Computer Modeling of the Level Curves using Matrix Calculation and BSP Tree

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Abstract
The update of topographic maps which thirty years ago was done manually is nowadays done interactively. The level curves of the topographic maps are originally obtained either by photogrammetric restitution or by a generalization of this restitution. Many research projects have been put forward with the aim to realize the topographic maps from topographic data base (Braun et al., 2007). These works imply the development of a new series of production of digital maps, by exploiting the vector database and various complementary sources of the cartographic level curves which offers the possibility of extracting other topographical elements, such as collars and peaks. Many papers focus on the production of the level curves such as (Riegler et al., 2006 and Wang, 2008). The scientific issues discussed in this paper focus on the generalization of the level curves by an interpolation method based on computer modeling, starting from one or more base lines with reference points allowing to generalize the set of level curves that have essential roles for the realization of a topographic map. The procedures and the computer objects that are put in the place integrate the cartographic constraints, and show the interest of object oriented modeling and the operations on sets (union, intersection, difference ...), and by manipulation of geometric transformations based on matrix operations. The data structures of the BSP tree have a crucial role for the conservations of geological constraints (Cheaito, and Cheaito, 2014), as well as the manipulation of set operations in two dimensions.

1. Introduction

The present work takes place deep in the set of researches on the computer modeling of geometric level curves which are at the origin of the composition of geographic maps. Since tens of years, the hydrographic and oceanographic service of the navy has been trying to create an automatic or semi-automatic system for the generalization of cartographic maps (Guilbert and Lin, 2007 and Guilbert and Saux, 2008). Most of the researches insist on polygonal lines, in which the data are connected by line segments extracted from the trigonalization and by B-spline curves (Lutterkort and Peters, 2000 and Matuk et al., 2006). The geologists have characterized the geological objects according to their geometric shapes (Suppe and Connors, 2004). Meanwhile, several research studies have indicated that the geometric shapes of geological objects are closely related to their history, i.e. to the series of processes which during the geological evolution have determined the characteristics of the object (Spraggins and Dunne, 2002). For this reason, the geometrical definition of objects is based on the analysis of successive transformations of their original shape. Some of the objects within the geological scene have a certain form resulting from their appearance by more specific processes (sedimentation of a stratum, injection of granite ...) (Cheaito, 1993 and 2013). This form should be the subject of subsequent changes (folding, dislocation caused by faults, partial digestion by recent intrusive bodies. The geologists establish a link between the forms of objects that sometimes are very complex and the sequence of natural processes which affect the objects during their history. The characteristics of some simple geological processes induce deformations. Our work aims to establish a geographical map, a geological section or a block diagram based on the technic realized by the software PCheGeol (Cheaito, 2013). The matrix calculations and data structures have been encouraging towards establishing a geological scene not only by its border but with all its internal specifics that allow us to explore the richness existing inside a geological object.

2. Curves Levels

We define a level curve to be the set of the points of a topographical surface having same altitude, i.e., the points of intersection of the topographic surface with a horizontal plane. The topography of the surface of the land is established via **level curves**. It abides a set of points of same altitude.

The difference of altitude between horizontal planes is called the **equidistance** of level curves.

3. Principle of Establishing Level Curves

Consider a series of parallel and equidistant horizontal planes (H1, H2 and H3) (Figure 1), which ideally cuts a topographical surface, for example a hill. The intersections of the hill with these planes are reported on the plane P. These projections are called, level curves.

4. Properties of Level Curves

4.1 Geometric Properties

The level curves are formed with non-intersected polygonal outlines.

4.2 The Density

Taking into account the reliefs: the strong slopes are characterized by numerous tight curves; to fewly numerous separated curves corresponds a flat region or a region with weak slope. The more the slope is strong, the more the curves levels are approach. The more the slopes are weak, the more the curves are separated.

4.3 Top

The level curves levels are concentric; the altitude of the central point is higher than that of the curves surrounding it (Figure 2).

