Computer Modeling of Geological Objects by Matrix Calculation

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Abstract
This work represents a computer method for the modeling of a 3D geological object. A lot of work and software have been realized in this domain, but we tried to use the matrix calculation to simulate the state of an object representing the result of one or more deformation processes. By a simple matrix calculation, the geological object is kept within the determined limits which make it possible to fix the points that result the deformation of the object, and the object will be represented in a fixed memory place. We have classified the set of basic transformations as three types (translation, rotation, and scale change); as in computer graphics there is a correlation between rotation, translation, scaling and a matrix of order 4, then a modification of the object will be a multiplication of the several base matrixes. The visualization of 3D object on a screen of 2D computer consists on making a projection of the object on a flat surface taking into consideration its cache part, and then framing this 2D object by a resultant matrix into the frame of the computer screen.

1. Introduction

This work can be applied in the modeling of geological objects domain using the tools of computer development; the goal of our work is to realize 2D or 3D geological objects by a determined number of memory based on a matrix calculation, making it possible to manipulate the interior and the exterior of this object, and to know all the riches that are inside or outside the geological object. A geological object has an initial geometrical form that will has along the time many deformations due to physical processes (Cheaito M., 1993, Cheaito, M. and Cheaito M., 2014, 2014a and Cheaito et al., 2016). In this work we classify certain processes of deformations as three types of geometrical transformations which are either a "translation" along a fixed vector V (a, b, c), a "rotation" around the origin, or "On scale change" (e_x, e_y, e_z). Along its history, geological objects do several types of transformations (T_i) i = 1... n which are linked in form by a chronological structure, so if A is the geometrical form at a specific time, then B is obtained by the formula:

$$B = \left(\prod_{1}^{n} T_i\right) * A$$

On the other hand, the visualization of a geological object consists of making a transformation from the real world to the world of the screen which is a problem we represent its solution, as indicated in the tools of computer graphics (opengl), by a composition of four basic transformations, by definition of a frame by a unit of measurement such as meters or kilometers, based on the use case. This frame can be simulated as a rectangle in the real world that is called window, and as a rectangle of pixel measurement in the world of the screen that is called fence.

All the deformations applied on a geological object (Spraggins and Dunne, 2002) must be realized in the real world, and then the final object will be visualized on the screen by a transformation of the transfer from the real world to the world of the screen. The storage of the set of points as well as the correlation structure between them, which reflect either the topography or the object interior, must be all recorded in an information system, based on a Databases Management System, which represent this set of points and structures by a set of tables related to each other and at least in the third normal form (Worboys, 2013, Guillemot and Le Meur, 2014, Slusarczyk et al., 2017).

2. The Basic Transformations

The three types of transformations that are the base of our work are: translation, rotation, change of scale. These three basic types are well studied in the domain of computer graphics and there is a link between matrix writing and each one of them, in other words translation along a fixed vector V (a, b, C) Will be described by the following matrix and calculation:

$$M = \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The matrix of rotation of an angle θ around an axis passing through the origin and having a vector $\vec{V}(a,b,c)$ in an orthonormal system axis is:

Having

$$d_1 = \sqrt{a^2+b^2} \ , \ d_2 = \sqrt{a^2+b^2+c^2}$$

$$\alpha = \mathrm{acos}(b/d_1) \ , \ \beta = \mathrm{acos}(c/d_2)$$

$$M_1 = \begin{bmatrix} cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) & 0 \\ 0 & \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} cos(\alpha) & \sin(\alpha) & 0 & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\beta) & \sin(\beta) & 0 \\ 0 & -\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation matrix will be: $M_4 * M_5 * M_3 * M_2 * M_1$. The scaling matrix along the three axes x, y and z is:

$$M = \begin{bmatrix} ex & 0 & 0 & 0 \\ 0 & ey & 0 & 0 \\ 0 & 0 & ez & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that for scaling only the point O (0, 0, 0) is a fixed point. On the other hand, when two transformations T_1 and T_2 are played on an initial geological object G_0 to have a final geological object G_2, then if M_1 is the matrix of the transformation T_1 and M_2 is the matrix of the transformation T_2 then $M_2 * M_1$ is the Matrix of the resulting transformation $T = T_1 \text{ o } T_2$ (Figure 1).

