Elementary Mathematics Teacher Preparation in an Era of Reform: The Development and Assessment of Mathematics for Teaching

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Teachers' understanding of the elementary school mathematics curriculum forms part, but not all, of the newly emerged field of mathematics for teaching, a term that describes the specialised mathematics knowledge of teachers. Pre-service teachers from a one-year teacher preparation program were studied in each of three years, using a pre-test/post-test survey of procedural and conceptual knowledge of mathematics required by elemtary teachers . Beliefs about mathematics were also examined through post-test interviews of 22 of the participants from one of the cohorts. Each cohort of teacher-candidates was consistently found to be initially weak in conceptual understanding of basic mathematics concepts as needed for teaching. The pre-service methods course, which included a strong focus on specialised mathematical concepts, significantly improved pre-service teachers' understandings, but only to a minimally acceptable level. Program changes, such as extra optional course in mathematics for teaching, together with a mandatory high-stakes examination in mathematics for teaching at the end of the methods course, have been subsequently implemented and show some promise.

Keywords: mathematics teacher education, pre-service teacher education, teacher mathematics knowledge, conceptual knowledge, teacher preparation, mathematics for teaching

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La compréhension qu'ont les enseignants du curriculum de mathématiques au primaire fait partie d'un nouveau domaine de recherche – les mathématiques en pratique d'enseignement – axé sur les notions mathématiques spécialisées dont les enseignants ont besoin. Des étudiants en pédagogie inscrits dans un programme de formation à l'enseignement d'un an pour le primaire ont fait l'objet d'une étude sur trois ans à l'aide d'une enquête pré-test et post-test portant sur leurs connaissances des méthodes et concepts liés aux mathématiques au primaire. Les croyances d'une vingtaine des participants au sujet des mathématiques ont également été analysées à l'aide d'entrevues post-test. L'auteure a constaté qu'au départ la compréhension des concepts mathématiques pour enseigner au primaire était faible dans chaque cohorte enseignant-étudiants. Le cours de méthodologie, fortement axé sur des notions mathématiques spécialisées, a amélioré nettement la compréhension des étudiants, mais seulement à un niveau tout juste acceptable. Des changements ont été par la suite apportés au programme, comme un choix plus vaste de cours optionnels de mathématiques en pratique d'enseignement et l'ajout d'un examen de mathématiques en pratique d'enseignement obligatoire et à enjeux élevés à la fin du cours de méthodologie. Ces changements semblent prometteurs.

Mots clés : formation à l'enseignement des mathématiques, formation à l'enseignement, connaissances mathématiques de l'enseignant, connaissance conceptuelle, mathématiques en pratique d'enseignement

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A wave of curricular changes in elementary school mathematics teaching, both in Canada and the United States, has been more or less influenced by the research described in the revised *Principles and Standards for School Mathematics* document re-released in 2000 by the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). In Ontario, for example, changes to the curriculum were released in 1997, with the most recent and current version emerging some years later (Ontario Ministry of Education, 2005). The Ontario curriculum, which is often referred to as a *Standards-based* or *reform-based* curriculum, reflects the five content strands described in the NCTM's *Standards* document; this recent curriculum is a much broader mathematics curriculum than was used prior to 1997. As well, changes inherent in the *Standards* make recommendations as to *how* students are to learn mathematics, with a considerable emphasis placed on the development of conceptual understanding through problem solving (NCTM, 2000). A similar emphasis is described in Ontario's elementary-level mathematics

curriculum (Ontario Ministry of Education, 2005). These changes in content as well as recommended teaching style represent a significant departure from traditional mathematics teaching.

Scholars have argued that the knowledge of classroom teachers is critical to the success of reform-based classroom practice (Barbeau & Taylor, 2005; Stein, Remillard, & Smith, 2007), and they have argued that teachers' knowledge of mathematics is a fundamental prerequisite for student achievement (Ball, Hill, & Bass, 2005; Heck, Banilower, Weiss, & Rosenberg, 2008; Wong & Lai, 2006). Yet in Ontario, in-service opportunities for teachers of mathematics have been uneven at best (Kajander & Mason, 2007), leaving the responsibility for teacher professional development largely up to pre-service programs. Hence, pre-service mathematics programs must address changes to content as well as pedagogy in an effective way so that they encourage new teachers to continue to actively seek opportunities for further professional learning.

