Philosophical Reflection and Cooperative Practices in an Elementary School Mathematics Classroom

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Following Matthew Lipman (Lipman, 1991; Lipman, Sharp, & Oscanyan, 1980), we introduced philosophical dialogue (PD) about mathematics in an elementary school to help pupils consider mathematical and meta-mathematical matters. This article describes the social and cognitive activity when pupils engage in PD and some pedagogical conditions necessary to foster the development of PD. Changes in pupils' discussions from the beginning to the end of the test period showed that the dynamic evolved from *monological* exchanges to *dialogical* exchanges. Whereas early pupil responses could be characterized mainly as simple *answers*, later responses displayed more *lower-order* and even *higher-order* thinking skills. The data suggest that for PD about mathematics to develop, the teacher must be proficient in the role of mediator.

À la suite de Matthew Lipman (Lipman, 1991; Lipman, Sharp et Oscanyan, 1980), les auteurs ont introduit le dialogue philosophique (DP) dans une école primaire afin d'aider les élèves à réfléchir aux questions d'ordre mathématique et méta-mathématique. Cet article décrit l'activité sociale et cognitive à laquelle donne lieu le DP ainsi que certaines des conditions pédagogiques qui doivent être réunies pour favoriser le développement du DP. L'analyse des discussions des élèves du début à la fin de la période d'essai a révélé une évolution de la dynamique, des échanges *monologiques* aux échanges *dialogiques*. Si en premier les élèves donnent de simples réponses, par la suite ils démontrent des capacités de raisonnement élémentaires et même de plus haut niveau. Ces données semblent indiquer que pour que le DP au sujet de la mathématique se développe, l'enseignant doit devenir un médiateur efficace.

Some studies suggest that the mathematics curriculum is not sufficiently meaningful for pupils (Baruk, 1992), does not foster critical thinking (Lipman, l991), and allows no place for philosophical discussion (Smith, 1995). On this last point, in mathematics classes the use of discussions among pupils is increasingly widespread, to the point that these contribute to improvement of pupils' mathematical comprehension (e.g., Bauersfeld, 1980; Hoyles, 1985; Sfard, Nesher, Streefland, Cobb, & Mason, 1998). However, we judge that discussions among pupils are generally organized from the perspective of problem-solving. The goal is to find solutions to mathematical problems submitted by the teacher or to discover the most efficient methods for solving these problems (Cobb, Wood, Yackel, & McNeal, 1992; Lampert, 1990). "Analyzing the adequacy of methods and searching for better ones are the activities around which teachers build the social and intellectual community of the classroom" (Hiebert et al., 1996, p. 16). Although this type of discussion improves pupils' mathematical performance (Cobb et al., 1992; Hiebert & Wearne, 1993; Lampert, 1990; Wood & Sellers, 1996), we judge that the emphasis placed on solving problems risks becoming too utilitarian and, for this reason, insufficiently stimulating of reflection and ideas among the pupils. "When useful mathematics becomes synonymous with learning strategies for solving problems, attention shifts to procedures and away from ideas" (Prawat, 1991).

Inspired by the works of Matthew Lipman and Ann Margaret Sharp (Lipman, Sharp, & Oscanyan, 1980), which has its foundations in the "reflexive inquiry" proposed by John Dewey (1933), we consider that significant teaching of mathematics should promote not only the acquisition of disciplinary knowledge but also the development of pupils' curiosity about the field, their critical thinking, and their social sense. Schools should give as much importance to ideas and concepts as to procedures for solving problems. That is, we suggest that regular mathematics teaching in primary school be complemented (for one hour each week) by reflection and dialogue among pupils on philosophico-mathematical concepts that interest them and meta-mathematical ideas that concern them. *Philosophico-mathematical* describes concepts whose essence is philosophical and whose content is mathematical; *meta-mathematical* is used here to describe events of everyday life that have a more or less direct relationship to mathematical notions. Basically, according to Dewey, schools should not distinguish between scientific problems and everyday problems (Dewey, 1929/1960).