It should be noted here that the transfer from G0 to G2 passes by the two matrices M2 * M1, which results that the retro transfer from G2 to G0, passes by the inverse matrix $M^{-1}=M_1^{-1} * M_2^{-1}$

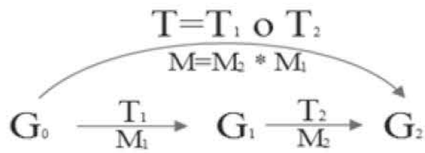


Figure 1: Composition of two transformations

3. Framing the Geological Object in the Real World

A such geological object existing in the real world must be framed in an initial cube Ci defined by two points vmin (xvmin, yvmin, zvmin) and vmax (xvmax, yvmax, zvmax) (see Figure 2). When the geological object does many deformations, its frame will be modified. That is why to use the object, it is necessary to frame the resulting object in another final cube Cf defined by the two points wmin(xwmin, ywmin, zwmin) and wmax(xwmax, ywmax , zwmax) (see Figure 2). Mp, the matrix of passage of cube Ci to the cube Cf, will be obtained by the following matrix of calculation:

M_1 : Translation by (xwmin,ywmin, zwmin)
M_2 : Change of scale by e_x=1/(xwmax-xwmin), e_y=1/(ywmax-ywmin),e_z=1/(zwmax-zwmin)
M_3 : Change of scale by e_x=(xvmax-xvmin), e_y=(yvmax-yvmin), e_z=(zvmax-zvmin)
M_4 : Translation by (-xvmin,-yvmin, -zvmin)

So $M_p=M_4*M_3*M_2*M_1$

$$Mp = \begin{bmatrix} \frac{xvmax-xvmin}{xwmax-xwmin} & 0 & 0 & -xwmin\frac{xvmax-xvmin}{xwmax-xwmin}+xvmin \\ 0 & \frac{yvmax-yvmin}{ywmax-ywmin} & 0 & -ywmin\frac{yvmax-yvmin}{ywmax-ywmin}+yvmin \\ 0 & 0 & \frac{zvmax-zvmin}{zwmax-zwmin} & -zwmin\frac{zvmax-zvmin}{zwmax-zwmin}+zvmin \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