The attainment of these goals is even further challenged in provinces such as Ontario that have a one-year teacher certification program. For example, the mathematics methods course at my institution is 36 hours. In fact, if the real classroom time is calculated by factoring in teacher candidates' travel to the next class as well as class breaks, the total actual learning time in the methods course is closer to 24 hours. Of these 24 hours, I, as instructor, also need to discuss non-mathematics topics such as classroom management and assessment. Thus with even the most careful twinning of mathematics content with pedagogical aspects, the reality is that not much more than 12 hours are available to significantly focus on mathematics understanding for teaching during my methods course, the focus of this article. These circumstances suggest the need for very careful use of this instructional time, as well as reconsideration of how much time should in fact be allotted to teacher preparation in mathematics.

The purpose of the research in the present study was to analyse what pre-service teachers entering our program might be expected to know about mathematics as needed for elementary school teaching, as well as to document growth in their understanding from their methods course. I studied three separate cohorts over three years. From my results, I have

developed recommendations for program change. Preliminary outcomes of these program changes show promise.

FRAMEWORK

Teacher Mathematics Knowledge

There continues to be lack of consensus in the literature as to what teachers need to know about mathematics to teach it well. In-service opportunities that provide teachers with experiences similar to what their students would receive in reform-based classrooms, such as those described in Langham, Sundberg, and Goodman (2006), are arguably founded on the premise that a deep understanding of the curriculum is sufficient mathematical knowledge for teachers. Others have argued that "special" mathematics for beginning teachers is not required and that "the starting point for mathematics education for *both* students and teachers should be a sophisticated and deep exploration of mathematics" (Gadanidis & Namukasa, 2007, p. 17). The assumption here is that the mathematical topics are those appropriate for teaching at the elementary school level. Alternately, a growing body of work (e.g., Hill, Sleep, Lewis, & Ball, 2007; Philipp et al., 2007; Stylianides & Ball, 2008) argues that the kinds of mathematical understandings teachers need include knowledge that goes beyond a deep curricular understanding. Ball, Hill, and Bass (2005) state that "knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably" (p. 21). However, the nature of this depth is not well defined (Ball, Bass, & Hill, 2004), which is problematic in determining how teachers' knowledge of mathematics for teaching might be best developed in teacher education programs.

Shulman (1986) initially described the concept of a specialised subject-matter knowledge for teachers, or "pedagogical content knowledge." More recently, Leikin (2006) has proposed a model to describe teachers' knowledge including *kinds of teachers' knowledge* (p. 2), which refers to teachers' subject matter knowledge, teachers' pedagogical content knowledge, and teachers' curricular knowledge. Moreira and David (2008) argue that the integration of academic mathematics (i.e., the kind of mathematics typically learned in school and used in formal settings) with the mathematics needed for school teaching practice is taken too

much for granted. They further describe how the values and principles of academic mathematics differ from those needed to teach students and may even be in conflict. Strategies may be needed to support the evolution of teachers' initial knowledge as developed during their own schooling to knowledge more appropriate for teaching, an evolution that may include changes in beliefs and values.

Chamberlain (2007) argues for a progression in content-based professional development courses from teachers taking the role of student, to one reflecting on that knowledge from the point of view of a teacher, to develop "pedagogical strategies that support students' making sense of the material" (p. 895). Ball and her colleagues have put forth strong arguments to support the development of a specialised knowledge of mathematics for teaching as part of teachers' development (Ball & Bass, 2003; Ball, Bass, & Hill, 2004). Ball et al. (2005) have described teachers' knowledge under the headings *common content knowledge*, described as "the basic skills that a mathematically literate adult would possess," and *specialized content knowledge*, or "a sort of applied mathematical knowledge unique to the work of teaching" (p. 45). An example of the former might be knowledge of the definition of a prime number, while the latter might include knowledge of appropriate models for learning fractions operations. They further argue that "effective teaching entails knowledge of mathematics that is above and beyond what a mathematically literate adult learns" (p. 45) and suggest that "there is a place in professional preparation for concentrating on teachers' specialised knowledge" (p. 45).

Two recently released reports in the United States further describe current issues related to teacher preparation in mathematics. The *Report* of the National Mathematics Advisory Panel (2008) states that "teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching" (p. xxi) and note the need in teacher preparation programs for "special emphasis on ways to ensure appropriate content knowledge for teaching" (p. 40). Similarly, the report on teacher preparation by the National Council on Teacher Quality (2008) states the need for teachers to "acquire a deep conceptual knowledge of mathematics" and further argues that "teacher candidates

should demonstrate a deeper understanding of mathematics content than is expected of children" (p. 40).

