Geometric and Statistical Assessments on Horizontal Positioning Accuracy in Relation with GNSS CORS Triangulations of NRTK Positioning Services in Thailand

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Abstract
Thailand has implemented GNSS Continuously Operating Reference Stations (CORS) and Network Real-Time Kinematic (NRTK) Positioning services collaborated by several government agencies and universities for various applications. This research includes 114 GNSS CORSs; providing 143 triangle loops and covering 63 provinces nationwide, to determine and analyse the obtained horizontal positioning accuracy and their ground station geometries. Triangulation networks are categorised in terms of shapes and baseline distances. Well- and ill-conditioned triangles are defined in accordance with different baseline lengths. The horizontal positioning accuracy is computed using static and NRTK positioning determinations whilst static positioning results are defined as the ground truth. Student's T tests are used to ensure the significance of calculated results according to each test case. The results show obtained horizontal positioning accuracies are within the specified accuracy of less than 4 centimetres and do not show any significant differences based on the defined significant level of 0.05. This GNSS CORS triangulation network geometry scheme does not influence the computed horizontal positioning accuracy obtained from NRTK-Virtual Reference Station (VRS) GNSS positioning services; however, the positioning accuracy is still directly due to distances between nearest GNSS CORS or its triangulation network, therefore, further suggestions are provided.

Keywords: GNSS Triangulation Network Characterisation, Horizontal Positioning Accuracy, Hypothesis Test NRTK-VRS GNSS CORS

1. Introduction

Global Navigation Satellite System (GNSS) positioning determination using NRTK techniques has become popular and widely adapted in many countries, because it allows users to obtain higher accuracy of a few centimetres of more precise positioning solutions with quicker field procedures and more efficiency [1] [2] [3] and [4]. NRTK algorithm is introduced as the conventional RTK faces the baseline limitations by using the multiple reference stations. Observations from these multiple stations provide regional error corrections that could be used to increase the spacing between reference stations.

Thailand has implemented a network of CORSs and NRTK GNSS positioning services to underpin national developments in several dimensions, including social, economic and environmental. National CORS Data Center (NCDC) obtains GNSS signal measurements, processes and provide robust services gained from multiple reference stations through internet communication linkages. Measurements from multiple CORS are combined to generate error corrections throughout the nations based on network RTK data processing components of kinematic ambiguity resolution determinations at the reference stations, estimations of correction coefficients and computations of corrections at the rover stations. The NCDC is collaborative works amongst government agencies, research organizations, universities and international collaborations includingthe Department of Lands (DOL), Department of Public Works and Town & Country Planning (DPT), Royal Thai Survey Department (RTSD), Hydro – Informatics Institute

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(HII), Geo-Informatics and Space Technology Development Agency (GISTDA), National Metrology Institute of Thailand (NIMT), Chulalongkorn University (CU), King Mongkut's Institute of Technology Ladkrabang (KMITL) and Japan International Cooperation Agency (JICA) [5]. These agencies conduct mapping and surveying, geodesy as well as other related GNSS applications. They are responsible for the setting up, operation and mount their receivers therefore the location selections a national GNSS ground marks were considerable and challenging to establish the GNSS CORS in the widely distributed patterns in order to obtain the well-triangulation network condition.

Extensive research based on NRTK GNSS positioning performances has been conducted and pointed out that the obtained positioning accuracy and reliability directly are due to distances between nearest GNSS CORSs or its triangulation network, also known as a loop [6] and [7]. This similar performance test has also been studied in Thailand led by Charoenkalunyuta et al., [8] and by applying VRS retrieved from the early stage of DOL GNSS CORS network containing 11 stations in practice, whereas its NRTK coverage is within Bangkok metropolitan region. Each station spacing ranges from 27.8 kilometres to 125.6 kilometres, and its averaging distances are around 60 kilometres [8]. This test has been continued once again in 2019 by increasing the number of GNSS CORS to 99 and integrating GNSS observations from DPT CORS network of 15 stations; therefore, providing an accumulating CORS of 114, comprising 143-triangle loops and covering 63 provinces nationwide, has been tested successfully. This CORS network covers a vast area in Thailand with station distances of 25.1 km towards 198.6 km and their averaged distances of 82.7 km which formulates a triangulation network with average distances between each reference station ranging from 37.2 to 110.0 km whereas their averaged distances are 72.5 km [9]. This study has shown that shorter distances from reference stations, or dense triangulation networks, result in significantly better positioning accuracy and reliability compared to sparse distances or networks.