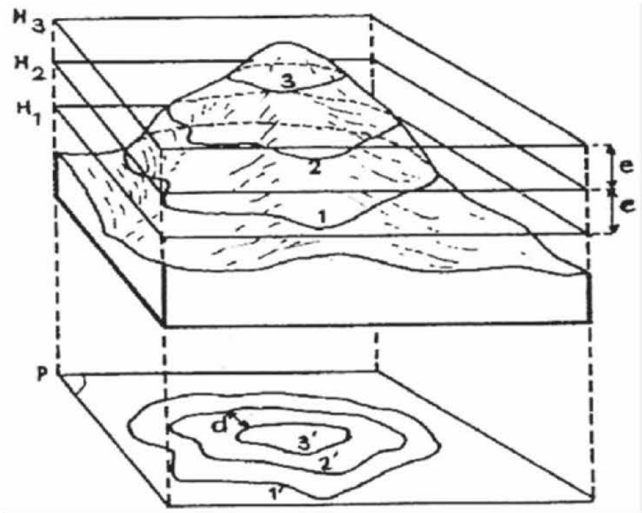


Figure 1: Principle of obtaining level curves

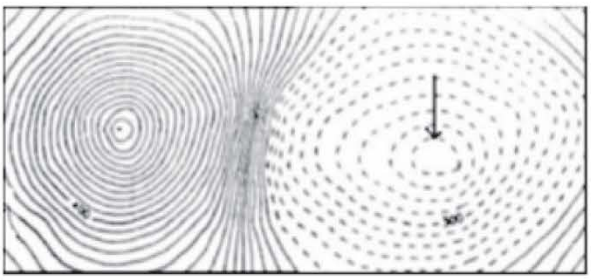


Figure 2: Top and Bottom

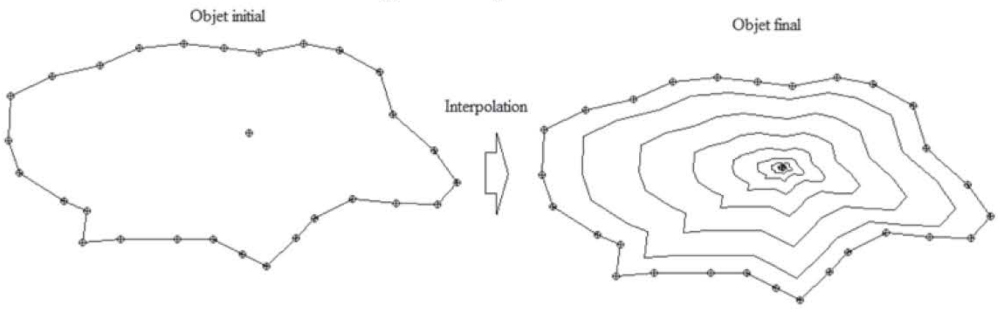


Figure 3: Level curves by interpolation realized by PCheGeol

4.4 Bottom

The level curves are also concentric, the altitude of the central point is lower than that of the curves surrounding it, sometimes an arrow indicates the center of depression (sometimes occupied by a lake) (Figure 2).

5. Computer Modeling

The modeling of level curves consists of: starting from an initial polygonal outline with a reference point representing the top or the deep point of the valley and by interpolation we can obtain the set of all level curves (Figure 3). These modes of modeling do not only allow the construction of a topographic map but also allows the establishment of the geometric form of the geological object in 3D facets and the construction of the corresponding BSP tree (Cheaito and Cheaito, 2014) (Figure 4).

6. Computer Objects

The data model that represents the level curves consists of establishing a set of points that are simulated by a set of computer objects able to accomplish our aim, hence, we integrate all the points of the segments describing the outlines by a linked list of segments that establish the geometry of the geological object. On the other hand, the address or pointers provide a simplicity of programming allowing a correspondence between real geological object and the data object as an entity describing the real object as a whole.

