

# Development of Spatial Uncertainty-Aware Visualization of Historical Traffic Queue Length

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## Abstract

*Traffic queue length at intersection is one of the most important measures of traffic signal performance and the traffic signal optimization. The spread of the ride-hailing services makes GPS data widely available with large area coverage, allowing us to quantify the queue length in a wide area without the need for additional installation cost like conventional sensors. Hence, queue length can be observed simultaneously and the relationship between intersections can be captured. Due to the current myriad of available queue length data, an analysis tool is needed to turn the data into useful knowledge to support traffic operations and management. However, the current queue length visualization only focuses on a real-time fashion, which cannot provide information about the change of the queue or its uncertainty. Such information is crucial for understanding the probability of the queue to be spillover, as well as determining the lengths of turning lanes to minimize blockage. In this research, we developed a spatial uncertainty-aware visualization by applying a conventional technique called box plot to visualize queue lengths on a map derived from taxi GPS data. A prototype of a web mapping application has been implemented and tested with users. The results indicated that participants with a statistics background have an advantage over those without a statistical background to understand the box plot. However, all of them were able to identify the uncertainty of the queue lengths as well as to compare the uncertainty between intersections and times.*

**Keywords:** Congestion, Delay, GPS, Queue Length, Traffic Management

## 1. Introduction

Traffic queue length at intersection is one of the most fundamental performance measures that is crucial to traffic management as it can reflect the performance of traffic signal control [1] [2] and [3]. Especially, when human control is in place, traffic signal managers must keep an eye on the length of the traffic queue to see whether the queue spills over to subsequent junctions.

The length of the queue can be observed by eyes, or via CCTV cameras. However, when the queue is longer than the line of sight, it cannot be easily observed. Alternative ways to measure queue length are available. Over-the-road sensors such as inductive loop detectors, videos, and license plate recognition [4] are commonly used to measure real-time traffic queue length. However, the additional installation cost of such sensors is expensive, making it difficult to observe queue length in a wide area. The current spread of ride-hailing services has enabled trajectory data from smartphones equipped with GPS. Whether it be a short or long reporting interval,

GPS data can be used to estimate the maximum queue length, i.e., the maximum length between the stop line and the last vehicle affected by the traffic light [5].

It is inevitable that the availability of GPS data will continue to grow, thus it is possible to derive queue length covering a wide area. This led to an analysis of the traffic queue from a whole network view. Hence, the interaction between intersections can be observed simultaneously. The queue length information is important to traffic signal management when synchronizing traffic signals between intersections. So, such information is crucial to those who manage traffic signals especially when fully automatic traffic control is not available.

Due to the current myriad of queue length data, an analysis tool is needed to help decision making in traffic light management. However, current analysis tools on a network wide are limited to simulation system such as VISSIM [6], which display traffic queue length in a real time fashion.

Such system, which focuses only on representing the current situation of the queue length, neglected information about the change of queue — its uncertainty information. Although the system can display the historical queue length as an animation, it can easily cause information overload when all intersections are displayed all together, leading to change blindness (people cannot notice changes in an animation) [7] [8]. The uncertainty of the traffic queue length is essential information for improving traffic light control strategy. As mentioned in [9], the probability of queue to be spillover should be controlled within the 95th and 99th percentile of queue lengths, especially of turning lanes, to avoid blockage.

In this research, queue length visualization is developed to support traffic signal management. Box and Whisker plot (aka. box plot) technique was applied to visualize the length of the queue derived from observed taxi GPS data. Conventionally, the box plot has been employed to visualize the uncertainty of univariate data. Here, the box plot is laid on top of a road map as a glyph to depict the queue-length uncertainty, i.e., its median with its quartiles. Visualization has been implemented in a web-based application. The technique attempted to display spatial uncertainty of the queue length on the road network.

## 2. Literature Review

### 2.1 Queue Length and Traffic Management in Bangkok

Traffic queue length plays an important role in traffic management in Bangkok where traffic lights are typically controlled manually especially during peak hours. The traffic police, who monitored traffic queue length in a police box at an intersection, observe the queue both by their eyes and by CCTV cameras. Each area is controlled by different police stations, sometimes causing communication failure.