Based on, Thailand's geography, western areas starting from Rachaburi and Prachuabkhirikhan to the south are part of a narrow and relatively long peninsula, laid from north to south, and distinctively different from other regions, as presented in Figure 1.

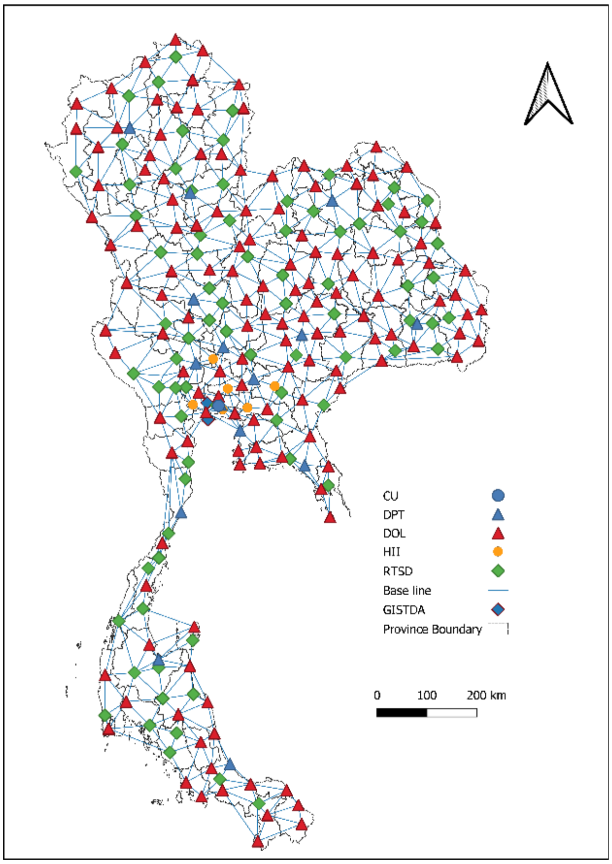

Figure 1: Geographical distributions of GNSS CORS and their triangulation networks providing NRTK-VRS GNSS positioning services

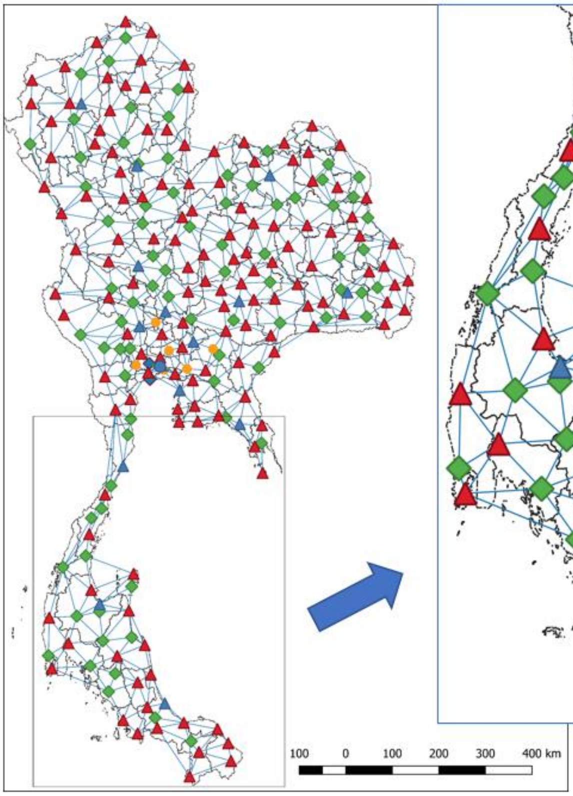

Figure 2: Illustrations on the triangulation network of GNSS CORSs in western and southern Thailand

