

# Modeling of Continuous Fields: Coverage Mapping Based on Dynamic In-situ WLAN Measurements

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## Abstract

*Various authors have investigated different approaches to modeling of EM (Electro-Magnetic) field maps, for the purpose of WLAN (Wireless Local Area Network) positioning. However, none of them considers coverage mapping based on dynamic WLAN measurements, taking surrounding mobile objects' positions into account. This research introduces a method for reference point measurements identification, out of dynamic in-situ measurements, which are enabled by spatio-temporal filter strategies. Furthermore, it proposes a spatial database model for processing and storage of EM field maps. The paper starts with a revision of theoretical concepts and methods of wireless network positioning metrics and field modeling approaches. Available data sources of static and dynamic characteristic will be evaluated and modeled for the use in a GIS (Geographical Information System) storage and processing framework. Based on this data, deterministic and probabilistic field modeling approaches have been implemented and qualified against a measurement reference track. These tests have shown that Ordinary Kriging is best suitable for EM field modeling in non-obstacle free environments. These findings provide an excellent starting point for spatio-temporal field modeling.*

## 1. Introduction

Continuous fields are mostly based on physical phenomena and in nature their constitution is very often dependent on the variables of time and space. The modeling of such phenomena faces several challenges: (i) an appropriate discretization approach is needed, which allows efficient numerical processing of the continuous phenomenon of interest. (ii) Highly organized data structures are necessary for storage and processing of modeled field coverage. (iii) Ideally modeled field phenomena underlie by nature unforeseen physical influence with inherent statistical occurrence. The modeling of many physical phenomena in nature and technology faces the same problems, as for instance acoustic noise and atomic or EM (Electro-Magnetic) radiation. The phenomenon of EM radiation is used in NPS (Network Positioning Systems) (Groves, 2008). In urban or indoor environments, where globally available GNSSs (Global Navigation Satellite System) fail due to lack of availability or accuracy, they can be replaced or complemented by local NPSs (Vossiek et al., 2003). Local NPSs have their own signaling infrastructure, and they operate by different location sensing technologies and location metrics. These metrics can be classified into

propagation based, AoA (Angle of Arrival) and RSS (Received Signal Strength) methods (Pahlavan et al., 2002). For AoA or propagation based location metrics highly specified technology is necessary to enable accurate angle and time measurements in wireless networking infrastructures. For these positioning methods existing standardized off-the-shelf technology is not yet available. Among stall metrics, RSS is the easiest adaptable to already existing WLAN communication infrastructures, as it does not claim additional hardware implementation effort. Moreover, RSS positioning has the inherent capability to consider multipath effects induced by natural and artificial obstacles (Haykin, 2001), as these pieces of information can be modeled in the underlying RM (Radio Map). The RM represents the EM field coverage, better known as signal power strength, of available WLAN APs (Access Point) and constitutes the basis for deterministic (Honkavirta et al., 2009) or probabilistic (Widyawan, 2007) positioning algorithms. Since this RM delivers the direct signal strength input to the positioning algorithm, the accuracy and actuality in dynamic non-obstacle free environments is vital for a reliable positioning solution. Much research has been made on the principles of RM modeling in



WLAN infrastructures. These principles can be divided into two different classes: *deterministic* and *probabilistic* modeling approaches. The radial propagation model (Parodi et al., 2006) is the most straightforward deterministic field modeling approach based on parameter of the radial symmetry in EM field distribution. This concept is extended in the MWM (Multi Wall model) (Parodi et al., 2006), considering the influence of walls to the EM field propagation in indoor environments. The ray-tracing model (Wölfle et al., 2005) is based on ray-optical propagation modeling, where all possibilities of optical ray propagation between radiation sink and source are taken into account. The DPM (Direct Path Model) (Wölfle et al., 2005) lies between the MWM and ray-tracing model, considering 2 to 3 dominant signal paths between AP and MU. However, all of these three models need an additional BIM (Building Information Model) (Schlueter and Thesseling, 2009) for RM computation. Probabilistic field modeling applies geo-statistical Kriging interpolation (Konak, 2010) based on empirically gathered reference point measurements. Although multiple field modeling approaches have already been discussed in the literature, strategies for ad-hoc RM generation in dynamic non-obstacle free environments have not been elaborately investigated yet. For this approach only a minimum of infrastructural information should be required to allow the maximum flexibility in adaptability to different infrastructures. This target asks for a generic, fully integrated spatial data and processing structure, which will be addressed in this research paper. The central research question is though the following: What modeling method is suitable for EM fields in non-obstacle free dynamic environments? In order to answer this, first of all a GIS data and processing framework will be proposed. Based on this framework then, a suggestion is made for how to extract reference point measurements out of a bulk of dynamically gathered in-situ observations. Using these reference points an ERP (Extended Radial Propagation) model and OK (Ordinary Kriging) interpolation approach will be applied for modeling of the continuous EM field. Finally both methods will be compared against empirical reference measurements.