Ma (1999) states that what teachers need "is the awareness of the conceptual structure . . . inherent in elementary mathematics, and the ability to provide a foundation for that conceptual change . . . in students" (p. 124). A focus on conceptual development, a particularly important aspect of the kinds of mathematical concepts teachers need to know, forms a starting point for the current study.

Conceptual Knowledge

Moving beyond procedural knowledge to support the development of conceptual knowledge is a particular challenge at the pre-service level (Adler & Davis, 2006). An important aspect of mathematics for teaching may be the relationship between procedural knowledge and conceptual knowledge because in reform-based learning the procedures should be developed from, and built upon, students' understanding of the underlying concepts (Ambrose, 2004; Hiebert, 1999; Hill & Ball, 2004; Lloyd & Wilson, 1998; Rittle-Johnson & Kroedinger, 2002). This approach may be particularly challenging for pre-service teachers who have been schooled in a more traditional or procedural mode, which may include simply learning formulae by rote.

Procedural knowledge may be thought of as a sequence of actions, or the computational skills needed to negotiate set methods, while conceptual knowledge, rich in relationships, requires an understanding of the underlying structure of the ideas (Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Hiebert, 1992; McCormick, 1997). In reform-based learning, the development of generalisations and procedures should evolve from students' reasoning about mathematical ideas. Often these ideas are represented with examples, models, or with manipulatives, and ideally students consider multiple representations, leading to a flexible understanding. To support such learning, teachers need to (a) probe stages of student understanding, (b) comprehend multiple student solutions and methods, and (c) provide powerful classroom models with which to work (Hill & Ball, 2004). Clearly, such reform-based classroom environments are highly dependent on the quality of the mathematical contexts offered by a teacher.

The Role of Beliefs

Teachers' beliefs and values may also be an essential aspect of their classroom practices (Ambrose, Clement, Philipp, & Chauvot, 2004; Cooney, Shealy, & Arvold, 1998; Ernest, 1989; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003; Stipek, Givvin, Salmon, & MacGyvers, 2001), and hence considering teacher knowledge without examining their beliefs may yield an incomplete picture. Further, recent research has shown that teacher beliefs about mathematics had a stronger effect on teachers' practice than beliefs about teaching (Philipp et al., 2007; Wilkins, 2008). Because much research on beliefs has been of a case study nature, investigating the relationship among these variables on a larger scale is important (Adler, Ball, Krainer, Lin, & Novotna, 2005).

Based on their own previous classroom learning experiences, pre-service teachers may have come to believe that mathematics is mainly comprised of rules, formulae, and equations, and that they present these routines to students. In other cases, however, pre-service teachers may believe that mathematics is about interacting with problems, being creative, finding solutions without following a fixed classroom structure (Boaler, 1999). Hence beliefs may be a factor in the development of pre-service teachers' mathematical knowledge (Philipp et al., 2007; Wilkins, 2008).

GOALS AND METHODOLOGY

Goals

My main goal in the present study was to examine teacher candidates' knowledge and understanding of mathematics as needed for elementary school teaching, and how these might develop during a teacher preparation program. My second goal, through interviewing a subset of the participants at the post-test, was to examine their beliefs about the importance of developing conceptual understanding of elementary school mathematics as needed to teach well.

I investigated teacher candidates' mathematical understandings based on the constructs of *procedural knowledge* (PK) and *conceptual knowledge* (CK), which are supported by the notions of *common content knowledge* and *specialised content knowledge*. For the purposes of the present

study, I defined procedural knowledge as participants' knowledge of traditional elementary school content such as standard computational procedures. I expected that participants would demonstrate reasonably strong performance in this area, and I intended to use results to support teacher candidates' self-confidence by sharing the individual results with them.

I defined conceptual knowledge as the ability to provide alternative approaches, such as through models, justifications, or explanations of the fundamental mathematical procedures addressed by PK, a definition I based to a large extent on the similar construct described by Ma (1999), which she terms *profound understanding of fundamental mathematics.* I expected that this type of mathematical knowledge would be weaker initially. I had expected that participants, who were interested in their students' conceptual learning and who saw themselves as weaker conceptually but strong procedurally, might be willing to focus on their own conceptual learning, without feeling that they were "bad" at "everything" in mathematics. I was also interested in how beliefs about mathematics might evolve during such mathematical development.