It is a task amongst collaborated government agencies during the planning stage to install the widespread CORS in these particular areas. The measurement baseline between pairs of simultaneously operating receivers exceed the predesign distances to be around 70 kilometres and not widespread in the equilateral triangle. This currently operating NRTK-VRS GNSS CORS along these peninsular forms a relatively long and narrow isosceles triangle as specified in Figure 2. This should be worth considering in order to specify whether these geometrical patterns and its triangular sizes will result in and correlate to its horizontal positioning determinations and its corresponding performances; namely, accuracy and reliability achieved from this service network

This research is based on observations that have previously been explained. It is necessary to understand this GNSS CORS triangulation network in terms of its geometry and size to determine its relationship with the horizontal positioning results obtained, as well as their accuracy and reliability as assessment indices.

2. Experiments

The experimental steps are explained as shown in Figure 3.

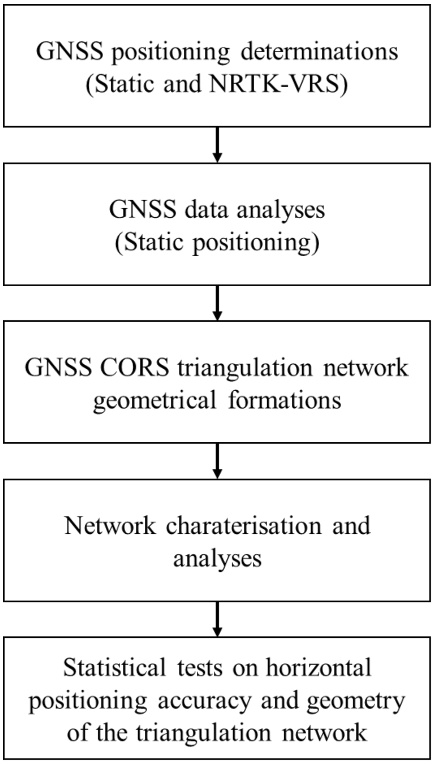

Figure 3: Computation steps on positioning performance analyses

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2.1 GNSS Positioning Determinations

GNSS observations are kindly provided by DOL as this study is part of a joint research collaboration project between CU and DOL to analyse and evaluate positioning accuracies achieved from the DOL NRTK-VRS GNSS positioning services, focusing on the GNSS CORS spatial distributions [9]. The field work of the DOL officers and their dedication to collecting GNSS observations across the country is greatly appreciated. The total number of 2,122 marker points of these 143 loops is observed and tested, retrieved from 114 GNSS CORS jointly collaborated from both DOL and DPT of 99 and 15 stations accordingly as illustrated in Figure 4. GNSS observations are made two folds as follows.

2.1.1 Static GNSS positioning

It is carried out by making 90-minute GNSS measurements with a 30-second interval and has to be measured in an open space without any obstacles.

2.1.2 NRTK-VRS GNSS positioning

It is carried out by making 15-minute GNSS measurements with a 1 second interval, where each observation takes 1 minute. The last two measurement sessions; the eighth and ninth round, have to be 30 minutes apart so that these measurements will be simultaneously with the previously defined static GNSS observations.

2.2 Static GNSS Data Processing

In this 90-minute static GNSS data processing, the solutions are assigned to be reference points (ground truth) by applying precise orbits and satellite clock error data; retrieving these GPS, GLONASS, Galileo and BeiDou precise products from Crustal Dynamics Data Information System (CDDIS) to each calculated triangle network.They were calculated using commercial processing software; Trimble Business Center (TBC) version 4.1, where, as two nearest GNSS CORS are assigned as preferable base stations according to previous research, it has been found that static positioning estimates from GNSS CORS with shorter baseline lengths provide better accuracy [10] as well as their horizontal and vertical precision to be better than 2 centimetres and 4 centimetres, respectively, at 95% confidence level.