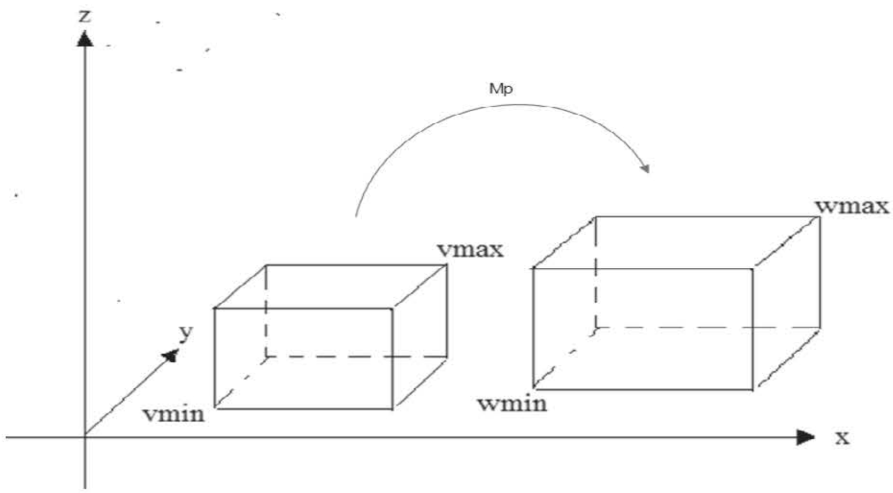


Figure 2: Framing of the geological objects and the passage matrix

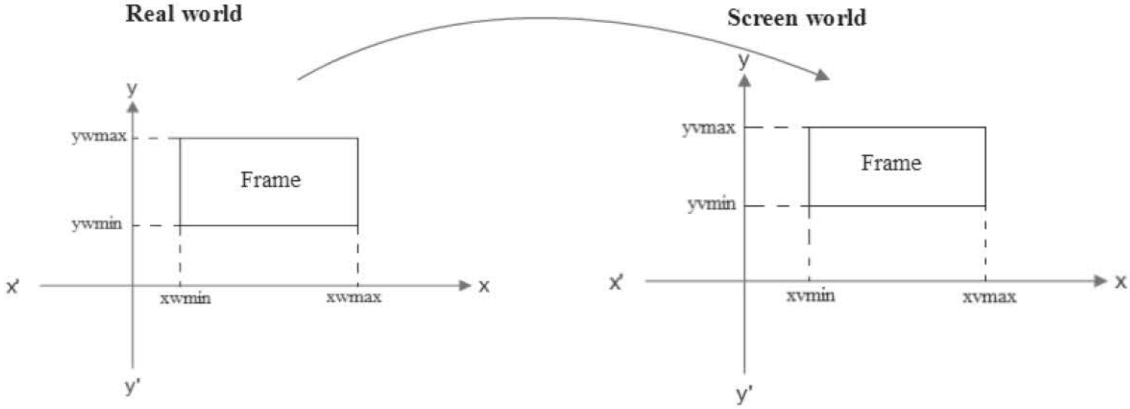


Figure 3: Transition from the real world to the screen world

4. Framing the Object in the World of the Screen

To see a three-dimensional geological object on a computer screen, you have to project this object on a two-dimensional surface, then frame it in two dimensions (xwmin, ywmin, xwmax, ywmax), choose a frame on the (xvmin, yvmin, xvmax, yvmax) and then find the matrix of the transfer from the real world to the world of the screen see Figure 3 (Tian et al., 2009 and Zhao et al., 2013). The transition matrix M_p of the real world to the screen world is defined by the following transformations:

M_1 : Translation into the real world by the vector (xwmin,ywmin)

M_2 : Change of scale by the values e_x=1/(xwmax-xwmin), e_y=1/(ywmax-ywmin)

M_3 : Scaling in the world of the screen by ex=(xvmax-xvmin), ey=yvmax-yvmin

M_4 : Translation into the world of the screen by (-xvmin,-yvmin)

So $M_p=M_4*M_3*M_2*M_1$

$$M_p = \begin{bmatrix} \frac{xvmax - xvmin}{xwmax - xwmin} & 0 & -xwmin\frac{xvmax - xvmin}{xwmax - xwmin} + xvmin \\ 0 & \frac{yvmax - yvmin}{ywmax - ywmin} & -ywmin\frac{yvmax - yvmin}{ywmax - ywmin} + yvmin \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix of transfer from the world of the screen to the real world is M_p^{-1}

$$M_p^{-1} = \begin{bmatrix} \frac{xwmax - xwmin}{xvmax - xvmin} & 0 & -xvmin\frac{xwmax - xwmin}{xvmax - xvmin} + xwmin \\ 0 & \frac{ywmax - ywmin}{yvmax - yvmin} & -yvmin\frac{ywmax - ywmin}{yvmax - yvmin} + ywmin \\ 0 & 0 & 1 \end{bmatrix}$$