Methodology

The context of the study was a Canadian pre-service teacher education program situated at Lakehead University, where I teach. Ontario teacher education programs are of one-year duration. The mathematics methods course taken by participants in the study is a 36 hour half-course that spans both semesters. Participants were elementary teacher candidates in the "junior-intermediate" (grades 4 to 10) cohort; more than 300 participants were studied over three years, during which I taught 12 separate class groupings of the methods course.

Measuring Mathematical Knowledge

Several instruments are available to measure teachers' understanding of mathematics as needed for teaching, with the best known one being *the Learning Math for Teaching* instrument (Hill et al., 2007). Although tasks and open-response items may have higher face validity than multiple choice items, they have seldom been studied regarding their validity or reliability (Hill et al., 2007).

The *Learning Math for Teaching* (LMT) instrument, a test with teaching-related, multiple choice items, was developed to measure the knowledge of large groups of teachers based on their ability to use mathematical content (Hill & Ball, 2004). The validity of the LMT instrument has been studied and the results show evidence of construct validity (Hill & Ball, 2004). The reliability of the LMT instrument was also examined with alpha coefficients for a classical test theory of reliability measures, and the reliabilities within each scale ranged from 0.71 to 0.84. However its multiple choice format may not allow examination of all the mathematical concepts teachers need to know (Hill et al., 2007) or how they might develop.

The LMT instrument (Hill & Ball, 2004) includes multiple-choice items related to "common" content knowledge, and those related to ". . . teachers' grasp of representations of content" (Hill et al., 2007, p. 130). Both aspects are important for teaching.

Instrument Development

An instrument was needed to assess knowledge at the beginning and end of the two-semester, mathematics methods course for elementary teacher candidates. Because of the time constraints of the program, brevity of the instrument was important; I did not use other existing beliefs instruments such as the IMAP [Integrating Mathematics and Pedagogy] Web-Based Beliefs Survey (Ambrose et al., 2004). As well, as described, because the dual foci of procedural as well as conceptual learning were of interest, I needed an instrument that separated these two constructs, yet was brief to administer. For these reasons, I developed the *Perceptions of Mathematics* (POM) survey.

The survey consists of separate sets of (open-response) knowledge items that separated the provision of procedures leading to correct answers (procedural knowledge or PK) from the ability to provide alternative solutions, explanations, justifications, or models (conceptual knowledge or CK) which is the hallmark of teacher understanding as needed for teaching. Face validity of the CK items draws heavily on the items used in Ma's (1999) landmark study.

I conducted a one-year pilot study to test and improve the survey with a preliminary group of 100 teacher candidates (Kajander, 2005) as

well as with a group of in-service teachers (Kajander, Keene, Zerpa, & Siddo, 2006). Based on item analysis, I revised the initial POM survey, lengthened it, and retested it on a group of 30 grade-7 teachers who were participating in a mathematics-related, professional development initiative at their Board (Kajander et al., 2006). I used the Middle School LMT instrument (Hill & Ball, 2004) in parallel with the POM. Both tests utilized a paper and pencil pre-test/post-test format. I again performed an item analysis, and correlated the POM measures of mathematical knowledge (conceptual and procedural knowledge) to the LMT measures of mathematical content knowledge (number and operation, algebra and geometry) for the two administrations of the instruments (pre-test and post-test). I used the correlations to show evidence of concurrent validity of the POM instrument when measuring mathematical knowledge (Zerpa, 2008). I again performed item analysis and shortened the instrument to 20 items (10 per variable, hence a maximum score of 10 points) and slightly revised them based on this second analysis. I used the third and current version of the instrument (Kajander, 2007) with the three cohorts of pre-service students at the beginning and end of their one-year mathematics methods courses as I further discuss in this article.

I surveryed about 100 teacher-candidates each year for the three years of the present study, using the final version of the POM survey (see Kajander, 2007, or Kajander & Mason, 2007, for the actual survey items). The survey was administered during the very first class of the mathematics methods course and again at the second last class. A team of two graduate students and me, as researcher and course instructor, scored the survey each year, working together until we achieved consistency. We developed a detailed scoring guide, which included sample responses of the open-response knowledge items to standardize scoring. A graduate student then scored participants' answers and I did not view individual survey results during the course. In each case, graduate students shared individual results of the survey with participants in the class immediately following the administration of the survey. I encouraged participants to use their scores for self-reflection and for setting personal goals. I encouraged participants to think about their procedural knowledge scores as what they had learned thus far, and reminded them

that developing deeper conceptual knowledge would further support their ability to teach others.