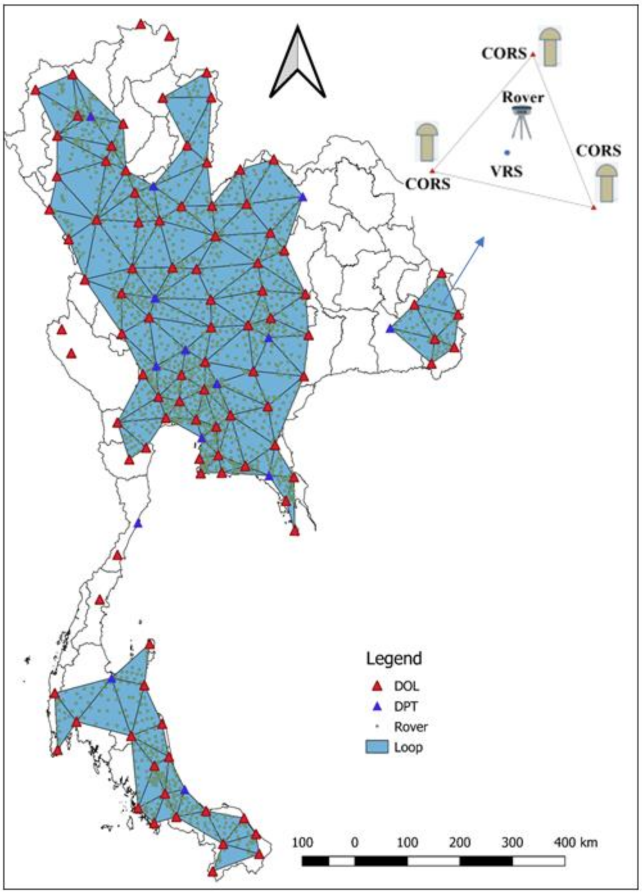

Figure 4: Locations and distributions of measured and tested marker points with their corresponding GNSS CORS network, as of June 2018

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2.3 Geometrical Characterisations of the GNSS CORS Triangulation Network

GNSS CORS triangulation networks are geometrically characterised by determining GNSS CORS baseline distances, angles, and shapes, as described below.

2.3.1 GNSS CORS baselines

They are determined from their Universal Transverse Mercator (UTM) coordinate systems: east and north coordinates (E, N) as specified in Equation (1);

$$D = \sqrt{[(N_1 - N_2)^2 + (E_1 - E_2)^2]}$$

Equation 1

Where:

- D is the distance of each GNSS CORS baseline (meter)
- N is GNSS CORS north-south coordinate (metre)
- E is GNSS CORS east-west coordinate (metre)

2.3.2 Triangulation angles

Angles of each GNSS CORS baseline are determined using trigonometry based on the law of cosines as described in Figure 5 and Equation (2) to Equation (4):

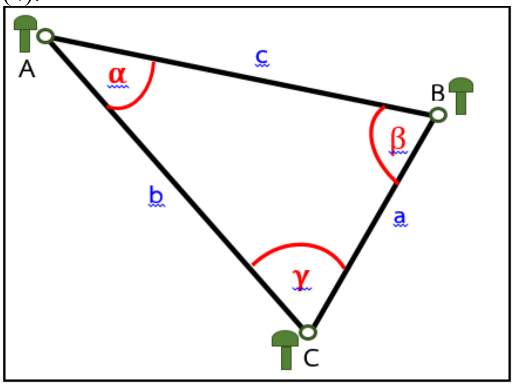

Figure 5: GNSS CORS baselines and their related angles in trigonometry

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

Equation 2

$$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos\beta}$$

Equation 3

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$

Equation 4

Where $\boldsymbol{A, B, C}$ represent GNSS CORS, $\boldsymbol{a, b, c}$ are GNSS CORS lengths (kilometres), $\boldsymbol{\alpha, \beta, \gamma}$ are angles of this GNSS CORS triangulation network.