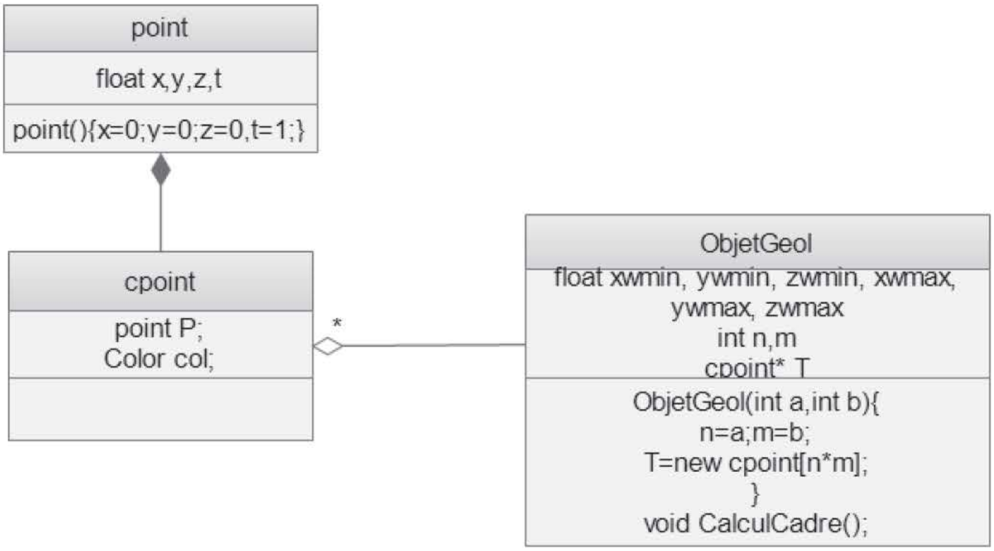


Figure 4: Class diagram for the representation of the geological object

5. Representation of the Geological Object

A geological object must be represented in the real world by a fixed number of points n * m which are a representation in a frame of n * m pixels screen. On the other hand, at each transformation step of the geological object, it is mandatory to calculate the new frame where the object in the real world exists, to ensure a simple transition of the object from the real world to the world of the screen.

The representation of the geological object in the computer world must be described by the following class diagram (Figure 4). The geological object is a set of points where each point is defined by its coordinates and its color indicating the corresponding geological layer, a pointer cpoint * T allowing a dynamic allocation of the set of points describing this object. The function CalculateCadre() allows to calculate the frame of the geological object and assigns the values of the following members; xwmin, ywmin, zwmin, xvmax, yvmax, zvamx.

6. Algorithm for Projection and Visualization of the Object

To display on a two-dimensional computer screen a geological object, that exists in the real world in a three-dimensional orthonormal system, it is necessary to make a projection of the 3D object in an orthonormal system 2D by eliminating the hidden parts, that is why we use the Z-Buffer method (Kolivand et al., 2011 and Ize et al., 2008), we retain in 2D the point (i, j) having the largest z of the object with its color (Figure 5).

7. Algorithm for the Calculation of the Frame of the Projected Object

After projecting the 3D geological object to a 2D projected object as indicated in the paragraph 6. It is necessary to translate this 2D object in a frame of the computer screen; that is why, it is necessary to find the passage matrix indicating this translation. To find the frame of the object in 2D it is necessary to pass the set of the points of the object and at each step we change the value of xwmin, ywmin and xwmax, ywmax as indicated in the following flowchart (Figure 6).