As stated in [10], the main problem of traffic congestion in Bangkok is that no lead agency in traffic management. Although traffic police control center has been established (<http://www.trafficpolice.go.th/read/160>), no data has been employed to traffic management currently. This research attempts to address the issue of traffic signal management by introducing queue length visualization, which designs to collect and display queue length data from numerous intersections, allowing traffic police to see queue length as a whole.

### 2.2 Visualizing Uncertainty

Uncertainty is defined “to include statistical variations or spread, errors and differences, minimum-maximum range values, noisy, or missing

data” [11] which can be categorized as *statistical*—the distribution of the data; *error*—the difference between correct datum and an estimate, or *range*—an interval of the data. Geospatial data consists of geographic features containing location and attribute information (characteristics about those features). Time is also considered an important part of geospatial data. The uncertainty of geospatial data is usually related to the three parts of it, whether it is an ambiguity/vagueness of location information of geospatial features, of attributes such as classes of objects, or of temporal part. The meaning of geospatial uncertainty can be categorized as accuracy/error, precision, completeness, consistency, lineage, currency, credibility, subjectivity, and interrelatedness. A sample of different types of uncertainty on attribute, location and time is given by [12].

Two popular techniques depicting uncertainty are error bars and box plots. The error bar is a line that its length usually represents a width of one standard deviation of the data. The box plot, given more information than the error bar, displays five values summarizing the distribution of data which are: median, lower quartile, upper quartile, minimum and maximum. With its popularity of box plot, many alternative techniques have been developed from box plots such as violin plots, notched box plot, and bee swarm plot. Such techniques are suitable for univariate data.

Visualization techniques developed for geospatial uncertainty usually involve location information. Therefore, most techniques applied on top of maps. One categorization is whether a technique is *extrinsic* or *intrinsic* [13]. *Extrinsic* is the use of glyphs or graphical objects to depict uncertainty. An example is Pang [14] developed uncertainty glyphs to represent uncertainty in directions and magnitudes of vector fields. Another example is from Cedilnik and Rheingans [15] and Kinkeldey [16], who put a grid on top of a map and applied distortion to represent the degree of uncertainty.

In contrast to the *extrinsic* approaches, visual variables [17] such as hue, transparency, and size are applied to geometries of spatial objects (*intrinsic* approach). An example is Clarke and Keuper [18] tested various intrinsic methods to represent uncertainty of urban growth from the simulation model, such as using bicolour and different shade schemes. Lim et al., [19] displayed flood uncertainty using different levels of lightness. Cheong et al., [20] compared 5 different visual variables including boundary, hue, value, transparency, and texture with using text to represent the likelihood of wildfire areas.

The application of 3D visualization can display uncertainty with time. For example, Kuijpers et al., [21] applied a 3D space-time prism to display the uncertainty of trajectories. Huang and Wong [22] displayed uncertainty of trajectories data using space-time cone paths on 3D. Another example is the work by Etienne et al., [23], which represented movement points from GPS data using the 3D box plot.

### 3. Methods

Research procedures and methods are divided into sub-stages from data collection, queue length estimation, visualization of uncertainty, and user evaluation.

#### 3.1 Data Collection and Queue Length Calculation

The data was a part of Rama IV model project, collected from the Grab application for drivers in Bangkok, Thailand, which focused on the Rama IV road consisting of many major intersections as shown in Figure 1. One of the busiest intersections on the Rama IV road is Sala Daeng intersection that crosses over Silom road, one of the major streets known for its financial and business neighbourhood. Another busy intersection is the intersection connected to an expressway network near Khlong Toei, which is very busy during peak hours as people are going to or

leaving the city centre. Khlong Toei is also an area of a big fresh market. Next to Khlong Toei, there are Ari and Kasemrat intersections which are popular routes to the Sukhumvit road, another major road in Bangkok. Next to the Ari intersection, there is the Bangkok University intersection, which is another shortcut to Sukhumvit Road. Although Grab had various services in Bangkok, including Grab Food and Grab Bike, only data from Grab Car and Grab Taxis was received. GPS data from Grab driver smartphones was originally collected for service purposes, but it can be applied to quantify traffic situations.