We do not pretend that a pedagogical approach emphasizing philosophical dialogue (PD) is more appropriate than the functionalist and structuralist approaches discussed above for fostering the learning of mathematics among pupils; both types of approach are socio-constructivist, and they seem more complementary than competitive.

PHILOSOPHY FOR CHILDREN ADAPTED TO MATHEMATICS

The Philosophy for Children (P4C) Approach

P4C, a philosophical approach to education pioneered in the 1970s by Lipman and Sharp, has been implemented in almost 50 countries. Its primary goal is to teach youngsters to think critically (Lipman, 1991; Lipman et al., 1980) along the lines of Dewey's (1933) reflective thinking. According to Lipman, critical thinking does not involve an exclusionary (for example, Platonic) perspective but rather seeks to establish a distinction between basic or lower-order thinking skills (description, explaining, giving examples, making statements, etc.) and higher-order thinking skills (justifying opinions, offering criticism, being sensitive to context, self-correction, following criteria, etc.) (Lipman, 1995).

P4C is distinguished by the philosophical quality of the dialogue that develops within a peer "community of enquiry." PD centres on philosophical concepts and ambiguous meanings; it deals with uncertainties, challenges, and compromises (see Richards, 1991). It presupposes the use of natural language (as opposed to formal language or mathematics) (see Sfard et al., 1998); its points of departure and arrival are found in daily experience (Dewey, 1916/1983). Yet it is not synonymous with conversation, in which participants' comments relate personal facts or events. Neither is dialogue synonymous with argumentation, the rhetorical game in which each participant's primary goal is "win the battle of wits." The dialogue in question emphasizes constructing ideas from points of view presented by peers to solve a common problem or to achieve a common goal (see Vygotsky, 1984). Also, in the dialogue, pupils learn together to question ideas, experiences, and events (Lipman et al., 1980). For the dialogue to become philosophical, regular practice in the classroom is required (Richards, 1991).

The community of philosophical enquiry, considered to be a micro-society, has a structure similar to that of other communities of enquiry (see Siegel & Borasi, 1996) but characterized by its dialectical nature and its aim of making participants think philosophically. By *philosophy* (and its various word-forms) we mean the deconstruction of assumed beliefs, prejudices, and concepts. Philosophy, on this view, rejects basic truths located in doctrinaire knowledge and promotes the subject's search for individual and social meaning. To say that pupils engage in the community of philosophical enquiry is to imply that they use higher-order thinking skills and cooperative behaviours (Splitter & Sharp, 1995).

Although the pupils lead the game (group discussions develop around their interests and their needs), the mediator assists the pupils to clarify and formulate their ideas and to set up dialogue and interaction. The mediator is usually the regular class teacher, who plays Socrates' role, asking pupils questions to help them deepen their ideas, find justifications for their opinions, and so on. The mediator believes that knowledge exists in the learner's ability to organize thoughts and actions, and values the learner's commitment to interpreting reality (Smith, 1995). The fact that the mediator has more experience and knowledge than the pupils does not make that person the sole source of authority; the mediator is a member of the community of enquiry that thinks and that questions itself (see Richards, 1991).

Briefly, the pedagogy of PD differs from the pedagogy of group discussions in that it is not teacher-directed but is instead centred on student problems. It takes students' questions and students' interests as the starting points for enquiry. Students' concerns set its agenda for discussion. PD works on open problems (philosophical concepts), not closed problems (problem-solving strategies). It is aimed not at training high-performance pupils but at educating individuals who are curious, autonomous, critical, and responsible at school and in everyday life.

Epistemologically, PD differs from group discussion in that it is dialectical. It is a method of enquiry that exploits both the complexity of a discipline and real-life situations, focuses on non-traditional topics where uncertainty and limitations are common, uses errors as springboards for enquiry among peers, exploits the surprise elicited when one works with new concepts or with anomalies, creates ambiguity and conflict by proposing alternatives to the status quo, and provides occasions for reflecting on the significance of one's enquiry (Borasi, 1992, p. 190).