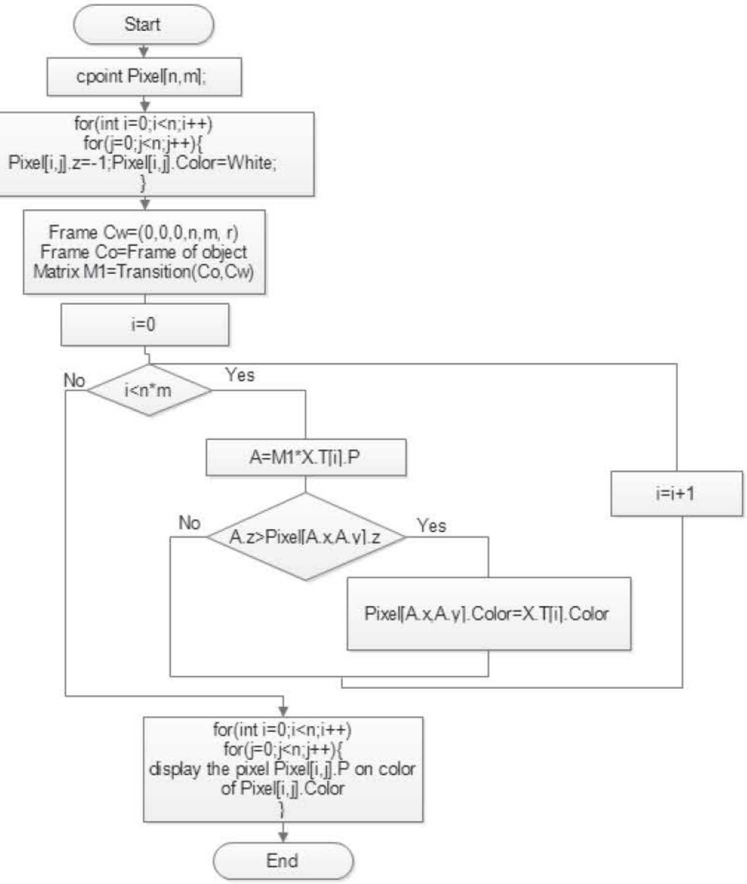


Figure 5: Flowchart for the projection and the display of a 3D geological object

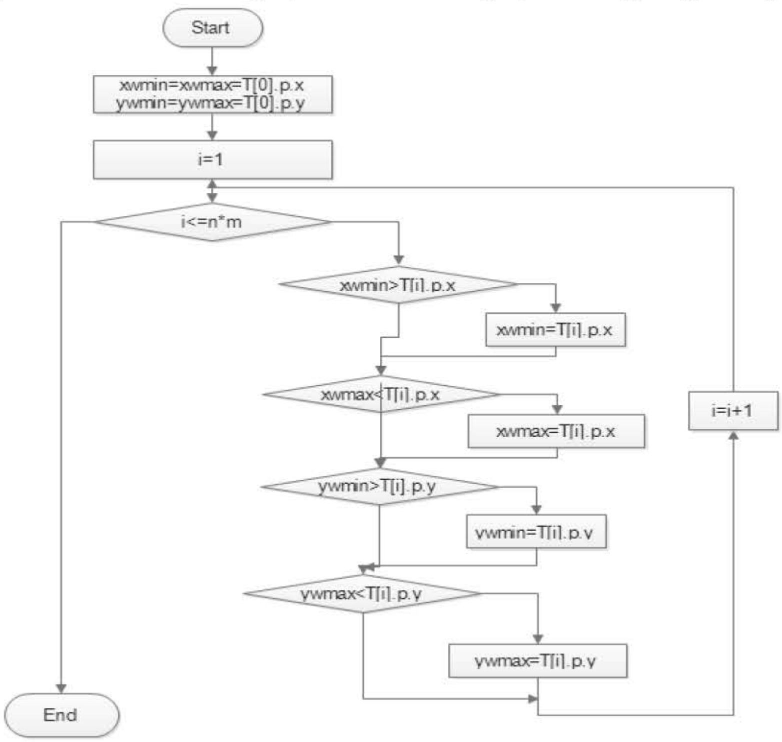


Figure 6: Flowchart to find the 2D frame of the object obtained by the paragraph 6

8. The Deformation of a Geological Object

A deformation of a geological object G by a function F (x) is a transfer of G to an object G'; for each point A (x, y) of G there is a point A' (x ', y') so that x '= x; y '= y + F (x) (Perrin et al., 1993, Suppe and Connors, 2004). There is several deformation functions that may be mentioned (Figure 7).

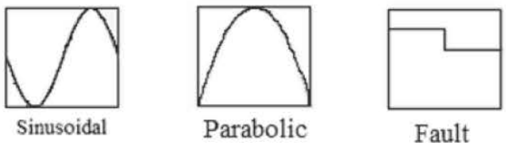


Figure 7: Example of training functions

9. Some Technical Results

The following figures are the technical results taken from the software PCheGeol. PCheGeol is a computer tool based on object-oriented programming that was developed and implemented within the IT department of the Lebanese University (Cheaito, 2013). It allows the modeling of 3D geological objects based on the dynamic modeling process; starting from a simple object we can obtain complex geological objects.