The Grab GPS data consist of geographic location (latitude and longitude), timestamp, encrypted driver id, along with direction and speed. The data has been map-matched and anonymized by Grab to protect users' privacy. The data set was collected between August 2019 and December 2019, approximately 56,539,344 points from taxis. Here, the queue length was estimated based on shock wave theory [24], which is the transition boundary of traffic that can be used to estimate the queue length. The method is similar to [25]. The shock wave theory can be shown as the space-time diagram in Figure 2, where the x axis is the time and the y axis is the distance to the stop line of a traffic signal.

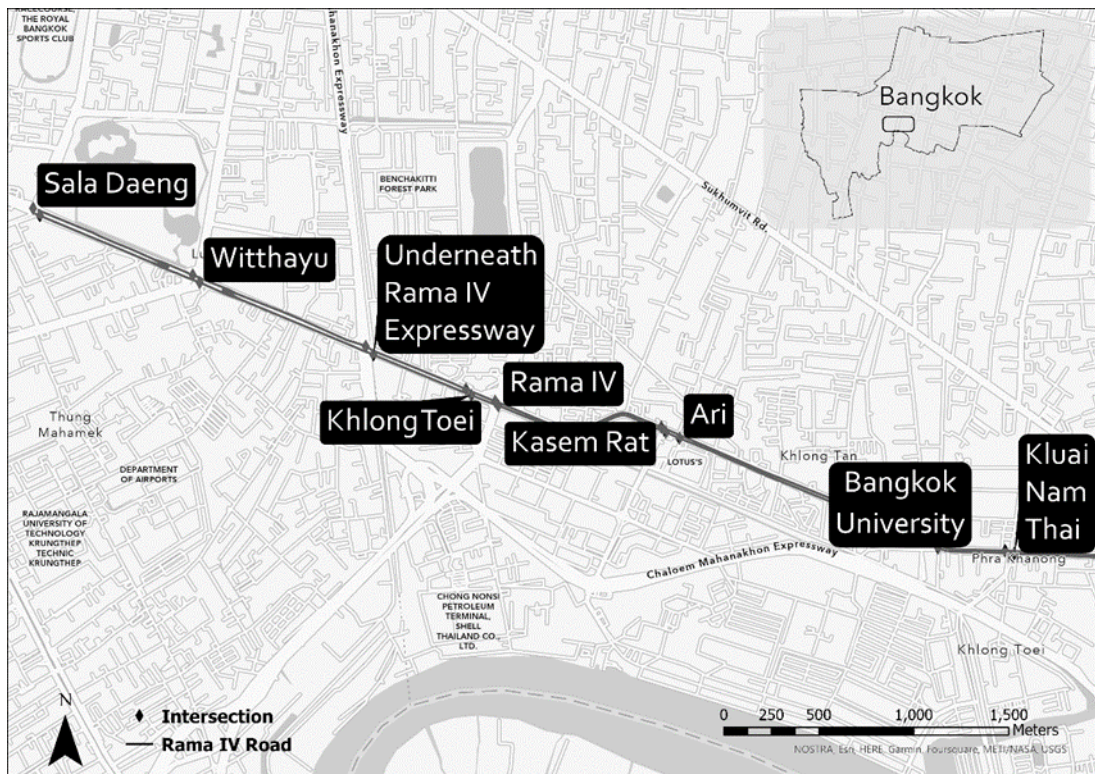


Figure 1: Study area

As the signal turns red, a vehicle stops in the queue. When the next vehicle reaches the queue, the queue accumulates. The area between two blue lines (shown as the grey area) in Figure 2 indicates the transition boundaries between moving and stopping, and also stopping and moving. The intersection of the two blue lines indicates the maximum queue length in a traffic light cycle.