2.3.3 Triangulation shapes

The shapes of each GNSS CORS triangulation network are separated into 2 main groups: namely well- and ill-conditioned triangles A well-conditioned triangle is a triangle that could define its angles causing less affect toward their lengths, almost equilateral or nearly so, where each angle is nearly 60° [11]. In practice, if its smallest angle is greater than 30° and its largest angle is less than 120°, this is a well-conditioned triangle [12]; whilst, if its smallest angle is less than 30° or its largest angle is greater than 120°, this is an ill-conditioned triangle as specified in Figure 6. GNSS CORS triangulation networks in this study could be geometrically identified as shown in Table 1. These GNSS CORS triangulation networks could be further separated based on their baseline lengths into 4 groups of 30-50, 50-70, 70-90 and 90-110 km [9] and their corresponding tested points are shown in Table 2.

3. Data Analysis

Analyses of data are on the GNSS CORS triangulation networks' geometrical characteristics and Statistical tests on relationships between horizontal positioning accuracy and geometrical characteristics of triangulation networks.

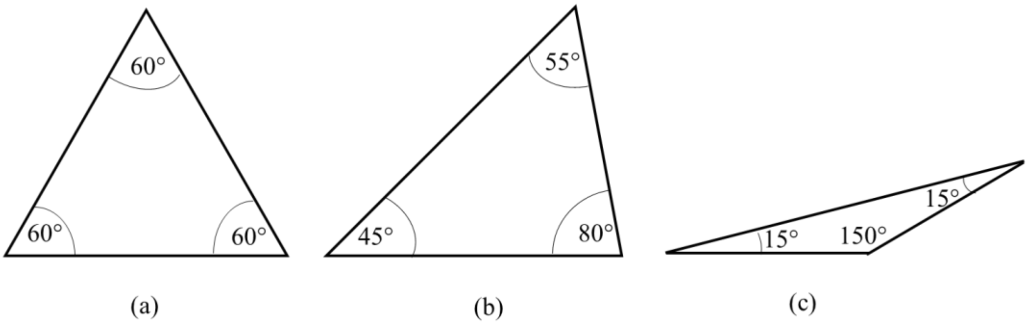

Figure 6: Definitions for the defined triangle shapes (a) an *ideal*-conditioned (b) *well*-conditioned (c) *ill*-conditioned

Table 1: Characteristics of the GNSS CORS triangulation network: geometric shapes, number of loops, and tested points

Category	Shape	Numbers of loops	Tested points
1	Well-conditioned triangle	109	1,614
2	Ill-conditioned triangle	34	508
Total number of		**143**	**2,122**

Table 2: Numbers of well- and ill-conditioned triangles according to each GNSS CORS baseline

Group	Averaged baseline (km)	Well-conditioned triangle		Ill-conditioned triangle	
		Numbers of loops	Tested points	Numbers of loops	Tested points
1	30-50	16	238	0	0
2	50-70	34	499	13	193
3	70-90	39	577	15	225
4	90-110	20	300	6	90
Total numbers		**109**	**1,614**	**34**	**508**
		143 loops		**2,122 tested points**	

3.1 Analyses of the Geometrical Characteristics of the GNSS CORS Triangulation Networks

This is statistically measured by horizontal positioning accuracy with a specified constraint of not more than 4 centimetres by determining its horizontal root mean square error (RMSE). The NRTK- VRS GNSS positioning estimates are compared with the static GNSS positioning solutions; previously defined as the ground truth, giving differences of horizontal RMSE as described in Equation (5) [13].