10. Results in Two Dimensions

The deformation of a geological object consists on the representation of the object in an initial state. For this purpose, the initial object is described by a set of horizontal layers represented one on the other, while retaining the chronological order of their representation. It is necessary to have a deformation function represented by a geologist, then this initial object does a deformation phase by this function, then it is rotated by an angle teta and the deformation is repeated by another phase, so the object will be obtained in its final state (Figure 8).

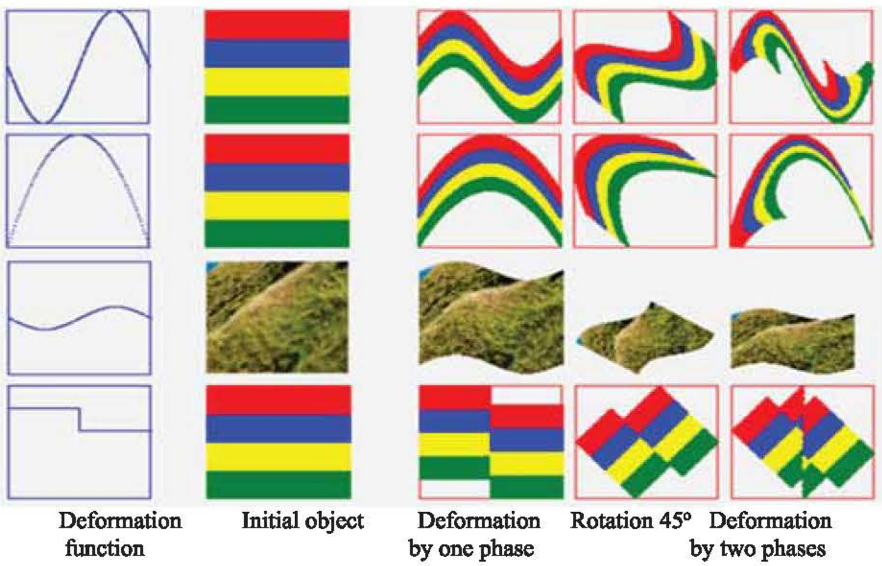


Figure 8: Example of different deformation results in 2D

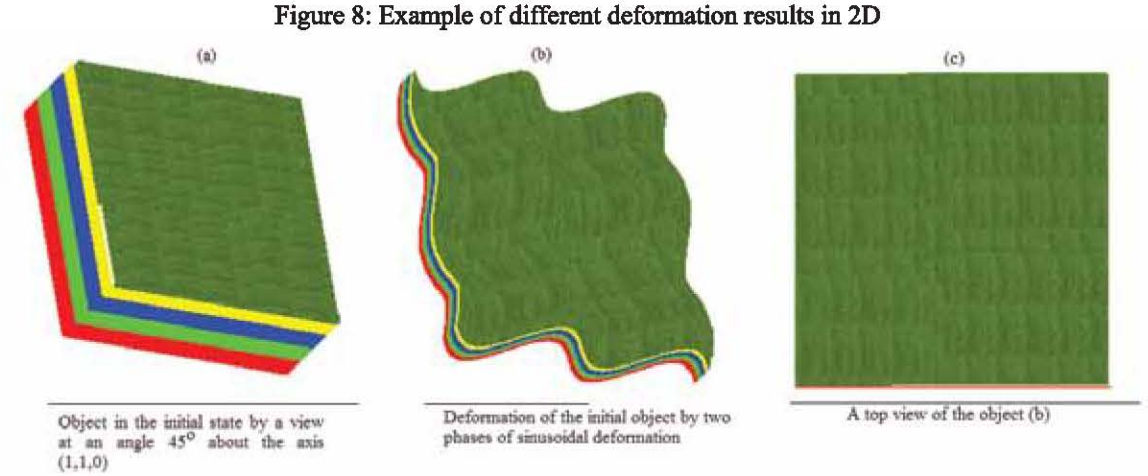
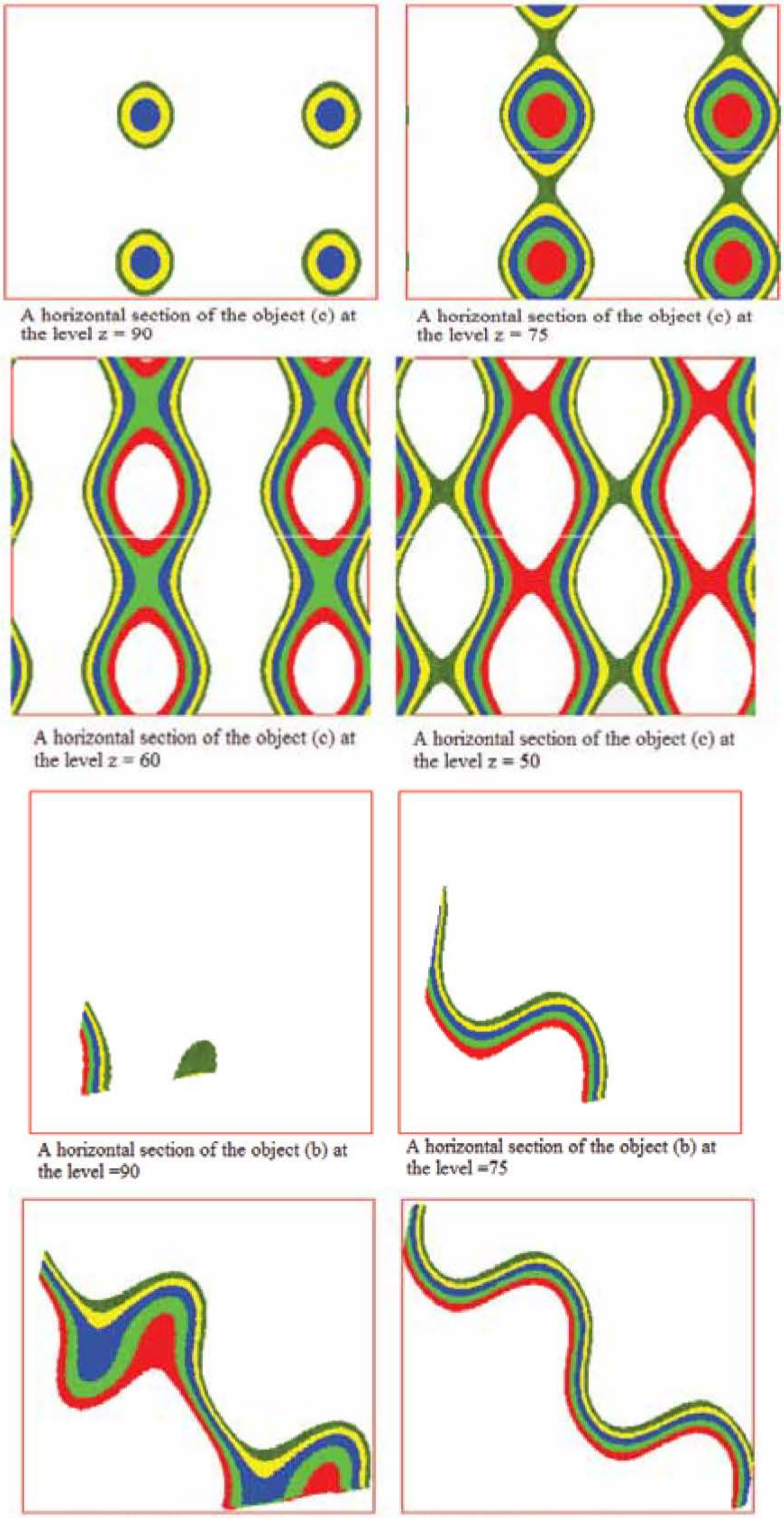