I used descriptive statistics obtained from the Perceptions of Mathematics survey (POM) to examine the effect of the intervention, namely the mathematics methods course, on pre-service teachers' mathematical knowledge. T-tests for repeated measures were then used to analyze the treatment effect between the pre- and post-test for each variable (conceptual/procedural knowledge). In the third year of the study, I conducted semi-structured exit interviews with 22 participants, which I transcribed and analysed.

Survey Validity and Reliability

I established initial face validity by basing the mathematics items in the POM instrument largely on Ma's (1999) interview questions. I obtained stronger evidence of validity measures of the POM instrument by comparing the POM with the LMT instrument (Hill & Ball, 2004) as a measure of concurrent validity. With this concurrent validity approach, I hypothesized that the two instruments – POM and LMT – would produce related measurements of mathematical knowledge in the pre-test and post-test because both instruments measured the same or similar constructs of mathematical knowledge. Participants in the year-two cohort wrote the LMT survey (Hill & Ball, 2004) at both the pre-test and the post-test concurrently with the POM survey. The results of the analysis showed positive significant correlations between the two instruments (POM and LMT) at the 0.05 level at both the pre-test and post-test (Zerpa, 2008). Furthermore, I obtained evidence of reliability measures of the POM instrument by analyzing the internal consistency of the POM with respect to measures of mathematical knowledge. The results of this analysis show a Cronbach's Alpha coefficient of 0.83 for both conceptual and procedural mathematical knowledge measures in the POM instrument from the pre-test and post-test (Zerpa, 2008).

RESULTS

I analyzed survey scores (from about 100 participants each year) for each of the three years of the present study; I also analyzed data for 22 individual interviews all conducted in year 3. Mean pre-test and post-test

scores respectively on the two survey variables were generally consistent over the three years of the study and from one class section to another. The survey revealed an interesting and consistent picture of pre-service teacher development. Results from the interviews support the data, suggesting that participants generally claimed to have developed stronger beliefs about the importance of conceptual learning for their students from the beginning to the end of the methods course, as well as developing significantly stronger conceptual understanding of the mathematical concepts themselves.

Survey Results Related to Knowledge of Mathematics

The knowledge variables also painted a strong picture of change. Procedural knowledge was initially reasonably strong, and increased during the methods course (see Table 1). In each of the three years, procedural knowledge increased significantly, as well as overall $t(310) = -15.41$, $p < .001$, $r^2 = 0.434$ (see Table 2). The conceptual knowledge of the pre-service teachers, while extremely weak initially (see Table 1), also showed a significant increase in each year and for the combined data, $t(310) = -25.02$, $p < .001$, $r^2=0.669$ (see Table 2).

The data indicate that participating teacher candidates' initial procedural knowledge was much higher than their conceptual knowledge; procedural knowledge scores of the teacher candidates as they began the methods course (and without any prior warning of the survey) showed means of about 70 per cent (shown as a value out of 10 in Table 1), and generally well over 80 per cent at the post-test. For a sample procedural knowledge response at the post-test, see Figure 1.

On the other hand, the means of the conceptual knowledge pre-test scores ranged from 1.0 to 2.6 out of 10, or 10 per cent to 26 per cent (see Table 1 for the values scaled out of 10). Means at the post-test ranged from 48 to 60 per cent on the same items (see Figure 2 for a sample CK response on a survey item at the post-test).

Table 1
Mean scores of POM survey, years 1-3

	Year 1	
	PK (*SD*)	CK (*SD*)
Pre-test	6.9 (2.13)	1.0 (1.42)
Post-test	8.4 (1.79)	4.8 (2.50)
	Year 2	
	PK (*SD*)	CK (*SD*)
Pre-test	7.0 (2.01)	2.6 (1.46)
Post-test	8.7 (1.99)	5.2 (2.50)
	Year 3	
	PK (*SD*)	CK (*SD*)
Pre-test	7.3 (1.75)	1.0 (1.40)
Post-test	9.4 (1.02)	6.0 (2.69)

Note. Change in all scores from pre-test to post-test is significant ($p<.001$). Each year included about 100 candidates, total over three years n=310.