## 2. Methods and Materials

### 2.1 Experimental Set-up and Data Sources

All experiments and data used for this research have been provided by the project context of SESAAM (Geo-Spatially Enhanced Situational Awareness for Airport Management) (Bretz et al., 2011).

Various static and dynamic data sets, in terms of their spatial behavior, have been provided from different sources and file standards. For data harmonization and integration a GIS framework with near-real-time processing capability was set-up: A spatial database for data storage and raster processing, a data processing framework for data handling and insertion into the database and a desktop GIS for data presentation. Within the GIS framework, data is organized into different static and dynamic layers. The static layer holds information of infrastructural information, WLAN AP position and the spatial boundary of the test bed area, covering approx. 108.000 m<sup>2</sup>. The RM is made available as additional static layer for the purpose of WLAN positioning. Positions of moving objects are further on regarded as dynamic datasets. These dynamic datasets are compound of MUs' data, including their GNSS position and EM measurements to APs. Furthermore these datasets hold aircraft's positions, which are gathered by a MLAT (Multi-Lateration) ground radar system. In this experimental set-up the MUs with WLAN and GNSS sensors on board allow the collection of reference measurements by acting as basis for the RM modeling. The aircraft positions provide valuable information about WLAN interfering obstacles in multipath environments.

### 2.2 Data Modeling

The RM model is the basis of RSS positioning and of the entire RM generation process. Though, first of all it has to be defined in its mathematical dimension. The RM describes the discrete representation of the continuous distribution of EM field strength. Each of the test-bed surrounding AP generates its own EM field, so multiple radio maps exist for a single positioning infrastructure. The positioning infrastructure can be stored in a matrix of spatially regularly distributed values. This way each cell of the matrix refers to a geographical location. The continuous field model can be either represented by a set of mathematical functions or by a discrete representation, which can be sampled from the continuous field phenomenon. In order to minimize the functional processing effort of (i) the expected field complexity in multipath environments and (ii) during map manipulation and in order to (iii) allow fast value accessibility a discrete mathematical map model is defined. Each sampling point  $x_{ij}$  and thus matrix element is defined as a tuple:

$$X_{ij} = (RSS_{ij}, \theta_{ij})$$

Equation 1

where  $RSS_{ij}$  is the radio signal strength and  $\theta_{ij}$  represents a universal parameter for additional use during the location estimation phase. This could be for instances used for the vehicles orientation in north, east, south or west direction, influencing the EM strength in RSS positioning. The sample point  $X_{ij}$  is now regarded as vector:

$$\vec{X}_{ij} = (RSS_{ij}^k, \theta_{ij}^k) \quad k \in 1 \dots q,$$

Equation 2

Where each element  $k$  of the vector  $\vec{X}_{ij}$  corresponds to the radio field of a dedicated access point AP $_k$  at a maximum count of  $Q$  APs. All that vectors are part of the radio map, defined by the matrix  $X$ :

$$X = (\vec{X}_{ij}) \quad i \in 1 \dots m, \quad j \in 1 \dots n.$$

Equation 3

Thereby, each element of matrix  $X$  represents a sample of the  $Q$ -dimensional radio map and refers to a geographical position by row  $i$  and column count  $j$  respectively. Note that each sample point  $\vec{X}_{ij}$  is generated at the center of a matrix cell in  $X$ . Figure 1 points out the multi-dimensional character of the RM matrix  $X$ . Each layer in that matrix represents a RM dedicated to an AP. Finally, after this pure mathematical definition of the radio map, from a spatial data type perspective it might be also regarded as raster coverage of the EM field strength. Each coverage cell has got its geographical center coordinate, in its defined geographic reference system, with certain spatial extent. That spatial extent together with the raster resolution implicitly defines the sampling width of the discretization process. This makes the mathematical raster definition perfectly suitable for the raster data type in spatial databases and allows the seamless integration of the data model to available DBMSs (Data Base Management System) with spatial extension, e.g. Oracle spatial or PostGIS.