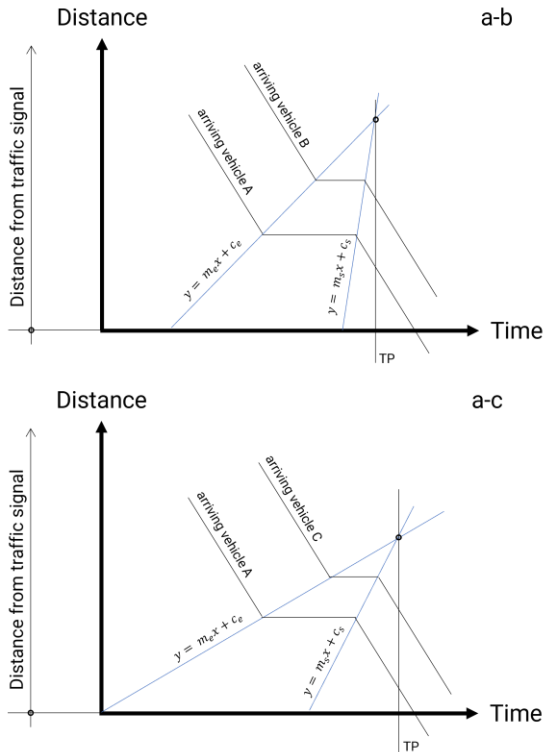


Figure 2: Space-time diagram showing the estimated queue length from vehicle A and B and from vehicle A and C.

To estimate the maximum queue length, the speed ( $s$ ) between two consecutive points of a vehicle was calculated by distance ( $d$ ) and time ( $t_1$  and  $t_2$ ) between the two points as follows:

$$d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

Equation 1

$$s = \frac{d}{t_2 - t_1}$$

Equation 2

Where:  $x_1$  and  $y_1$  = the Universal Transverse Mercator zone 47 position of points at the first location  $x_2$  and  $y_2$  = the Universal Transverse Mercator zone 47 position of points at the second location  $t_1$  and  $t_2$  = the timestamp at location 1 and location 2, respectively.

Points with 0 speed and 0 direction were filtered out because most of them were due to GPS loss signals. Therefore, points with a speed less than 1 meter per second were considered as stop points.

Since the traffic cycle time was not known, we estimated it by creating the blue lines for every pair of vehicles as shown in Figure 2. Suppose there were only three vehicles arriving at an intersection, namely a, b, and c, the transition boundaries between a and b and a and c were estimated. By extracting the first stop point at which a vehicle started to stop at an intersection, the first blue line can be represented as follows:

$$y = m_e x + c_e$$

Equation 3

The points that a vehicle starting to move of can be estimated with the second blue line as shown in Figure 2, and the line can be represented as follows:

$$y = m_s x + c_s$$

Equation 4

Traffic cycle time was estimated by trying to pair every possible vehicle within 30 minutes; only two lines that do not contain other moving points in their boundaries are possibly the maximum queue length. In Figure 2, the blue lines from vehicle a and c are given the farthest distance, so it was chosen to estimate the maximum queue length. The blue lines were recalculated again based on every vehicle within the cycle time using least square estimation as shown in Figure 3, and the maximum queue length is the intersection between the two new blue lines.

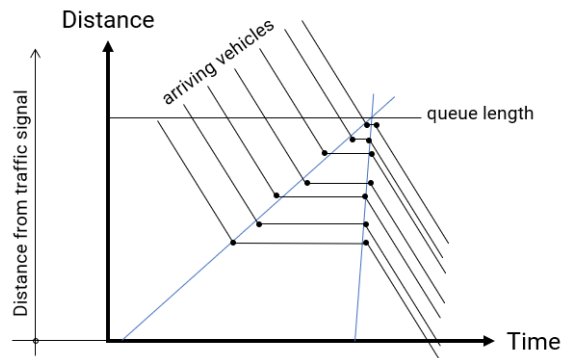


Figure 3: Queue length estimation of all arriving vehicles

All the queue lengths calculated from above within 30 minutes are averaged because no data were available in some cycles. The median, 25<sup>th</sup> percentile, and 75<sup>th</sup> percentile of all stop points within 1 month were calculated. Since this method estimated queue length from only taxi data. It is possible that the queue can be longer than estimated because there were possibly other vehicles behind.



### 3.2 Visualization Design

The design is based on the user requirements, where most intersections are controlled manually by traffic policemen, who manually observe queue length at signal intersection. Based on an interview with traffic policemen, they observed the queue lengths by comparing them with landmarks including roadside buildings or bus stops. When the queue is over the landmarks, they may change their traffic control strategies. When eye observation is limited by line of sight, CCTVs are used to help them to monitor traffic queue length. The problem of traffic control is that there is no real traffic authority to oversee traffic queue as a whole.

