left(e_{i+1}^{b}, e_{i}^{f}, N_{i}\right)\right], i=1, \mathrm{~L}, M-1 .
\end{align*}
$$

## 3. CASE STUDY

The case study considered in this work is a production system for automobile exhaust control units from an electronic plant, and its internal code is B26. This B26 line is a typical U-line operated by three workers per shift, and its details are shown in Figure 6. Line B26 includes sixteen different types of machines served by three or four operators. The machine processing time, loading time, and unloading time are listed in Table 1.

Table 1. Machine processing time and loading and unloading time in the B26 production line

|  | Machine | Machine processing time (s) | Loading and unloading time (s) |
| :---: | :---: | :---: | :---: |
|  | V-CUT=>Press fit | $10 \sim$ | 24 |
|  | ICT 1 | 47 | 9 |
|  | ICT 2 | 34 | 9 |
| $\stackrel{0}{0}$$\stackrel{0}{\pi}$$\stackrel{\rightharpoonup}{0}$$\stackrel{N}{N}$ | TIM | 25.5 | 3.5 |
|  | Silicon 1 | 22 | 1.5 |
|  | Test 1 | 6 | 1.5 |
|  | PCB mounting | 20 | 4 |
|  | Screw | 24 | 1.5 |
|  | Silicon 2 | 24 | 2 |
|  | Test 2 | 6 | 1.5 |
|  | Cover | 9.5 | 3.5 |
|  | Crimping | 8.5 | 7 |
|  | Leakage | 19 | 4 |
|  | F/T | 52 | 3 |
|  | EOL | 55 | 9 |
|  | Pin check output | 15.5 | 5.3 |

### 3.1 Analytical approach

To measure the performance of the production system before and after considering the influence of the man-machine interference, the production line efficiency was quantified in Eq. (10).

$$
\begin{equation*}
P E=\frac{\overline{\boldsymbol{P R}}}{\boldsymbol{P R}}, \tag{10}
\end{equation*}
$$

where $\overline{P R}$ is the production rate of the serial production system with the influence of man-machine interference, and PR is the production rate of the line consisting of the same machines without man-machine interference. We assumed that the line efficiency is $100 \%$, i.e., we did not consider a machine failure, and one operator was responsible for the operation of a machine.


Figure 4. Analytical model flow diagram


Figure 5. Working sequence diagram of B26 line
For convenience, we assumed that x machines were assigned to the first operator, y machines were assigned to the second operator, and z machines were assigned to the third operator. We used the notation $(\mathrm{x} / \mathrm{y} / \mathrm{z})$ to represent the current man-machine assignment strategy while satisfying the constraint of $x+y+z=16$.

Let us suppose that the machines are assigned in the order of the production processes shown in Figure 5. Namely, the first operator must serve the first machine, the third operator's service must include the last machine, and the second operator must serve the remaining machines in the middle. For example, the initial assignment of the B26 line is (3/9/4), i.e., the system buffer capacity is two. Under the abovementioned assumptions, we developed a C++ program with regard to the man-machine assignment strategy to estimate the performance of the production line based on the analytical model flow diagram shown in Figure 4.

The numerical calculation was divided into two groups. The first group took y as an independent variable, and x was the control variable used to control the changes in the dependent $P R$ variable. The PR values were calculated for $x \in[3,4,5,6]$, $\mathrm{y} \in[2,3, \cdots, 11]$, and $z=16-x-y$. The second group took y as an independent variable, and z was the control variable used to observe the change of the dependent PR variable. The PR values were calculated for $\mathrm{z} \in[2,3,4,5], \mathrm{y} \in[2,3, \cdots, 9]$, and $z=16-x-y$.

For each assignment, 20 repetitions of the developed code were performed. The resulting performance estimates are shown in Figures 6-7.