Figure 9: Process of deformation of the object by two phases of deformation



A horizontal section of the object (c) at the level z = 90

A horizontal section of the object (c) at the level z = 75

A horizontal section of the object (c) at the level z = 60

A horizontal section of the object (c) at the level z = 50

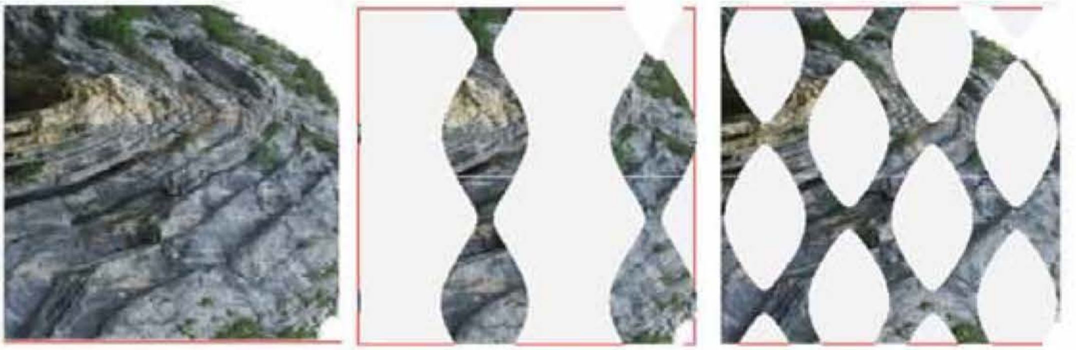
A horizontal section of the object (b) at the level =90

A horizontal section of the object (b) at the level =75

A horizontal section of the object (b) at the level =60

A horizontal section of the object (b) at the level =50

Figure 10: Set of horizontal sections for an object in different levels



Deformation of the object by two sinus-soidal phases | A horizontal section at the z=75 | A horizontal section at the z=50

Figure 11: Deformation and geological cutting of objects with a real topography

11. Results in Three Dimensions

The deformation of a geological object consists on the representation of the object in an initial state. For this purpose, the initial object is described by a set of horizontal layers represented one on the other, while maintaining the chronological order of their representation it is necessary to provide a deformation function which is repeated by a constant value along the axis of z and then this initial object does a deformation phase by this function, and then the object obtained must be projected in a 2D orthonormal system, so we calculate the transition matrix to the screen frame (Figure 9).

12. Geological Section

Since the modeling of our geological object is described by a set of points in 3D, we can obtain a geological section at an altitude h having a plane of equation $z = h$, or a plane section of equation: $ax + by + cz + d = 0$. To obtain the cut it is enough to traverse all the points of the objects in 3D and keep the points which belongs to the plane of the cut. Below is a geological section of object b and c described in Figure 9.

13. Result and Deformation of a Real Image

We can apply our model on an object having a real topography (plant, tree, rock, ...) and after deformation we can obtain result with real topographic texture (Figure 11).

14. Conclusion

In this work, we have represented a computer method for modeling 3D geological objects (Shin et al., 2008) based on a matrix calculation that always allows a certain number of basic operators to frame this object in a cube of fixed size. Then this provides the ability to fix the number of points affecting the deformation of an object, so it will be possible to fix the memory space representing the concerned object. First, the trial to represent the topography (Guilbert and Saux, 2008) by a simple correlation with an image (jpg, bmp, png, ...) of the reality, and second the fixing of the memory place taking into consideration the number of points of the object and framing it in a determined cube, then we will be able to apply an interesting analysis of the interior of the object. The presentation of the geological section by a cut in the flat surface $z = c$ and by making a simple rotation of the object gives an important result.

In the future, this work can be developed by taking into consideration a surface structure instead of being volumic, so we can have an optimization of the set of points that describes the interior of the object (Fuchs et al., 1983, Fuchs et al., 1980, Guilbert and Lin, 2007 and Lysenko et al., 2008). Since a deformation process is obtained by a resultant matrix, this matrix can be stacked and unfolded in order to have a back and forth of the deformation of the initial state. This work can be applied on the material modifications analysis.

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