Table 2
Paired Samples test of Changes in Procedural (PK) and Conceptual (CK) Knowledge

						95% CI				
			M	*SD*	*SE*	*LL*	*UL*	*T*	*df*	*p*
Pooled (Yrs 1 to 3)	Pair 1	prePK - postPK	-1.76	2.02	0.11	-1.99	-1.54	-15.41	310	0.000
	Pair 2	preCK - postCK	-3.88	2.74	0.16	-4.19	-3.58	-25.02	310	0.000
Year 1	Pair 1	prePK - postPK	-1.50	2.10	0.19	-1.89	-1.12	-7.80	118	0.000
	Pair 2	preCK - postCK	-3.81	2.60	0.24	-4.28	-3.33	-15.96	118	0.000
Year 2	Pair 1	prePK - postPK	-1.71	2.28	0.25	-2.21	-1.21	-6.85	82	0.000
	Pair 2	preCK - postCK	-2.58	2.49	0.27	-3.12	-2.03	-9.44	82	0.000
Year 3	Pair 1	prePK - postPK	-2.08	1.65	0.16	-2.40	-1.77	-13.17	108	0.000
	Pair 2	preCK - postCK	-4.95	2.64	0.25	-5.46	-4.45	-19.59	108	0.000

Note. CI = confidence interval; *LL* = lower limit; *UL* = upper limit.

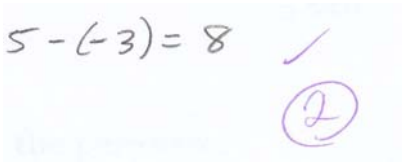

Figure 1

Sample Procedural Knowledge (PK) post-test item and response. Participants were asked to calculate the answer to 5 – (-3).

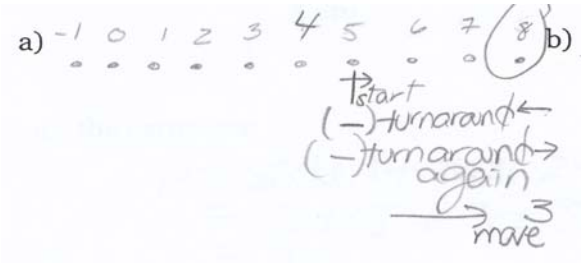

Figure 2

Sample Conceptual Knowledge (CK) post-test item and response. Participants were asked to use a model and/or reasoning to explain their calculation for 5 – (-3).

Overall, the data indicate that candidates generally had a far-from-strong ability, even at the end of the program, to provide explanations, models, or other evidence of alternative methods that indicated a deep understanding of standard mathematical procedures. In summary, although conceptual knowledge may have improved significantly during the participants' one-year teacher preparation program, many teacher-candidates graduated with what may be an inadequate understanding of the conceptual aspects of elementary school mathematics content, arguably the backbone of reform-based mathematics for teaching (NCTM, 2000).

Interview Data

Responses from the individual semi-structured interviews, conducted at the post-test in year three, support the survey data. Nearly all participants mentioned an awareness of their initial weakness in the conceptual aspects of their mathematical understanding at the beginning of the course, and some, to a varying degree, expressed anxiety and fear of mathematics. Virtually all participants described their own prior learning in mathematics as memorization. For example, one participant whose knowledge scores in both variables were above average at both the pre-test and the post-test said, "I learned the formulas, I got the right answers, I got a good mark, and I don't know anything about what it actually meant, [n]or could I use it in any way, shape or form at this point in my life." When asked about their prior conceptual learning, interviewees typical responses were "not very strong," or "I wasn't very sure even what that meant." A few mentioned that a particular teacher who had taught them more conceptually stood out in their minds in a positive way. When asked to recall their perceptions of mathematics teaching at the beginning of the teacher-certification year, many participants described intending initially to teach as they had been taught. For example, one candidate, who also had above average knowledge scores on the survey, responded, "Well I really didn't have any experience with the conceptual methods [prior to the course] so I just assumed it was okay to teach the formula and then off you go."

Virtually all interviewees described changes during the year in their point of view about what was important in mathematics teaching and learning. One respondent, whose initial knowledge scores were average, but who demonstrated above average growth in conceptual knowledge in particular, said, "It's almost like I've relearned things to think conceptually, rather than just follow the steps and do it that way. It's just like my eyes have been opened."

Many respondents indicated satisfaction with their experiences and growth. For example, one participant who began the course with average knowledge scores, but whose PK and CK scores both increased to above average by the post-test, said, "I'm excited about the different ways of teaching math." Another participant, whose initial conceptual knowledge score was below the mean, described the following experience:

> Yesterday in class . . . I spent my whole break working on a question, and I finally got it, and I realised, hey, I've actually got involved! I've been engaged by a math problem. Like that's, that's very odd for me, so it was a really nice moment for me.