As a result, a policeman has to coordinate with one another to synchronize traffic signals which can occasionally result in communication issues during rush hours. Another problem is that there is no quantifiable performance measure of traffic control signal strategy. Hence, the effectiveness of traffic control strategy created by police is still unclear. Based on the requirements mentioned above, the two important components should be highlighted. One is the traffic queue length that should be easily perceived. Another one is a level of uncertainty of traffic queue. If the queue is uncertain, traffic control strategy may need to be adjusted. The design was aimed to display the queue length as well as its uncertainty. The box plot technique was chosen because it is one of a long-development conventional visualization techniques that can be placed on a limited narrow area, that is, a road.

To apply box plot, the distance between the first and the third quartile is displayed as a thick red line on the road, similar to a box component. Following the conventional box plot, the median was displayed

as a yellow line within the box. The lower and upper whiskers were shown as dashed lines representing the lower 25% of the queue length and the upper 25% of the queue length, respectively. Arrows have been applied to the whisker part to indicate vehicle's running direction. The box plot was aligned on a base map allowing users to see where the end of the queue length was located. Figure 4 compares the designed box plot with the conventional box plot overlaying on a swarm plot, another visualization technique showing each individual data point.

### 3.3 Implementation

The visualization was implemented using ArcGIS Dashboard, which is hosted on ArcGIS online, a cloud-based mapping service. Fundamental shapes available in ArcGIS online were applied to create the box plot. The box plot is overlaid on the map to display spatial uncertainty of the queue length. A prototype has been implemented as a web-based interactive application with additional interactive features to allow users to explore queue-length data at different times. The application has been developed using ArcGIS Dashboard. The system is hosted in ArcGIS Online—a cloud-based mapping service. The fundamental shape available in ArcGIS online was applied to create a box plot. The box plot is overlaid on the map to display spatial uncertainty of the queue length.

The overall prototype is shown in Figure 5. The box plot was presented on top of the based map. The red bright dots represent the location of traffic lights while the pale red ones represent the location of pedestrian crossing lights. Users can specify day of week and time from the drop-down box on the top of the page.