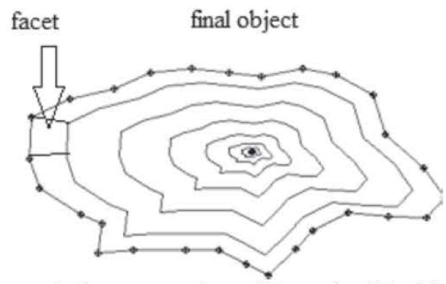
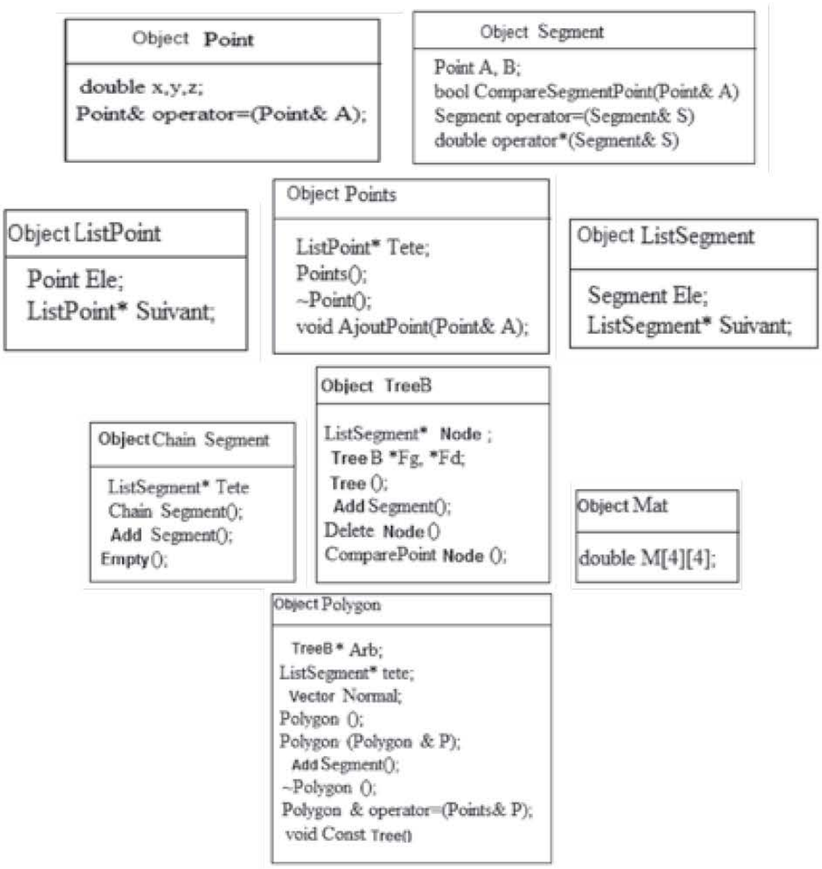


Figure 4: Representation of Facet by 3D object



7. Method of Passing from Polygon I to Polygon II

In our model we start from a basic polygon and a reference point to establish the level curves and the totality of all outlines or polygons describing the geological object as a whole to establish the polygon II from the polygon I and the reference point, we must enter the Polygon I and the top S (a, b, c), and the altitude of each point M'(x, y, z) of polygon II is obtained by a mapping φ (Figure 5).

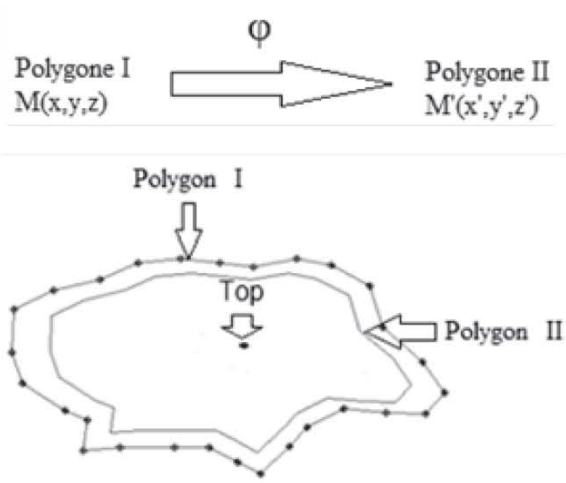


Figure 5: Passage from Polygon to Polygon II

For each point M (x, y, z) on Polygon I corresponds a point M' (x', y', z') on Polygon II. Polygon II is obtained by a multiplication of matrices of order 4 allowing the transition from real world to the world of the screen, then each point of polygon I must undergoe the following transformations:

Matrix M_1=Geometric translation of vector (-a,-b,-c)
Matrix M_2=scaling (1/e,1/e,1)
Matrix M_3= Geometric translation of vector (a,b,c)

The resulting transformation matrix $R=M_3*M_2*M_1$
Thus M'=R*M.

We need to indicate that the point S remains fixed and the Polygon II slowly approaches until it reaches the top S (see Figure 2).

Generality of φ:

$$\varphi = \left[\prod_{1}^{n} M_i * S_i * M_i^{-1}\right] * P$$

Equation 1

P= the transition matrix from real world to world of the screen
M_i: the translation matrix
S_i: the scaling matrix

Polygon $P_i = \varphi(P_{i-1})$

8. The Interior of the Polygon

The interior of the polygon is described by the construction of the BSP tree (Binary Space Partition). The principle of the binary partitions of the space. In its principle, the structure of a binary partition of the space (Cheaito, 1993) is based on the recursive use of plane separators dividing equally a given space. (Fuchs et al., 1980, Naylor, 1990 and Naylor and Thibault, 1986)

8.1 Definition of the Mixed BSP Tree, Representation of Objects

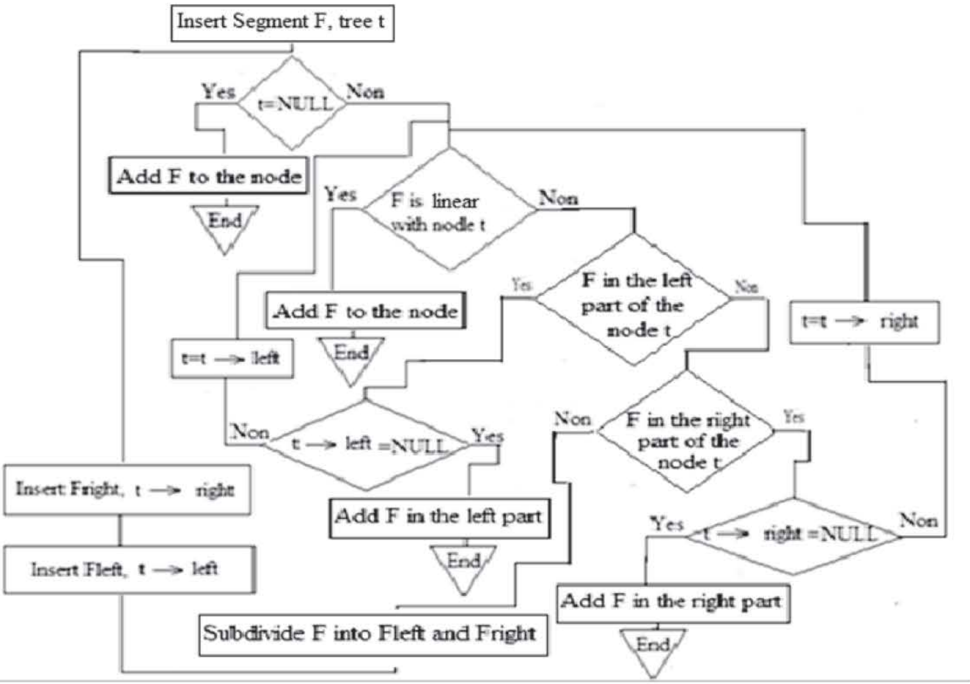
The representation of the boundary (Wang, 2010) of the described objects is not regarded as that in the available classic works done on the BSP tree (Wang and Manocha, 2013). We propose to include this by mentioning, the nodes of the BSP tree, the right sections or the plane separators that effectively correspond to the boundary of the sections as seen in (Lysenko et al., 2008).