$$\mathbf{RMSE}_{hor} = \sqrt{\frac{\Sigma[(N_{static} - N_{i(VRS)})^2 + (E_{static} - E_{i(VRS)})^2]}{n}}$$

Equation 5

Where:

RMSE$_{hor}$ is horizontal root mean square error (metre)

N$_{static}$, E$_{static}$ is static GNSS positioning solutions in northing and easting coordinates (metre)

N$_{VRS}$, E$_{VRS}$ is NRTK-VRS GNSS positioning solutions in northing and easting coordinates (metre)

i is 1-second measurement interval (observation epochs)

n is the number of points tested.

3.2 Statistical Tests on Relationships between Horizontal Positioning Accuracy and Geometrical Characteristics of Triangulation Networks

Statistical processing software named IBM SPSS Statistics Subscription is used in this research data analysis. Student's t tests have been performed to determine the relationship between the horizontal positioning accuracy obtained and the geometry of the triangulation network, previously defined as triangles of well and poor condition, to demonstrate sample independence. Two defined hypotheses are described below.

H_0: $\mu_1 = \mu_2$ (Averaged horizontal positioning accuracy does not correlate with the geometrical characteristics of the GNSS CORS triangulation network)

H_1: $\mu_1 \neq \mu$ (Averaged horizontal positioning accuracy correlates with the geometrical characteristics of the GNSS CORS triangulation network)

When the null hypothesis (H_0) is not true when the 2-tailed sigma (or P-value) results less than or equal to the assigned statistical significance of 0.05 (Sig. (2-tailed) ≤ 0.05) [14]. These test samples are designed for sample groups 2 to 4 (see Table 2) of those well and ill condition triangulation formations comprised of 6 groups with station baselines of 50-70, 70-90 and 90-110 kilometres, respectively. The test results are explained in detail in the following session. It is noted that group 1 with the baseline length of 30-50 kilometres is not taken into account, as they are all well-conditioned triangles.

4. Results

The results of the geometry of the GNSS CORS triangulation network and their hypothesis test results are presented below.

4.1 Geometric Analyses

GNSS CORS triangulation networks have been formulated and processed based on CHC Precision Positioning Service Software (CPS) [15] in order to determine the NRTK VRS positioning results. Their geometric tests are shown in Table 3. The average horizontal positioning accuracy of the well and ill-conditioned triangles is pretty much the same as 0.034 and 0.035 meters, respectively. GNSS CORS triangulation networks and their baseline lengths as previously separated into 4 main groups according to their triangle shapes (see Table 2) of 30-50, 50-70, 70-90 and 90-110 km, respectively, as shown in Table 4. The results are relatively the same for every defined category, whereas the differences are at millimetre-level accuracy.

4.2 Statistics Analyses

This statistics analysis focuses on the three latter groups; 2 to 4 (see Table 4). The number of test samples and their statistics analysis results are presented separately in 3 test cases based on their GNSS CORS averaged baseline lengths of 50-70, 70-90 and 90-110 kilometers as shown in Table 5, Table 6 and Table 7 respectively.

Table 3: Average horizontal positioning accuracy based on GNSS CORS geometries

Category	Shape	Numbers of loops	Test simple (points)	Horizontal RMSE (m)
1	Well-conditioned triangle	109	1,614	**0.034**
2	Ill-conditioned triangle	34	508	**0.035**

Table 4: Average horizontal positioning accuracy based on baseline lengths of GNSS CORS and their triangle shapes

Group	Averaged baseline (km)	Well-conditioned triangle			Ill-conditioned triangle		
		Numbers of loops	Test simple (points)	Horizontal RMSE (m)	Numbers of loops	Test simple (points)	Horizontal RMSE (m)
1	30-50	16	238	**0.032**	0	0	-
2	50-70	34	499	**0.036**	13	193	**0.032**
3	70-90	39	577	**0.034**	15	225	**0.037**
4	90-110	20	300	**0.033**	6	90	**0.038**