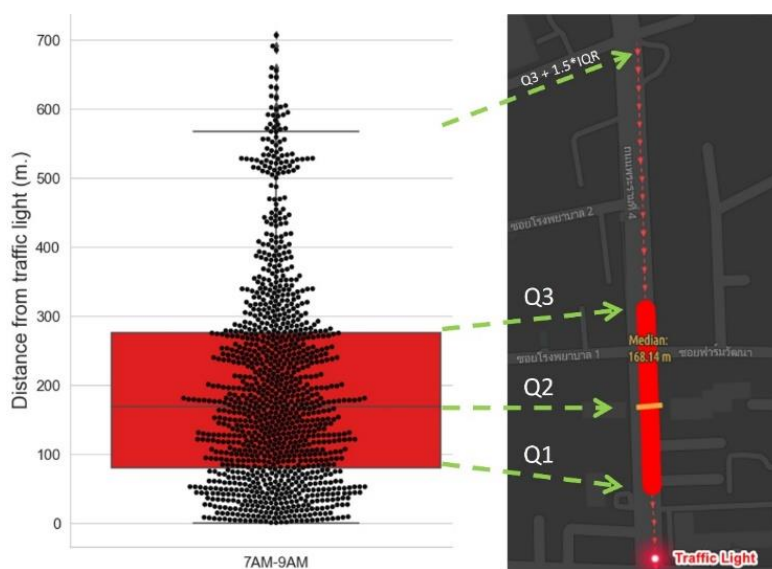


Figure 4: Box plot

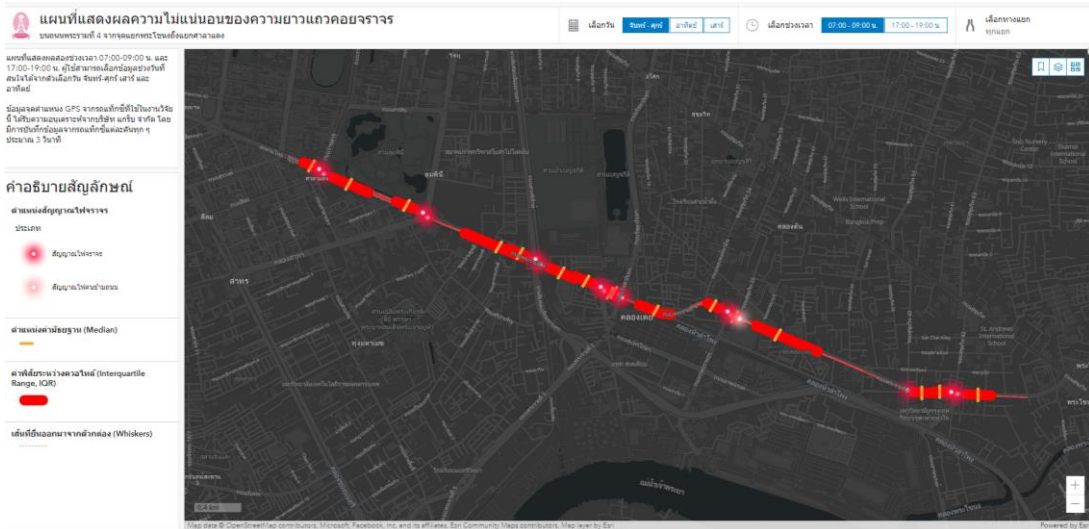


Figure 5: Interface

Saturday 7-9 AM



Saturday 5-7 PM



Figure 6: Overview of queue length on Saturday 7-9 AM and 5-7 PM

### 3.4 Interface

Due to data availability, not every hour of the day is available. So, the whole data (between August 2019 and December 2019) was aggregated into weekdays, Saturdays, and Sundays because we found that the queue behaviour is different between Saturdays and Sundays. Time of day (07:00h-9:00h or 17:00h-19:00h) can be also chosen. A sample of an overview result is shown in Figure 6. It can be observed that the queue length on Saturday evening (5-7 PM) was more uncertain than on Saturday morning (7-9 PM) as almost all the boxes in the evening were longer

than those in the morning. A zoom-in view of the box plot can reveal the trend of the queue within each intersection. By comparing different time periods, the result shows that the queue on weekdays is usually longer than on weekends. For example, Figure 7 shows the box plot on the map of the Ari intersection. During the morning rush hour (7PM-9PM) on weekdays, the median queue was longer, about 335.52 meters away from the traffic light, compared to those on weekends, which was just 80.89 meters on Saturday and 58.1 meters on Sunday.

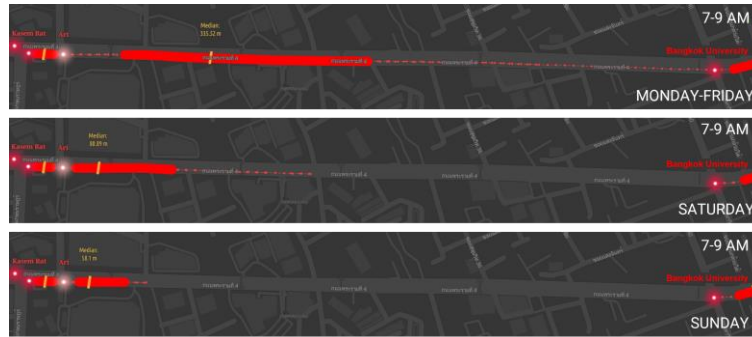


Figure 7: Comparison of the average queue length between weekdays, Saturday, and Sunday



Figure 8: Zoom-in view around Lumpini

The uncertainty of the queue on weekdays is also greater than on weekends. As shown in Figure 7, the thick red line of the box plot is longer than those on weekends, which means that more than 50 percent of the queue length on weekdays was longer than on weekends. The whisker of the box plot on weekday is beyond the next traffic light (Bangkok University intersection), which means that the queue could be over the next intersection (Bangkok University Intersection). However, the probability is less than 25 percent.

When users zoom in to an area, both directions of the queue length can be seen. Figure 8 displays the comparison between weekday morning and week evening. The queue towards Saladeang station on the left hand side in the morning was more uncertain than in the evening as the box is longer. However, the median queue in the morning is slightly shorter than in the evening. Similar trend to this, the queue towards the Witthayu intersection in the opposite direction on weekday morning is a bit more uncertain than in the evening. Compared to Figure 7, it can be seen that longer queues do not correspond to more uncertain ones.