As can be seen in Figure 6, the variation of several curves were approximately the same and independent of the number of machines assigned to the first operator. From the perspective of the third operator and by considering Curve (4) as an example, when the assignment changed from $(6 / 6 / 4)$ to $(6 / 7 / 3)$, the efficiency of the B26 line appeared to reduce significantly. In other words, when the number of machines allocated by third operators was less than four, the line efficiency would decrease. This result was independent of the number of machines assigned to the first operator.

In comparison with Figure 6, the curves tended to decrease first and then increase. However, the numerical gap of the total efficiency was high. This occurred regardless of the four lines shown in Figure 6 being similar to those shown in Figure 7. In Curve 1, the efficiency of the entire line reached the highest point ( $92 \%$ ) in the case of ( $1 / 10 / 5$ ). However, this does not agree with previously obtained results because, in this case, the model of the production line served by three workers turned into two operators serving 15 machines and a machine that did not require an operator, therefore, except for the case where the first worker was only responsible for the first machine, the efficiency of the


Figure 6. Performance measures as a function of the number of machines assigned to the second operator. Curve (1): operator 1 was assigned three machines; Curve (2): operator 1 was assigned four machines; Curve (3): operator 1 was assigned five machines; Curve (4): operator 1 was assigned six machines)


Figure 7. Performance measures as a function of the number of machines assigned to the second operator (Curve (1): operator 3 was assigned five machines; Curve (2): operator 3 was assigned four machines; Curve (3): operator 3 was assigned three machines; Curve (4): operator 3 was assigned two machines)

The production line was still the highest with the distribution strategy corresponding to Curve (1); i.e., the (5/6/5) assignment scheme was the most reasonable assignment in all cases.

### 3.2 Discrete event simulation experiment

The objective of the simulation was to verify the effectiveness of the production line performance estimation method.

### 3.2.1 Establishment of the simulation model

The eM-Plant software (developed by Tecnomatix Ltd.), which is standard commercial simulation software, was used in the simulation experiment. This is a discrete event simulation (DES) package that incorporates a graphical user interface and preestablished object-oriented modeling elements. The method used here was in accordance with the technique proposed by Law (2007); i.e., it carried out a preliminary simulation of the system under investigation and chose one output variable to observe, which was PR in this case, as suggested by the abovementioned study.

As has already been mentioned, the B26 line was a U-type production line with three operators and sixteen machines. The DES model was built as shown in Figure 8, and its particular setting details are shown in Figure 8.


Figure 8. eM-Plant DES model of B26 line
The beginning of the line, now renamed as PCB, generated parts according to Table (1). Because an assembly line or conveyor belt did not exist in the B26 line, the products were manually loaded, unloaded, and moved by the operators. In the eM-plant DES model, both the loading and unloading of products required effort from the workers. When a worker operated a machine, the other machines could only wait, even if they required service. Therefore, this process was timeconsuming.

PCB clamping and crimping were the primary assembly processes and only occurred when the assembly of all parts as required (i.e., all parts had arrived). Buffer 13 was a buffer between workers 1 and 2 . Buffer 29 was a buffer between workers 2 and 3 .

In the simulation model, Drain is the export of parts (now renamed as Finished) and was embedded in the DES model. The global variable, namely, Variable 1, was used to calculate the output of the production line, which could then be used to calculate the production rate. The simulation period was one month (that is $2,592,000 \mathrm{~s}$ ).

### 3.2.2 Simulation Experiment

The processing entailed in the discrete simulation was carried out in the same manner as the theoretical part. The system performance was evaluated by using the following production efficiency equation:

$$
\begin{equation*}
\mathrm{PE}=\frac{\text { Production rate } 1 \text { of line under specific assignment strategy }}{\text { production rate } 2 \text { of line corresponding to the non }- \text { worker service }} \tag{11}
\end{equation*}
$$

Additionally, we assumed that x machines were assigned to the first operator, y machines were assigned to the second operator, z machines were assigned to the third operator, and ( $\mathrm{x} / \mathrm{y} / \mathrm{z}$ ) represents the current man-machine assignment strategy satisfying $x+y+z=16$. The resulting performance estimates are denoted as PR.