Table 5: Hypothesis tested results from the GNSS CORS network with an averaged baseline of 50-70 kilometres

Shape	Number of test simple (points)	T-tests for the equality of means				
		Average	Standard deviation	t	df	Sig. (2-tailed)
Well-conditioned triangle	85	0.026	0.024	0.217	173	0.828
Ill-conditioned triangle	90	0.025	0.017			

Table 6: Results tested for hypotheses from GNSS CORS network with averaged baseline of 70-90 kilometres

Shape	Number of test simple (points)	T-tests for the equality of means				
		Average	Standard deviation	t	df	Sig. (2-tailed)
Well-conditioned triangle	91	0.023	0.018	0.079	179	0.937
Ill-conditioned triangle	90	0.022	0.019			

Table 7: Hypothesis tested results from the GNSS CORS network with an averaged baseline of 90-110 kilometres

Shape	**Number of test simple (points)**	**T-tests for the equality of means**				
		Average	**Standard deviation**	**t**	**df**	**Sig. (2-tailed)**
Well-conditioned triangle	90	0.023	0.020	-0.547	178	0.585
Ill-conditioned triangle	90	0.025	0.029			

In the case of the averaged baseline length of GNSS CORS of 50-70 km, the obtained 2-tailed sigma is 0.828 (see Table 5) which is greater than the significant level assigned of 0.05 (Sig. (2-tailed) > 0.05) therefore, the null hypothesis test (H0) is true, which implies that the mean values tested of the well and ill conditions are not significantly different at the assigned significant level of 0.05. This also applies to the following test cases where the ground station baseline lengths are 70-90 and 90-100 kilometres, the 2-tailed sigma obtained are 0.937 and 0.585 (see Table 6 and Table 7) accordingly; therefore, the tested results are not significantly different.

5. Conclusions

This research divides GNSS CORS triangulation networks into two main groups of well and ill condition triangles according to their geometry. The computed horizontal positioning accuracy slightly varies from 0.034 to 0.035 metres; better than the DOL specified accuracy of 4 centimetres. Their station baselines are later taken into account, making 4 more groups of 30-50, 50-70, 70-90 and 90-110 kilometres and resulting in their averaged horizontal positioning accuracy ranges between 0.032 and 0.038 metres; also, better than the desired accuracy of 4 centimetres. This ensures that the GNSS CORS geometries; namely, their shapes and baseline lengths, do not affect their required horizontal positioning accuracy. However, their accuracy increases directly when the GNSS CORS baseline is longer, which is consistent with extensive research previously.

The relationships between horizontal positioning accuracy and the geometry of the GNSS CORS triangulation network using Student's T tests could be concluded that the well and ill conditioned triangles of every defined GNSS CORS baseline of 50-70, 70-90 and 90-110 kilometres do not show significant differences based on the defined significant level of 0.05. This could lead to the conclusion that the geometry of the GNSS CORS triangulation network does not influence the horizontal positioning accuracy calculated from the NRTK-VRS GNSS positioning services in Thailand.

6. Suggestions

The following suggestions are provided based on this research work on the relations between the GNSS CORS geometry and the horizontal positioning accuracy obtained from this NRTK-VRS GNSS positioning scheme (as of 2019) in Thailand.

- On GNSS positioning observations, they are not purposely designed to directly comply with the geometry of the GNSS CORS triangulation network. The research hypotheses and methodologies are designed from measurements provided at the time. Further works on data collection, research methodologies, data processing algorithms and analyses shall be highly considered to be done to ensure analyses results and for further references.
- On data processing software, this research uses only commercial software; Trimble Business Centre (TBC) 4.1. Research processing software: namely, Bernese or GIPSY-OASIS II, shall be further employed.
- On GNSS receivers, a single type of receiver; CHC i-80, is used; hence, more receiver types and models, especially the ones used by government institutes and private survey users, shall be taken into tests.

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