This structure allows a very direct classification of any point of the space as exterior, interior or border of the object. (Wang, 2010). This classification is obtained while inserting the point M in comparison with the existing nodes of the tree until to classify it as a leaf (Fuchs et al., 1983):

- If M does not belong to any delimiter of the tree, then it is internal or external to the object. Therefore, it is found at the end of the procedure in a leaf of the tree and single, and classifies as "in" or "out".

- If M belongs to a delimiter in the tree. In this case, M can belong to the border of the object. It is then classified as "in" or "out" according to the obtained value. If this value is a point on one of the segments, then it is classified as "in" otherwise it is classified as "out".

The node of the tree is a list of segments. Algorithm for constructing the BSP tree:



9. Method of Realization of Level Curves

The intersecting level curves between two polygons A and B is the set of segments that can be found outside A and B, hence the constructed level curve by two polygons A and B (Figure 6) is equal to:

$$(A - B) \cup (B - A)$$

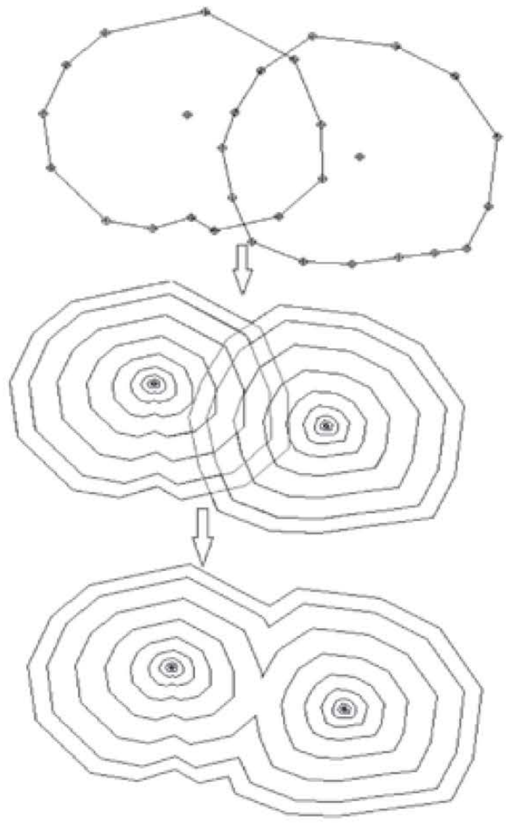


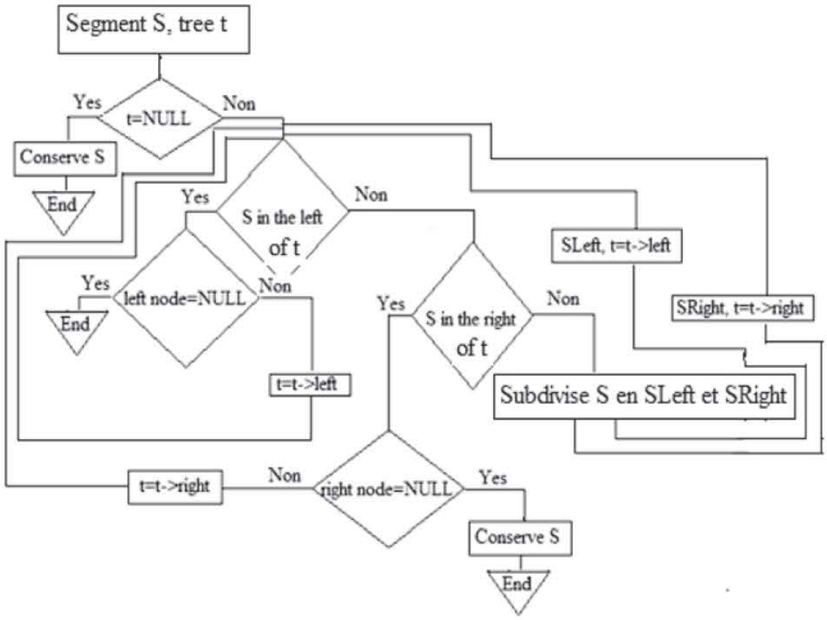
Figure 6: Realization of level curves

Generality: The number of level curves of n polygons P_i is equal to:

$$\bigcup_{i=1}^{n}\left[P_i - \bigcup_{j=1 \text{ and } j\neq i}^{n} P_j\right]$$