Philosophy for Children Adapted to Mathematics

Inspired by the program of Lipman and Sharp, we developed philosophico-mathematical course material (P4CM) for pupils aged 9 to 13 years.[1] As evaluation of the course material by experts and by pupils aged 9–13 was very positive (Daniel, Lafortune, Pallascio, & Sykes, 1994), we undertook a preliminary research project.

The material consists of two philosophical novels for pupils and a manual intended for P4CM mediators, usually the mathematics class teachers (Daniel, Lafortune, Pallascio, & Sykes, 1996a, 1996b, 1996c). The novels, each of which develops a narrative involving young people of the same age as the readers/pupils, present the thoughts of a group of friends on ideas about mathematics and learning mathematics. The mathematical content of the novels evolves in parallel with the school curriculum. The mediator's manual works in tandem with the two novels and is intended to assist P4CM mediators in their role by suggesting philosophical discussion plans and mathematical activities or exercises. The questions implicit in the narrative are dealt with explicitly in the mediator's manual. These questions are of two types: *philosophico-mathematical* (e.g., Does a perfect cube exist? Does zero signify nothing? What are the similarities and differences between a digit and a number? Was mathematics invented or discovered?) and *meta-mathematical* (e.g., What use is mathematics? Does the math teacher have to know everything?).

In the classroom, the P4CM approach amounts to pupils reading a chapter in the novel, collecting pupils' questions arising from the reading, and philosophical dialogue within a community of enquiry on one question chosen by the group.

RESEARCH METHOD

Experimental P4CM sessions took place from mid-October to the end of May. Weekly sessions lasted one hour and involved an entire Grade 6 class of 30 pupils from an underprivileged socio-economic background. Prior to the experiment, the class teacher participated in a two-day training course on the P4CM approach. For P4C to work well, teachers must learn to mediate a philosophical dialogue in class, that is, to question pupils in a way that revamps their ideas with a view to deepening them and exploiting them, without knowing all the answers.[2] Teacher training continued throughout the school year in that each week a resource expert in P4CM assisted the teacher to prepare her lessons, monitored the exchanges with the pupils, and presented the teacher with a review of the session.

Our research project is qualitative in orientation; it aims to describe “what is” in a group-class, not to verify a theory or even to state “what should be” in all communities of enquiry. Nevertheless, certain data were analyzed quantitatively to allow us to observe whether changes took place in the class between the beginning and the end of the school year (Van der Maren, 1995, pp. 85–87).

We examine two discussions using transcripts of two video recordings, one at the beginning and one at the end of the school year. For each transcript, we compiled the data and studied the percentage of interventions belonging to each of the three types. Two research assistants helped to validate and code the pupils’ interventions. The principal researcher suggested the definition of the codes for the transcripts and did the initial coding. Coding focused on the general character of the exchanges, the dynamics of the exchanges, the behaviour and roles of the mediator, and the thinking skills that pupils demonstrated. Conformity of coding between the two research assistants exceeded 85%.

General Character of the Exchanges: Anecdotal, Monological, Dialogical

To analyze the character of the exchanges, we classified pupils’ interactions in the transcripts into three types. *Anecdotal* exchanges are based on elaboration of anecdotes or of pupils’ personal experiences. *Monological* exchanges involve individual pupils discoursing on a particular theme. Pupils scarcely listen to each other; each pursues his or her own idea without being influenced by peers’ points of view. *Dialogical* exchanges are based on interrelationships. They emphasize the construction of ideas from points of view expressed by a group of peers while attempting to solve a common problem or to achieve a common goal.

Dynamics of the Exchanges: Pupil-Pupil or Pupil-Mediator

To study the dynamics of the exchanges, we examined the content of pupil interventions to determine whether each dealt with a question or a comment expressed by either the mediator or another pupil. The relationship may be explicit, such as "to answer so-and-so's question" or "in reference to what so-and-so just said," or implicit. In the latter case, we were guided by the flow of logic.

Behaviour and Roles of the Mediator

We examined the roles played by the teacher as mediator (management and control of discipline, fostering the community of enquiry, etc.) and the type of questions she asked the pupils (seeking simple answers or stimulating cognitive conflict).