Interestingly, these two participants had post-test scores that were three or more (out of a possible 10) points above the mean in CK. Yet another participant, who began the course with low knowledge scores in both knowledge variables, was asked whether her understanding of mathematics had changed during the course responded: "Oh like huge. I wouldn't say I'm fully over my phobia, but . . . with time and practice, I feel confident I'll learn everything."

Many participants expressed the desire for more time in the mathematics methods course: "The course does need to be longer." Another participant, whose knowledge scores were all above average, described herself in the post-test interview as "being on the verge of making those deep connections," also indicated the need for more course time.

Distressingly, most interviewees indicated that their associate teachers (teachers who supervised them during their classroom practica) did not use the kinds of teaching methodologies they had learned about and experienced in the methods course. One described her associate as "controlling," and another said, "She wanted everything out of the book." Only a few reported working with teachers who regularly used manipulatives and other hands-on materials. These observations point to issues with long-term collegial support for the newly graduated teachers in the study, as well as factors that may influence participants' growth during the year.

DISCUSSION

The survey and interview data illustrate some of the challenges faced by prospective teachers with largely traditional mathematics backgrounds as they struggled to change their perspectives and deepen their knowledge of mathematics in ways more appropriate for teaching. Candidates initially appeared to have had previous experience with procedural learning, and indicated very weak to non-existent understanding of conceptual aspects. However, describing the candidates as "weak" at math-

ematics produces an incomplete picture; participants generally demonstrated a reasonably strong procedural knowledge of basic mathematics concepts in the elementary curriculum, as well as a desire to help their own students to learn more conceptually. During the interviews, participants described how finding out about their level of conceptual knowledge helped them focus on this learning much more than previously: "it's like I've relearned . . . to think conceptually." The survey pre-test results, which were shared with participants during the second class of the methods course, may have allowed me, as instructor, to more readily concentrate on conceptual aspects of the subject with support of the teacher candidates. Participants described that learning early on in the course about their weaknesses on the more conceptual aspects of mathematics helped to support their willingness to have "their eyes opened"; whereas many felt that they did not really know prior to the course about the importance of conceptual learning or how it differed from procedural aspects. Several interview participants used the word "phobia" to describe their prior feelings about mathematics, and yet during the course most expressed interest rather than dismay about increasing their conceptual knowledge.

The results from the present study support other research (e.g., Wilkins, 2008), indicating that both beliefs and understanding play a role in teacher development. The increase in procedural knowledge during a methods course, which focused largely on conceptual aspects, also supports a premise of mathematics reform, namely that procedural skill will and should develop *along with* a focus on concepts (NCTM, 2000).

According to the interview data, participants generally claimed at the end of the program to have come to believe more strongly in the importance of the conceptual aspects of learning through problem solving and reasoning, using models and manipulatives. As well during these interviews, participants claimed to have a diminished desire to directly teach or to focus mostly on set procedures and methods. Both of these shifts align with ideas of mathematics reform as described by NCTM (2000).

Although the survey and interviews suggested a picture of growth and positive development, the data also indicate that much more understanding is needed. Many participants cited program length as an issue

for their learning in mathematics education. The conceptual understanding of teacher candidates as demonstratd by the post-test data at the end of the methods course, although much improved, was in general far from adequate to negotiate reform-based classroom environments independently. The survey and interview results support Ma's (1999) observations from her interviews of U. S. teachers, namely that most elementary school teachers were able to calculate correct answers using a standard method, but some were not able to provide alternative methods, such as with models or justifications.

Many of the interviewees claimed that the teacher with whom they worked during their practicum did not follow ideas espoused in their methods course, hence concerns remain as to how and where these new teachers will find like-minded and knowledgeable colleagues to support their continued personal growth. My work with professional learning groups of in-service teachers – a currently popular form of professional development – indicates that teacher groups that do not have access to a knowledgeable colleagues or a facilitator may have more trouble escaping from a traditional agenda (Kajander & Mason, 2007). All these factors place pressure on pre-service programs to prepare teachers as well as possible before they enter their own classrooms.