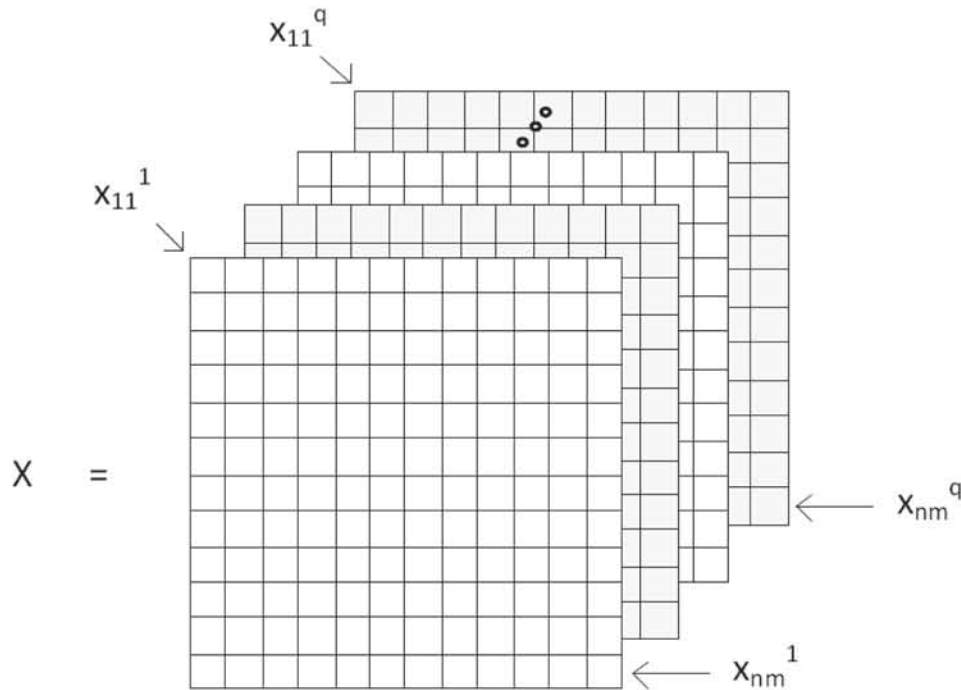


Figure 1: Graphical representation of the  $q$ -dimensional RM



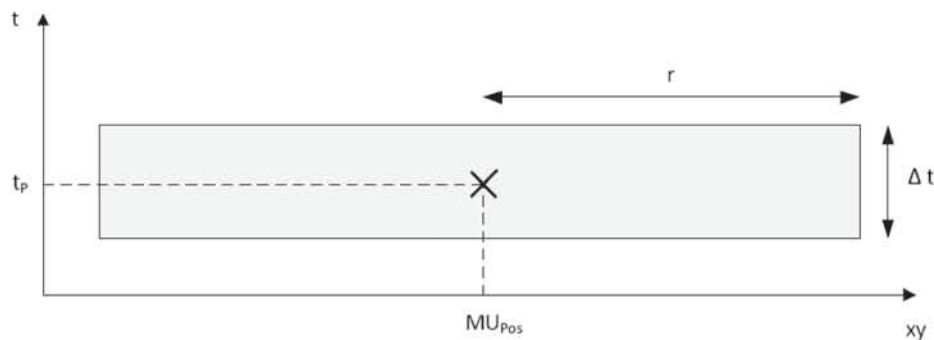


Figure 2: Cross section of the spatio-temporal buffer concept

### 2.3 Dynamic data Pre-processing

For the purpose of continuous field modeling, reference point measurements are often needed for model calibration (Parodi et al., 2006) or as direct input to spatial interpolation. This research shows though that certain spatio-temporal filter strategy allows extracting reference points and their corresponding measurement values out of dynamically collected measurements in non-obstacle free environments. As a first step of pre-processing, GNSS positions and WLAN measurement data have to be correlated over time. This is due to the fact that WLAN and GNSS sensors are neither synchronized in recording time nor in data rate. Since in this case the GNSS recordings have the lowest inherent data rate of the system, the GNSS track was interpolated via 2-dimensional cubic parametric spline interpolation (Späth, 1995) in order to gain an exact interpolation with segment wise computation capability. This allows generating a reasonable high density of GNSS track positions to correlate WLAN measurements and interpolated GNSS positions via nearest neighbor matching in the time domain. This results in a geo-referenced track of WLAN measurements, by acting as input variable to the reference point measurement extraction process. That process identifies spatially aggregated positions of reasonable quality as measurement cluster and determines their cluster parameters. The process starts with filtering the available measurement values by speed over ground and HDOP (Horizontal Dilution of Precision) of the GNSS position. Next, the filter algorithm divides the selected measurement values in clusters, based on a time gap criterion. The start and stop time of each cluster describes the boundaries in time; on that basis the cluster values can be selected accordingly. In the following step selected cluster values will be tested for whether a close spatio-temporal relationship to aircraft positions exists.