Pupils' Thinking Skills

We coded every pupil intervention (defined as any verbal utterance by a pupil) according to the thinking skills the pupils displayed. Interventions were classified into three types. *Answers* are interventions of one or a few words (as opposed to a complete sentence). They do not represent the statement of an idea as such but rather are simple responses to questions or comments. *Interventions displaying lower-order thinking skills* involve basic cognitive skills that emerge spontaneously among pupils of this age, including stating a point of view, questioning (to obtain a specific answer), description, explanation, illustration with examples, comparison, and providing a simple definition. *Interventions displaying higher-order thinking skills* involve complex cognitive skills that do not usually appear in Grade 6 pupils' discourse unless specifically encouraged. They include defining in a complex way, justifying, expressing a nuance, formulating a question (in a search for meaning or perspective), developing logical relationships (part-whole, means-end, etc.), searching for criteria, proposing hypotheses, and criticizing. To distinguish lower-order from higher-order thinking skills, we used the categorization proposed by Dewey (1933, 1916/1983) and Lipman (1991, 1995).

ANALYSIS

Comparison of the Character of the Exchanges

The first recording followed pupils' reading of the first chapter of one of the P4CM novels. The teacher collected pupils' questions. Then came the choice (by consensus) of the question the pupils wanted to discuss: "Why don't we like mathematics?" The teacher chose to have the pupils discuss their different strategies for learning mathematics, perhaps because she assumed that if pupils dislike mathematics, it is because they lack pertinent learning strategies.

During the first half of the session, she asked pupils to memorize a number of words, then verified pupils' recollections. The exchanges can be characterized as linear, following the traditional "question-answer" archetype:

Mediator: So, *P*, you remembered how many words?
P: 20.
Mediator: *A*?
A: Me? 19.
Mediator: *S*?
S: 12.
Mediator: *G*?
G: 15.

Later in the session, the mediator sought to determine what memorization strategy the pupils had used:

Mediator: Did you use a particular strategy?
K: I said "addition" in my head, so immediately I thought of addition, division, multiplication. When I said "square," I thought of all the shapes I know. When I said "denominator," I immediately thought of numerator.
Mediator: *K*?
K: I repeated [the words] in my head.
Mediator: That's the strategy you used. Did you repeat them in your head, *M*?
M: I repeated the words in my head, then hid the sheet and repeated them again. If I forgot, I started over.
Mediator: So you repeated them in your head, then said them again. *P*?
P: I looked at my paper, then repeated all the words in order. Then I covered the paper and tried to say them again. Not in order, but I tried to re-say them in my head.

We classify these communications as *monological* because the brevity of their answers does not allow the pupils to participate in a dynamic situation characterized by the co-construction of knowledge. This, plus the fact that they rarely interact with their peers, indicates that each pupil pursues his or her own idea without being influenced by peers' points of view and that they actually disregard peers' points of view when stating their own opinions. For example, when asked to define "strategy," each proposed an independent definition.

At the end of the school year, a second video recording was made. After reading a chapter from the same novel, the pupils chose the following question for discussion: "Why would we ever invite a math-brain to a party?" The nature of this question is meta-mathematical, even ethical, in that it deals with the prejudices the pupils have towards those who do well in school. To exploit this question, the teacher asked the pupils to break up into teams and to draw their representation of a math-brain; afterwards, the class came together again to engage in dialogue on the subject.