10. Algorithm for Constructing the Level Curves Between Two Polygons of Same Altitude

For each segment S of polygon P_1 we conserve in one instant chain of segments of the parts of S that fall in the outer region of the BSP tree that describes the interior and the exterior of the polygon P_2, and conversely for the segments of polygon P_2.



Algorithm for finding the parts of a segment S that are located outside a given polygon for a tree t

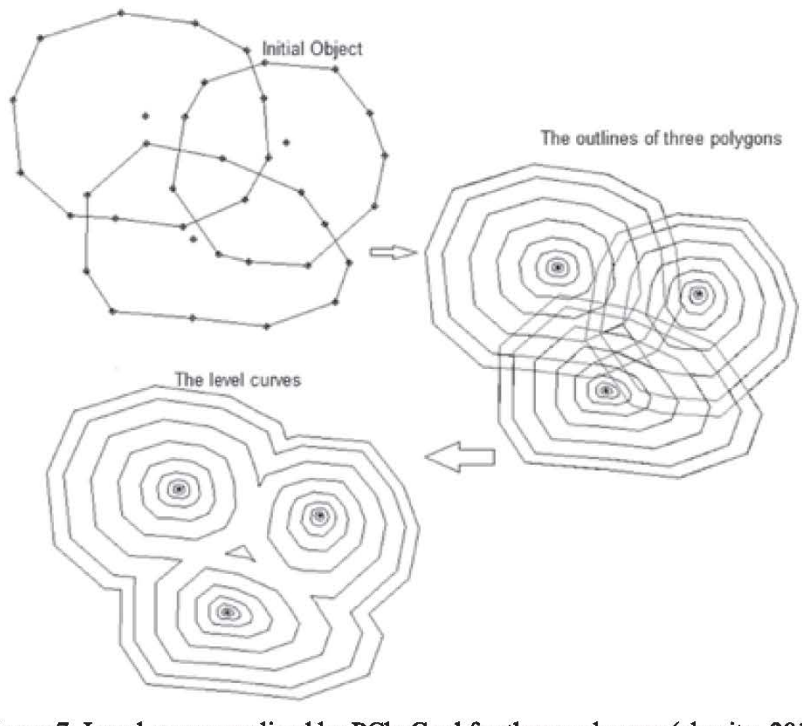


Figure 7: Level curves realized by PCheGeol for three polygons (cheaito, 2013)

11. Conclusion

In conclusion, the preliminary results obtained by this application are encouraging. The level curves constitute an effective mean for representing the form of a surface even if the procedures of analysis are more complex with this type of representation than a representation using mesh. The modeling of level curves plays an important role in achieving 3D geological objects (Perrin et al., 1993 and Cheaito and Cheaito, 2014a). Several types of approaches might lead the geometric modeling of level curves. In order to realize the level curves level we need an algorithm allowing to distinguish the types of relief. There are numerous methods like the method of (Chaudhry and Machanses, 2008) which uses the level curves, but we want a more precise method that is directly related to the field of digital modeling of land. The algorithm that we have developed consists of dividing the digital model of land by polygonal outlines then by interpolation we calculate the level curves. The use of BSP tree allows us to realize our objective but other methods can also play an important role in the objective to describe the interior and the exterior of such an object. In the coming future, it is possible to interpret the level curves by using three dimensional geological objects, as well as to realize the set of reliefs that play an important role for the estimation of the quantity of underground water. On the other hand, it should be pointed out that the information should be conserved in a data base model.

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