This is done to exclude multipath influences caused by moving obstacles. In order to achieve this, a 3-dimensional buffer criterion in the space-time plane was defined as shown in figure 2. Thereby, around the actual  $MU_{Pos}$  a radius  $r$  was defined in the space domain and a buffer height of  $\Delta t$  was selected on the time axis. This results in a flat cylinder buffer shape, allocated in time and space. For the final computation of cluster parameters, only those values will be taken into consideration that fulfill the buffer criterion in reference to aircraft positions. These parameters are well known statistical parameters as value count, average power and standard deviation  $\sigma_P$  of the EM field strength, as well as the positional standard deviation  $\sigma_{xy}$ . On the basis of these parameters, each reference point measurement gained out of this process can be evaluated in terms of its statistical constitution.

### 2.4 Field Modeling – Extended Radial Propagation Model

In the introduction of this paper a series of deterministic EM modeling approaches have been considered: The radial propagation model, MWM, ray-tracing model and DPM. For the latter three models accurate BIM information is required, arresting the flexibility of adaptation to different infrastructures. This research elaborates an ERP (Extended Radial Propagation) modeling approach, and takes into account the directivity characteristic of applied antennas beyond fairly simple isotropic radiation. The radial propagation model is based on the radial symmetry in electro-magnetic field distribution. Thereby, the assumption is made that the field strength of an isotropic radiator is exclusively dependent on the distance  $d$  between receiver and transmitter antenna, at a given wave length  $\lambda$ , (Haykin 2001). Atmospheric influences

have not been further considered in that approach, as they have minor influence at close receiver transmitter distances. The given concept of the radial propagation model (Parodi et al., 2006) is thereby extended by the antenna's gain directivity  $G_t(\delta)$  of the transmitter, as a function of the angle  $\delta$  in reference to the antenna's bore side direction:

$$PL(d, \delta) = -10 \log_{10}(G_t(\delta)G_r) + 10 \log_{10} \left( \frac{4\pi d}{\lambda} \right)^2 \quad [\text{db}]$$

Equation 4

Consequently, the path loss PL between the MU and the AP is dependent on the distance  $d$  between both. The receiver antenna gain  $G_r$  is regarded as a constant due to its omnidirectional antenna characteristic. The geometric relationships in correspondence to the RM are shown in figure 3. Therein, each raster cell of the  $k$ -dimensional RM matrix  $X$  corresponds to a discrete sampling vector  $\vec{x}_{ij}^k = (r_{SS_{ij}^k})$  of the field model. The distance  $d$  can be determined by the Euclidian norm of  $\|\vec{d}\|_2$ , where  $\vec{d}$  is given by:

$$\vec{d} = x_{ij}^k - AP^k$$

Equation 5

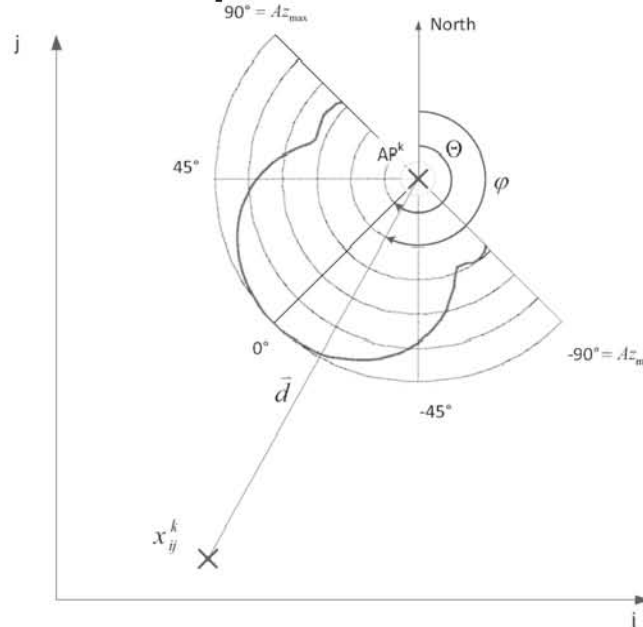


Figure 3: Geometrical relationships of the ERP model

The angle  $\varphi$  between vector  $\vec{d}$  and geographic north can be determined by well-known Trigonometric functions and finally gives the angle  $\delta$  by subtraction of  $\Theta$  and  $\varphi$ . These equations deliver the input for the map generation algorithm of the ERP model for implementation into a spatial DBMS. Additionally, this model is calibrated on the basis of empirically gathered reference point measurement values. In order to meet the best suitable calibration of the field model in the power domain, least square approximation for over determined systems is applied. The calibration parameter  $a$  is best fitted if the error function  $E(\Delta p_i, a)$  reaches its minimum. Therein,  $\Delta p_i$  describes the delta between a reference point measurement and the corresponding value of the field model. This approach is supplemented by a weight criterion  $w_i$ , which acts as quality parameter of reference point measurements:

$$E(\Delta p_i, a, w_i) = \sum_{i=0}^N (\Delta p_i - a)^2 w_i$$

Equation 6

The numerical solution of  $|E(\Delta p_i, a)|_{\min}$  gives the exact value of the calibration parameter  $a$  in the power domain.



### 2.5 Probabilistic Interpolation

As an alternative approach to a parametric field modeling, this research considers a 2-D interpolation technique in order to overcome expected local field variations due to static obstacles, influencing the EM field by multipath effects. Various authors have shown that probabilistic Kriging leads to reasonable results in WLAN infrastructures (Konak, 2010). Kriging is a geostatistical interpolation method for producing an interpolation surface, and incorporating the statistical properties of the measured data. Besides the resulting prediction surface, the Kriging techniques produce an error surface, by indicating the statistical residuals of the predictions. The Kriging process is divided into two main tasks, the quantification of spatial structure of data and the production of a prediction surface. In the structural quantification, known as variography, a spatially dependent model is fitted to the data. For the prediction of unknown values at specific location, Kriging utilizes that spatial model, the spatial data configuration and the values of measured data around the prediction location. The configuration describes spatial autocorrelation amongst measured points around the prediction location. Dependent on the fitted model, Kriging is either an exact or inexact interpolator. If the regression model in variography starts at the origin of the coordinate system the Kriging prediction is exact. That means

the error in measurement data is zero. This research uses the exact interpolation technique for comparison to other modeling approaches.

## 3. Results

### 3.1 Dynamic Reference Point Measurement Extraction

For testing the reference point measurement extraction process, a MU data set consisting of 234,806 WLAN measurements, captured over a recording time of 18 hours, has been taken. The key parameters  $\Delta t$  and  $r$  of the spatio-temporal filtering process to moving obstacles were adjusted to  $\Delta t = 2s$  and  $r = 100m$ . These allow in the time domain a guaranteed intersection to aircraft positions, depending on their acquisition rate. The same is valid in the space domain accordingly; there at 100m a sufficient signal attenuation of  $\sim 92$  dB is given in loss-free signal reflection. The computational result of the dynamic extraction process, spatially delimited on the test-bed area, counts 24 reference point measurements. Table 1 shows the exemplary statistics of the reference point measurement parameter  $\sigma_P$ ,  $\sigma_{xy}$  and the count of measurements per cluster to a single AP. This result acts as a reasonable basis in size and precision for further model calibration purposes.