The experiment was divided into two groups. The first group took y as an independent variable, and x was the control variable used to control the changes in the dependent PR variable. The PR values were calculated for $\mathrm{x} \in[3,4,5,6], \mathrm{y} \in$ $[2,3, \cdots, 11]$, and $z=16-x-y$. The second group took $y$ as an independent variable, and $z$ was the control variable used to observe the change of the dependent PR variable. The PR values were calculated for $\mathrm{z} \in[2,3,4,5], \mathrm{y} \in[2,3, \cdots, 9]$, and $z=16-x-y$.

For a given assignment strategy ( $\mathrm{x} / \mathrm{y} / \mathrm{z}$ ), we first simulated the production rate of the entire production line for one month (labeled PR1). The human service operations were then removed from the simulation, with the loading and unloading times considered as part of the machine processing time. The simulation was carried out again, with the other conditions remaining unchanged (labeled PR2). These results were used to calculate the production line efficiency PE for the specific assignment strategy.

We first simulated the production line for one month. DES code was used to simulate the experiment with 20 repetitions for each assignment under the same experimental conditions. In each repetition, we used the first $2 \cdot 104$ time slots as a warm-up period, while the subsequent 4 , 105-time slots were used to statistically calculate the average performance. Then, we calculated the performance estimates by using the DES model and compared them with those obtained by the analytical model according to the following metrics:

$$
\begin{equation*}
\Delta_{\mathrm{PE}}=\frac{|\overline{\mathrm{PE}}-\mathrm{PE}|}{\mathrm{PE}} \times 100 \% \tag{12}
\end{equation*}
$$

Table 2. Comparison of average accuracy of line efficiency theoretical and simulation values under the condition of y being the independent variable and x being the control variable

| Assignment strategy | Line efficiency theoretical values | Line efficiency simulation values | Relative error (\%) |
| :--- | :--- | :--- | :--- |
| $(3 / 2 / 11)$ | 0.895847 | 0.9040 | 0.90 |
| $(3 / 3 / 10)$ | 0.89585 | 0.9040 | 0.90 |
| $(3 / 4 / 9)$ | 0.895848 | 0.8421 | 6.38 |
| $(3 / 5 / 8)$ | 0.895849 | 0.8116 | 10.38 |
| $(3 / 6 / 7)$ | 0.895848 | 0.8788 | 1.94 |
| $(3 / 7 / 6)$ | 0.895848 | 0.8631 | 3.79 |
| $(3 / 8 / 5)$ | 0.895831 | 0.9167 | 2.28 |
| $(3 / 9 / 4)$ | 0.885688 | 0.9495 | 6.72 |
| $(3 / 10 / 3)$ | 0.882229 | 0.9658 | 8.65 |
| $(3 / 11 / 2)$ | 0.891255 | 1.0000 | 10.87 |
| $(4 / 2 / 10)$ | 0.896458 | 0.9040 | 0.83 |
| $(4 / 3 / 9)$ | 0.896456 | 0.8421 | 6.46 |
| $(4 / 4 / 8)$ | 0.896457 | 0.8116 | 10.45 |
| $(4 / 5 / 7)$ | 0.896456 | 0.8788 | 2.01 |
| $(4 / 6 / 6)$ | 0.896457 | 0.8631 | 3.86 |
| $(4 / 7 / 5)$ | 0.896448 | 0.9167 | 2.21 |
| $(4 / 8 / 4)$ | 0.884438 | 0.9495 | 6.86 |
| $(4 / 9 / 3)$ | 0.881027 | 0.9658 | 8.78 |
| $(4 / 10 / 2)$ | 0.892331 | 1.0000 | 10.77 |
| $(5 / 2 / 9)$ | 0.899663 | 0.8421 | 6.84 |
| $(5 / 3 / 8)$ | 0.899669 | 0.8116 | 10.85 |
| $(5 / 4 / 7)$ | 0.899668 | 0.8788 | 2.37 |
| $(5 / 5 / 6)$ | 0.899669 | 0.8632 | 4.23 |
| $(5 / 6 / 5)$ | 0.89965 | 0.9167 | 1.86 |
| $(5 / 7 / 4)$ | 0.878019 | 0.9495 | 7.53 |
| $(5 / 8 / 3)$ | 0.875222 | 0.9658 | 9.38 |
| $(5 / 9 / 2)$ | 0.893981 | 0.9697 | 7.81 |
| $(6 / 2 / 8)$ | 0.884109 | 0.8117 | 8.92 |
| $(6 / 3 / 7)$ | 0.884109 | 0.8781 | 0.68 |
| $(6 / 4 / 6)$ | 0.884109 | 0.9100 | 2.84 |
| $(6 / 5 / 5)$ | 0.884107 | 0.9100 | 2.84 |
| $(6 / 6 / 4)$ | 0.875526 | 0.9100 | 3.78 |
| $(6 / 7 / 3)$ | 0.87344 | 0.9100 | 3.02 |
| $(6 / 8 / 2)$ | 0.882485 |  |  |
|  |  |  |  |