General analysis of the second transcript shows that the exchanges were conducted in the *dialogical* mode, with each pupil developing his or her own intervention based on the previous intervention, thus adding to the complexity of the exchanges. So *K* states that she would never invite a math-brain to a party, and justifies her remarks as follows:

K: . . . Because math-brains, if they go to a party, they talk about nothing but math.
J: Me, if I had a party, I would invite the math-brains anyway, since *P* and *M* [two pupils in the class] are good in math, and . . .
E: So we don't really know if they're math-brains.
J: So what if they're math-brains. I would still invite them to a party because they're no different from others in personality. Maybe they like math, and do a lot of math, but I really doubt if they'd be thinking of math at a party. I don't think so.
T: But math-brains aren't the only ones listening to Céline Dion, because *K* listens to Céline Dion too, and she's not a math-brain.
P: I agree with *A*, a math-brain is a person like everyone else. Me, I would invite them to my party. That wouldn't bother me. . . . They don't think about nothing except mathematics. Me, I'm good in mathematics, but when I'm at home, I don't only think about that.
Mediator: I see a drawing here of a math-brain [shows a drawing by one of the pupils]. Would you invite him to a party? And what difference is there between a math-brain and another person?
T: There are math-brains who think only of math. And there are math-brains who are good in math but who are like everybody else. But some of them are really, really math-brains. It's mostly the ones with glasses, you know.
Pupils: That [wearing glasses] doesn't make any difference!
M: Me, I don't think it's just math-brains who wear glasses. Look at *G* [a pupil in the class]. . . .
J: Glasses are just for appearance, because even in the movies when you see math-brains in class, they add them. . . . It makes them look intellectual, but really, it's just something they add for the intellectual look, that's all.
A: You know, that doesn't make any difference. Math-brains who look the most intellectual could also go to parties because when you go to a party, it means that the person who's having the party is your friend if they invited you.
J: They're your friend, so you invite them. So, if a math-brain in your class isn't your friend, you sure wouldn't invite them to your party. I mean, it's just a matter of friendship.

In this extract, *J* is not afraid to state a criticism that runs counter to the opinion of *K*. Her criticism is nevertheless weak, in that she bases it on two specific examples. In doing so, *J* takes for granted that there is an implicit relationship between her definition of a math-brain and the two pupils in the class she used for her example. *E* points out her lack of critical sense, and *J* accepts the remark. She starts over, basing her new criticism on justification of the distinction between people and their academic abilities. From a philosophical point of view, this distinction shows the fundamental difference between *being* and *doing.*

Then *T* supports *J*'s criticism, using an example that could be considered a concrete syllogism: K listens to Céline Dion, K is not a math-brain, therefore math-brains aren't the only people who listen to Céline Dion. *P* also reinforces *J*'s criticism by presenting his point of view and giving a personal example.

Next, the mediator asks the pupils to use their drawings to support their arguments. *T* states a preconceived idea: "Math-brains . . . it's mostly the ones with glasses," which provokes howls of protest from the class. *M* contradicts him using a concrete syllogism: G doesn't have glasses, G is a math-brain, therefore not all math-brains wear glasses. *J* goes even further in this sense with a criticism that highlights the distinction between "being" and "seeming." The truth, the real truth, is in being; seeming is only an "add-on," an illusion. Finally *A*, perhaps inspired by *J*'s interjection about being, reorients the problem and injects the idea of friendship into the discussion, an idea taken up and abstracted by *J*, who maintains that the central element in the question is "friendship," not whether someone is a math-brain.

The use of the *dialogical* mode of communication between peers is seen as pupils question each other, request nuances, base their statements on criteria or examples, criticize one another, and correct themselves. In other words, the exchanges were based on the construction of ideas arising from points of view enunciated by peers to solve a common problem. Such exchanges presuppose reciprocity and cooperation among pupils, and because the meta-mathematical dimension is involved, both the origins and the conclusions can without doubt be found in young people's everyday experiences.

Comparison of the Dynamics of the Exchanges

The final set of exchanges appear much more autonomous than the first, with pupils typically addressing their peers rather than the mediator (see Table 1). At the beginning of the school year, just over 87% of pupils' interventions were directed to the mediator and barely 13% to peers. At the end of the school year, the exchanges occurred mostly among peers, with slightly more than 71% of total pupil interventions made in light of a peer's comment or directed to peers and just under 29% directed to the mediator.