#### 4. User Evaluation

The user evaluation process has been designed to measure whether the system is usable for expert users. Usability is a quality of the interface when the user interacts with it [26]. It consists of five quality components: learnability, efficiency, memorability, errors, and satisfaction [27]. In this research, efficiency and satisfaction have been selected to measure for each task. Efficiency is how fast the users can accomplish their tasks using the system. Satisfaction is an overall opinion to the system. The study was conducted as Verbal Protocol Analysis (VPA), which users were interviewed when they used the system. The amount of time it takes the users to complete the specified tasks is measured to evaluate the efficiency.

The tasks were designed based on spatio-temporal task categorization by Andrienko and Andrienko [28]. Tasks are possible questions related to spatio-temporal data components (location, time, and attributes) users need to answer by seeing the visualization. In the test, three tasks related to our visualization were selected: lookup, comparison, and relation-seeking. Lookup is a question related to seeking a specified element or characteristics. Comparison implies a comparison between two or more elements.

Relation-seeking is a question related to relations between characteristics. In this research, three questions are set up during the design process as follows:

- **Lookup:** Which intersections have a spillover queue?
- **Comparison:** Which intersections have the most uncertainty?
- **Relation-seeking:** Is there any differences between weekdays and weekends in terms of uncertainty? Is there differences between the morning rush hour and the evening rush hour?

#### 4.1 Setup

The evaluation was divided into three main parts. In the first part, participants were asked about their background information and whether they were familiar with the box plot. Since four out of five participants were not familiar with the box plot, the concept of the box plot was explained to them. The objectives of the developed system and its various components such as buttons and the box plot map were also explained to them. In the second part, the participants began to perform the three tasks assigned to them. The time used to complete each task was recorded to measure the efficiency of the system. In the third part, the participants were asked about their overall satisfaction with the system. We also discussed their opinion about the improvement of the system. This part will help us to understand the overall satisfaction of the system.

The evaluation took approximately 20 to 30 minutes for each participant, depending on the interest of the participants. Due to the COVID-19 situation, the evaluation was processed on video conferencing (Zoom). The interview was recorded to measure time and to make further observations.

#### 4.2 Participants

Five participants between 29 and 31 years of age were invited for the user evaluation process. The number of participants (5) was based on [13], which focused on low-level user evaluation rather than quantitative results. All participants were not paid for participating in the evaluation. Three of them graduated and are currently working in the field of geography. The other two were not Geography related. All of them had taken a statistical course during their undergraduate studies. Two of them had known the box plot. The rest of them were not familiar with it. Although all of them said they realized the concept of uncertainty, only one participant could be able to describe the detail of box plot and how it related to uncertainty. All participants were familiar with the study area—Rama IV Road;

and one of five had a residence near Rama IV Road and was very familiar with the area.

#### 4.3 Results and Discussion

Four out of the five participants were satisfied with the overall prototype and they understood the purpose of the system. However, one of the five participants, who has no previous GIS background with limited understanding of the box plot composition, suggested making the system simpler for users who have no prior knowledge of statistics.

Regarding the lookup pattern, we asked users to identify the spillover queue on Rama IV Road. All the subjects were able to identify the spillover. However, it took about 1-2 minutes to understand all the parts of the map and answer the question. One participant, who was a GIS expert, said that the map had to be zoomed in to see individual box plots; otherwise they were overlapping. However, when the map was zoomed in, the participant can see the individual box plot and can answer the question easily.

The comparison task was assigned to the users to examine the use of maps to compare queue length and uncertainty to other intersections. The participants were asked to identify the longest queue and the most uncertain queue. One of the participants with an expertise in statistical especially Box Plot, was able to identify the longest and the most uncertain queue quickly and easily in less than a minute. Others were also able to identify the longest and the most uncertain queue, but they were not confident on their answers so the researcher was asked whether their answers were the correct one. However, after all participants had received additional instructions, everyone could identify the uncertainty range of the queue on the map by looking at the length of the solid red line displayed on the map.