Table 3. Comparison of average accuracy of line efficiency theoretical and simulation values under the condition of y being the independent variable and z being the control variable

| Assignment strategy | Line efficiency theoretical values | Line efficiency simulation values | Relative error (\%) |
| :--- | :--- | :--- | :--- |
| $(2 / 12 / 2)$ | 0.891614 | 1.0000 | 10.83 |
| $(3 / 11 / 2)$ | 0.891255 | 1.0000 | 10.87 |
| $(4 / 10 / 2)$ | 0.892331 | 1.0000 | 10.77 |
| $(5 / 9 / 2)$ | 0.893981 | 0.9696 | 7.80 |
| $(6 / 8 / 2)$ | 0.882485 | 0.9100 | 3.02 |
| $(7 / 7 / 2)$ | 0.877131 | 0.8767 | 0.05 |
| $(8 / 6 / 2)$ | 0.883038 | 0.8591 | 2.79 |
| $(9 / 5 / 2)$ | 0.885254 | 0.8426 | 5.06 |
| $(10 / 4 / 2)$ | 0.878073 | 0.8101 | 8.39 |
| $(11 / 3 / 2)$ | 0.8675 | 0.8542 | 1.56 |
| $(12 / 2 / 2)$ | 0.824265 | 0.8205 | 0.46 |
| $(2 / 11 / 3)$ | 0.878754 | 0.9658 | 9.01 |
| $(3 / 10 / 3)$ | 0.882229 | 0.9658 | 8.65 |
| $(4 / 9 / 3)$ | 0.881027 | 0.9657 | 8.77 |
| $(5 / 8 / 3)$ | 0.875222 | 0.9658 | 9.38 |
| $(6 / 7 / 3)$ | 0.87344 | 0.9100 | 4.01 |
| $(7 / 6 / 3)$ | 0.870139 | 0.8767 | 0.75 |
| $(8 / 5 / 3)$ | 0.859935 | 0.8591 | 0.10 |
| $(9 / 4 / 3)$ | 0.816957 | 0.8440 | 3.20 |
| $(10 / 3 / 3)$ | 0.833153 | 0.8101 | 2.84 |
| $(11 / 2 / 3)$ | 0.851106 | 0.8542 | 0.36 |
| $(2 / 10 / 4)$ | 0.883001 | 0.9495 | 7.01 |
| $(3 / 9 / 4)$ | 0.885688 | 0.9495 | 6.72 |
| $(4 / 8 / 4)$ | 0.884438 | 0.9495 | 6.86 |
| $(5 / 7 / 4)$ | 0.878019 | 0.9495 | 7.53 |
| $(6 / 6 / 4)$ | 0.875526 | 0.9100 | 3.78 |
| $(7 / 5 / 4)$ | 0.871166 | 0.8767 | 0.63 |
| $(8 / 4 / 4)$ | 0.85348 | 0.8591 | 0.65 |
| $(9 / 3 / 4)$ | 0.759385 | 0.8426 | 9.88 |
| $(10 / 2 / 4)$ | 0.785318 | 0.8101 | 3.06 |
| $(2 / 9 / 5)$ | 0.90353 | 0.9167 | 1.44 |
| $(3 / 8 / 5)$ | 0.895831 | 0.9167 | 2.28 |
| $(4 / 7 / 5)$ | 0.896448 | 0.9167 | 2.21 |
| $(5 / 6 / 5)$ | 0.89965 | 0.9167 | 1.86 |
| $(6 / 5 / 5)$ | 0.884107 | 0.8767 | 2.84 |
| $(7 / 4 / 5)$ | 0.877788 | 0.8591 | 0.12 |
| $(8 / 3 / 5)$ | 0.884512 | 0.8440 | 5.96 |
| $(9 / 2 / 5)$ | 0.888152 |  | 3.24 |
|  |  |  |  |