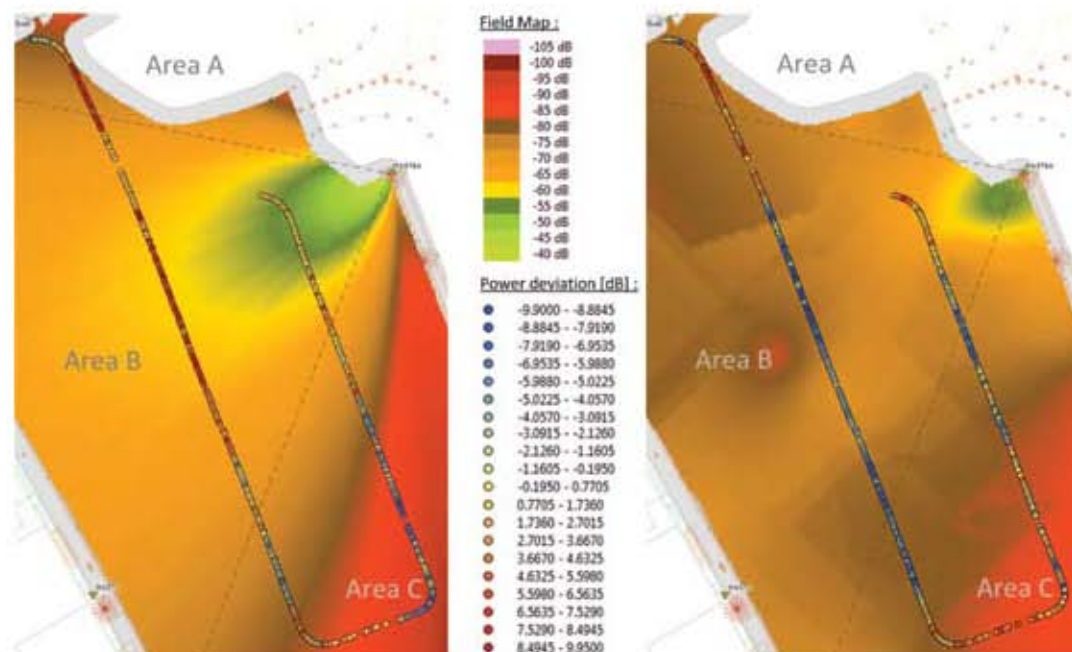


Figure 4: Verification of the ERP model (left) and OK interpolation (right) against a measurement reference track

Table 1: Exemplary reference point measurement statistics

| parameter     | min   | max   | mean  | median |
|---------------|-------|-------|-------|--------|
| $\sigma_p$    | 0     | 1.496 | 1.034 | 0.999  |
| $\sigma_{xy}$ | 0.023 | 0.266 | 0.123 | 0.112  |
| count         | 5     | 144   | 34    | 18     |

Table 2: Statistical analysis of test-bed area A, B and C

| Area      | parameter        | Ext. rad. model | Ord. Kriging |
|-----------|------------------|-----------------|--------------|
| A         | $\Delta p_{min}$ | -9.00           | -8.60        |
|           | $\Delta p_{max}$ | 9.65            | 9.46         |
|           | $\mu$            | 4.3021          | 4.5361       |
|           | $\sigma$         | 4.5482          | 3.4536       |
| B         | $\Delta p_{min}$ | -9.10           | -9.85        |
|           | $\Delta p_{max}$ | 9.95            | 8.68         |
|           | $\mu$            | 3.6961          | -3.3377      |
|           | $\sigma$         | 3.3334          | 4.4714       |
| C         | $\Delta p_{min}$ | -9.23           | -8.48        |
|           | $\Delta p_{max}$ | 8.20            | 7.96         |
|           | $\mu$            | -1.7243         | -1.3466      |
|           | $\sigma$         | 3.9102          | 4.2824       |
| A & B & C | $\Delta p_{min}$ | -9.23           | -9.85        |
|           | $\Delta p_{max}$ | 9.95            | 9.90         |
|           | $\mu$            | 1.4059          | -1.8218      |
|           | $\sigma$         | 4.6029          | 4.8153       |

### 3.2 Comparison between Extended Radial Propagation Model and Ordinary Kriging Interpolation

For the final results of the RM field modeling, a 16-bit, six channel raster file was allocated in the spatial DBMS. This allows storing of a six layer counting RM with a numerical quantization of  $2^{16}$  steps per cell value. The exemplary RM modeling results of AP Mast6A, for the ERP model and OK interpolation, are shown in figure 4. The measurement reference track gives the basis for an empirical qualification and comparison of both RM modeling approaches. The sub-division of the test-bed is done in reference to (i) the geographical location of the AP and (ii) the antenna's field directivity characteristic. These result in a main-lobe area (Area B), in reference to antenna AP mast6a, and two side-lobe areas (Area A, Area C). While the measurement reference track is influenced mainly by the induced multipath effect of surrounding buildings inside of side-lobe Area A, Area C is least affected. As the pure visual interpretation of the power deviation in figure 4 is hard to quantify, it is additionally underpinned by statistical parameters in table 2, for each of the exploration Area A, B and C.