Regarding the relation-seeking task, participants were asked to compare the difference in terms of uncertainty between weekdays, Saturdays, and Sundays, also the difference between morning and evening. To answer the question, the participants used the tools provided on the system to change the time. As a result of the experiment, all participants could indicate a significant difference between days. However, two out of five discussed that the difference in queue length between morning and evening is small, so it was quite difficult to observe. One participant who lived near the Rama IV area mentioned that the result was similar to his experience on the road, i.e., morning peak on weekdays had longer queue than on weekends. Table 1 summarizes the result of the user evaluation.



Table 1: Summary of user evaluation

Task	Question	Results
Lookup	Which intersections have a spillover queue?	-Everyone could answer the question but it took 1-2 minutes. -1 of 5 who had a strong background in statistics and box plot could answer the question.
Comparison	Which intersections have the longest and the most uncertainty queue length?	-The rest of them asked for more explanations and they were not confident in their answers.
Relation-seeking	Is there any differences between weekdays and weekends in terms of uncertainty? Is there differences between the morning rush hour and the evening rush hour?	-Everyone could see the difference between days and times. - 2 out of 5 mentioned a little difference between uncertainty which was difficult to observe.

Regarding feedback on the improvement, one of the participants suggested describing the range of uncertainty that is shown on the map to make it easier for the general user to understand the system. One participant suggests making the system more responsive to support various screen sizes since the current version only supports desktop, and some fonts cannot be clearly seen. Participants were also suggested to add more the time interval option since there are only two time periods available in the current system.

In summary, users can specify where the queue spilled over, and they can indicate the difference of queue lengths between times. However, uncertainty may be a difficult concept to understand, as evidenced by users who seek further explanations. Although box plots are a common visualization technique used in many applications and are included in many popular visualization and statistical software packages, the result suggests that audiences without a background in statistics may need a brief explanation of how a box plot can be used to show the median and variance of the data.

## 5. Conclusion

This research developed a visualization tool to display the length of the traffic queue with its uncertainty. A box and whisker plot technique was applied to visualize queue length with its uncertainty because it can be laid on a narrow space. The box was on a base map displaying the median, with the first (Q1) and the third (Q3) quartile of the queue length. Its whisker of the box plot displayed  $Q1 - 1.5IQR$  and  $Q3 + 1.5IQR$  ( $IQR=Q3-Q1$ ), indicating a less probable queue length, yet it was possible to occur. Arrows have been applied to the whisker to represent the direction of traffic. As the box is on the map, the length of the queue can be referenced to point of

interests (POIs), helping users to comprehend the length of the queue. Moreover, as the user zoomed out the map, the box plot of all the intersections can be compared. Because box plot can visualize both the length of the queue and its uncertainty, the relationship between them can also be revealed, allowing users to further explore the cause of it.

To determine whether users can easily understand the visualization, a prototype was created and user testing was done. The visualization prototype was implemented as a web-based application, which box plots have been applied to intersections on Rama IV road. Interactive features i.e. buttons have been added to allow users to choose date and time. User evaluation was conducted to measure efficiency and overall satisfaction of the visualization based on three selected tasks: look-up, comparison, and relation seeking. All tasks are questions related to queue length and its uncertainty. Five participants with a basic statistical background were individually asked to perform tasks. The study indicated that the prototype can inform the queue length with its uncertainty although the efficiency of the system depended strongly on their background knowledge. The concept of uncertainty may be difficult to understand for people who have only basic knowledge in Statistics.

We estimated the queue length from taxi GPS data. Our proposed method to estimate the length of the queue may not be accurate since the data was collected from only taxis—the queue may be longer than estimated. However, this research mainly aims to develop a visualization technique rather than the estimation technique, which can be applied to queue length data from any source.

This research tried to implement conventional box plot in order to find the potential in displaying the queue length. The result of user evaluation

suggests the visualization can be used to communicate queue length with its uncertainty to users. However, uncertainty should be described to those users who are not familiar with the concept. Therefore, alternative ways to visualize queue length should be explored to improve learnability. There are variations of the box plot available, and it can be interesting to further see which variation is more suitable for visualizing the queue length. It is also interesting to see if the technique can be applied to a wider area once the queue length data is more available.

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