As can be seen from Tables 2 and 3, the errors in the performance estimates were controlled within $5 \%$ in more than half of the cases. One can also see that the errors of some performance estimates during the experiments tended to be larger. The main reasons for these results are as follows:

In the simulation, we found that the eM-Plant software was not sensitive to the size and position of the buffer. Specifically, the size of the buffer did not affect the final output of the simulation experiment. However, when the machine could operate without workers, the output was always the same regardless of how the location of the two buffers changed and equal to the output of the bottleneck machine EOL (penultimate machine, up to 55 s of processing time). Thus, some simulation results deviated greatly from the theoretical values. For example, the production efficiency from DES reached $100 \%$ for the $(3 / 11 / 2)$ and (4/10/2) strategies presented in Table 2 and the (2/12/2) strategy presented in Table 3. Under careful observation, these results revealed that the third operator should only be responsible for the last two machines. In this case, man-machine interference did not appear in the bottleneck machine EOL group. Therefore, the output was the same as that in the case where the machine could be operated successfully without workers, and the efficiency reached $100 \%$.

The DES experiment was different from the analytical approach. The proposed analytical method applied to queue theory to each subsystem to estimate the efficiency loss caused by operator-induced machine interference. The arrival rate of the machine and the worker's service rate were obtained by averaging. Thus, the processing time between the different subsystems was different, and the fluctuations in the theoretical results were relatively modest. However, because the DES experiment was completely different and each machine was running independently, the concept of subsystems in the simulation was not obvious. Therefore, when the number of machines assigned to one operator was reduced, the performance estimation difference between the analytical and DES approaches became more pronounced. When each operator was assigned to more than three machines, the average number of machines for each operator, the typical error, was approximately $3 \%$.

Moreover, the time required for calculation was significantly shorter than that required by DES. Additionally, despite the lack of guaranteed convergence, the procedure was still convergent with a probability of approximately $100 \%$ under practical parameter ranges. Finally, the machine and buffer parameters are rarely known on the factory floor, with an accuracy better than $5 \%-10 \%$. We concluded that the recursive procedure shown in Figure 4 could be used to approximate the performance of the production system effectively and efficiently.

## 4. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we have described an analytical model that can be applied to analyze the performance of a lean production line composed of multi-process handling subsystems in series. This approach can also be used to assign the number of machines to each operator throughout the production line, while operators can be assigned to multi-process handling cells such that the total production rate is optimized. This is a novel approach and is not well addressed in the literature. It solves an industrial case study effectively and efficiently. The case study and DES experiment results revealed that the analytical method proposed in this paper approximated the discrete event simulation method with regard to the analysis of production line performance. In the analysis of the influence of operators on the performance of the production system, the analytical approach produced better results than the discrete event simulation method.

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