The maximal positive and negative deviation  $\Delta p$ , the arithmetical mean value  $\mu$  and the standard deviation  $\sigma$  of the power deviation is considered. The spatial distribution of  $\Delta p$  compared amongst both models shows that the ERP model counts the biggest deviation on the very north end of the track, where the multipath effect by surrounding buildings comes into account. In contrast, the OK interpolation shows significantly better performance in that region. Near the border to Area B, the OK approach seems to meet that physical phenomenon slightly better as well. The statistical figures out of table 2 underpin these observations. Although the ERP model has the lowest mean value, the high variation of  $\Delta p$  relativize this advantage. In the main lobe area of the reference track, Area B, we can distinguish in figure 4 between the near field and the far field of the antenna. In the near field area the ERP model shows a rather positive result with small variation and low deviation from the measurement reference track. The OK model meets the measurement track partially even better, however with slightly higher variance on the near field boundary areas.



In the far field center area the ERP model significantly over-estimates the EM field, whereas the OK interpolation shows the opposite. Nevertheless, the ERP model meets the EM field well at the boundary areas of the far field. In Area C, the ERP model shows high variation in model deviation close to the northern boundary of Area C. In the southeastern part of the measurement reference track, that model clearly underestimates the EM field. In this area the OK approach succeeds with lowest deviation from the EM field. By the means of statistical parameters OK shows the lowest deviation, however combined with high variation. In the statistical overall consideration the ERP model shows the best performance in terms of absolute deviation from the measurement data set in combination with lowest variance. The OK approach clearly underestimates the EM field with higher variation in field deviation. Despite of the statistical overall results, the areal analysis above has shown that each model has strengths and weaknesses in certain areas against different influences. This shows that the evaluation of field models cannot simply strap down statistical overall analysis. In order to take the visual analysis into account a result matrix summarizes the areal assessments in table 3. The final result of that analysis shows that OK applied on reference point measurements seems to be the better EM field modeling approach. This is mostly due to the overall robustness of OK regarding the multipath effect and the fact that it allows compensating field discontinuities best.

Table 3: Result matrix of the visual statistical areal analysis

|        | Ext. rad. model | Ord. Kriging |
|--------|-----------------|--------------|
| Area A | --              | - +          |
| Area B | ++              | - +          |
| Area C | - +             | ++           |
| Result | - +             | ++           |

#### 4. Discussion and Outlook

The findings of data modeling, together with the actual implementation of the object relational data model inside a spatial DBMS, have shown that this approach is perfectly suitable for the application of multi-layer field mapping. The utilization of spatial indexing in a spatial DBMS allows fast data access within the RM in near real-time applications. Moreover, the export of RMs in compressed file formats e.g. GeoTIFF supports the seamless integration of RMs in position algorithms, GIS and

many other services. The elaboration of the reference point measurement extraction process has introduced a filtering strategy in the spatio-temporal domain. This approach makes it possible to take the reference point measurements into consideration without the interfering multipath influence of mobile obstacles in the vicinity of the measurement unit. Therein, the quality of extracted reference point measurements is highly dependent on the availability of positioned objects inside the area of interest. The Ordinary Kriging interpolation applied on reference point measurements has the potential to model EM fields in non-obstacle free environments on a global scale, without considering field effects caused by mobile obstacles. This is mostly due to the fact that multipath effects of static obstacles can be considered. However, this approach depends (i) on the quality of measurement reference values and (ii) the interpolation point density of this data set. Unfortunately, both criterions were only partially given in scope of this research project. The extension of the recording time-frame would increase the density of measurement reference values and thus the quality of the RM as a whole. This research paper provides a sustainable basis for modeling of continuous fields. It does not provide a solution for dynamic field mapping, considering spatio-temporal field influences of mobile objects. Though, the proposed GIS framework constitutes the basis for that. This should be considered from a data structure and management point of view, as well as for implementation of advanced field modeling approaches. Once the static field model is in place and immune against interfering multipath effects, the spatio-temporal influences could be modeled on top of it. This point out the author's recommended direction of future research: The proposed GIS framework would allow the extraction of field influences caused by mobile objects of different shape. Once those influences are known, they could be modeled and incorporated in a spatio-temporal field model. This could be realized by means of geo-statistical simulation (Isaaks and Srivastava, 1989), depending on mobile objects' positions. On that basis a dynamic field map could be provided, e.g. to a positioning algorithm, in near real time by OGC defined web coverage services.

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