Applications

Based on the data presented from the present study, I made two program changes in my methods course just prior to the third year of the study. In considering year one and two data, I noted that some participants were still attaining very low conceptual knowledge scores at the post-test. To compensate, I developed a 20-hour optional, non-credit course entitled "Mathematics for Teaching," which an elementary teacher with strong mathematics and teaching knowledge taught in the third year of the study. About 20 participants (about 20% of the year-3 cohort) completed the course, which was offered during six weekly sessions in each of the two semesters of the methods course (4 sessions of one and one half hours, and 2 two-hour sessions, in each semester). The mathematics content of the optional course was purposely aligned with the topics in the methods course. Participants in this course, who were interviewed as part of the present study, were universally happy with the

courses, saying that the extra time was very helpful to them to work through the ideas: "I know there's not a lot of time in the [methods] course to cover a lot of the material in depth . . . a lot of the time it was rushed but . . . [the extra course] gave you the time to learn it." Although the sample of participants who took the extra course is small, the greatest growth in CK took place during the year when optional course was offered (see Table 1).

The second change, which was not established without considerable deliberation, was the initiation of a final written exam for all participants on mathematical concepts needed for effective teaching. The exam was worth 25 per cent of the methods course grade. The Lakehead program now requires all teacher-candidates at the "junior-intermediate" (grades 4 to 10) level to pass this exam with at least a 50 per cent score to receive a passing grade on the methods course.[1] The exam, which I initially set, was edited and discussed by two other mathematics educators, two elementary teachers including the instructor of the optional course, and a mathematician. For example, candidates were asked to use a particular calculation of the product of two proper fractions to illustrate a model that would demonstrate the operation as well as show how the answer made sense, and to explain and justify the traditional fraction multiplication procedure using their model. The other questions on the exam which were similar to the CK POM survey questions focused on models, reasoning, and explanations of methods (not unlike the concept of *specialised content knowledge)* which are arguably the backbone of the mathematical content needed for teaching. A similar exam had existed in years one and two of the study and was also worth 25 per cent of the methods course grade, but participants were not required to pass it as long as they passed the course overall.

Examination of the data indicates that all teachers candidates who took the optional course easily passed the methods course mathematics exam, scoring 60 per cent or higher. The few teacher candidates who did not pass the exam had not taken the optional course, nor did their CK scores increase over the duration of the year. Preliminary analysis sug-

[1] Students who do not pass all their courses in the teacher education program are delayed in graduating until all the requirements are met, and so this additional requirement caused some anxiety.

gests that these program changes may also have reduced the number of very low achieving candidates in year three. Perhaps the larger increase from pre-test to post-test seen overall in CK in year 3 as compared with years 1 and 2 may be due at least in part to these program changes. Although a larger sample of students who took the optional course will be needed to determine definitively the benefits of the optional course, these preliminary indications are promising.

CONCLUSIONS AND RECOMMENDATIONS

The present study is supported by other research that suggests that the kind of knowledge teachers need for effective teaching may go beyond the mathematics they typically learned in school (Moreira & David, 2008). Teacher candidates generally arrived at my methods course with adequate recall of, and facility with, standard mathematical methods and procedures, but were generally unable to explain the methods they used, or provide a relevant model or example that might be needed for effective classroom teaching. Although most participants made significant progress in conceptual understanding, deep mathematical understanding as needed for teaching remained far from strong.

According to the interviews, participants' beliefs, which have been argued as a strong predictor of inquiry-based classroom practice (Wilkins, 2008), did appear to shift to a more reform-based conception. At the end of the program, interviewees claimed to be less concerned with classroom practices typically associated with traditional learning, and more interested in the kinds of practices to promote deep learning and problem solving.

Although interested teachers can, in the right professional development environment, continue to deepen their mathematical understanding (Kajander & Mason, 2007), professional development in mathematics is currently not readily available in Ontario to all practising teachers. The future of reform-based teaching, which is arguably at least somewhat dependent on teachers' deep mathematical knowledge, will no doubt be compromised if teachers are not supported in their own mathematical development. According to a recent report,

> Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding. Required coursework should be tailored to the unique needs of the elementary teacher. (National Council on Teacher Quality, 2008, p.11)

The report advises that a minimum of 115 hours be devoted to mathematics preparation specifically for elementary teachers (National Council on Teacher Quality, 2008). As well a recent Canadian policy statement, as found in Kajander & Jarvis (2009), calls for similar mathematics preparation. These recommendations far exceed the time available in the program described here. In conclusion, although the present study has provided evidence that teachers can significantly deepen their mathematical understanding and shift their beliefs during a pre-service teacher education program, more time in such programs appears to be required for conceptual understandings as needed for teaching to more fully develop. Further program changes, which require at least our concurrent education students to take a specialised mathematics course as part of their undergraduate degree, are in the planning stage at our institution.

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