We also see indications of a relationship between these results and the mediator's roles. During the first P4CM session (mid-October), the mediator adopted the following roles: managing or controlling the activity, the work methods, and the means to attain goals; managing discipline; and questioning pupils. Managing the class and the activity constituted a major part of the first session. The

TABLE 1

Comparison of the Dynamics of the Exchanges

Transcript	*Total Number of Interventions*	*Number (and Percentage) of Interventions Directed to the Mediator*	*Number (and Percentage) of Interventions Directed to Another Pupil*
First	109	95 (87%)	14 (13%)
Final	133	38 (29%)	95 (71%)

"Socratic art" was demonstrated only in mediator's reformulation of pupils' statements, as all questions asked were the question-answer type.

By contrast, complete analysis of the final transcript shows that the mediator suggested the activity and the work methods, managed discipline and ensured the class atmosphere was respectful, questioned pupils, asked them to define the terms they used, took advantage of divergence of opinion among pupils to foster discussion, confronted them with cognitive conflicts, fostered objectivity of remarks and intersubjectivity, questioned the consequences of pupils' remarks, and questioned pupils' biases. In other words, in this last session, the mediator, in addition to managing activities and the classroom, created a community of enquiry within the classroom.

Comparison of Intervention Types

Comparison of the two transcripts is also informative about the thinking skills pupils displayed during the two sets of exchanges (see Table 2). At the beginning of the school year, answers (Type 1) represented 61% of all pupil interventions, interventions displaying lower-order thinking skills (Type 2) represented 30% of interventions, and those displaying higher-order thinking skills (Type 3) represented 9% of pupil interventions. Analysis of the second transcript reveals a decrease in simple answers (Type 1) from 61% to 26%, an increase in interventions displaying lower-order thinking skills (Type 2) from 30% to 39%, and, of special note, a jump in interventions displaying higher-order thinking skills (Type 3) from 9% to 35% at the end of the school year, after some seven months of work with the P4CM approach.

Chi-square analysis of the results rules out a hypothesis of homogeneity of observed values; we conclude that there exist significant differences between the sets ($\chi^2 = 34.8$, df = 2, $p < .001$).

Measuring variations in terms of rates of increase and decrease for each type of intervention, we found that answers (Type 1) decreased by 42%, interventions

TABLE 2

Comparison of Intervention Types: Number and Percentage of Interventions

Transcript	*Total Number of Interventions*	*Number (and Percentage) of Different Types of Interventions*		
		Type 1	*Type 2*	*Type 3*
First	109	66 (61%)	33 (30%)	10 (9%)
Final	133	35 (26%)	52 (39%)	46 (35%)

displaying lower-order thinking skills (Type 2) increased by 13%, and interventions displaying higher-order thinking skills (Type 3) increased by 39%.

Specific analysis reveals that in the first transcript, three lower-order thinking skills occurred more often than others: statement of opinion (10 times), explanation (9 times), and description (7 times). We also noted the occurrence of simple definition (2), observation (2), precision (2), and example (1). Among higher-order thinking skills, formulation of a hypothesis and doubt appeared twice and three times, respectively. Appearing once were comparison, categorization, justification, criticism, and counter-example.

The final transcript is characterized mainly by interventions displaying lower-order thinking skills: statement of an opinion (18), use of examples (13), simple definition (9), description (4), simple question (4), explanation (3), and precision (1). Interventions displaying higher-order thinking skills in the final transcript were use of nuance (9), criticism (6), contradiction (6), use of criteria (6), concrete syllogism (4), categorization (4), comparison (3), hypothesis (3), counter-example (2), doubt (1), search for meaning question (1), and justification (1).

DISCUSSION

Did pupils philosophize about mathematics in the second transcript to the point where we can speculate about young people aged 11 or 12 entering into this mode of communication? What are the cognitive and social components inherent in this type of communication? What conditions favour development of a community of philosophical enquiry about mathematical or meta-mathematical matters?

Our comparison provided some evidence about these questions. This study shows once again the complexity of philosophy, which remains open and rooted in a perspective on learning. Because we define philosophical dialogue in terms of questioning beliefs, deconstructing experience and prejudices, and searching collectively for meaning, we claim that in the second class pupils philosophized more than in the first. Basically, although the themes that pupils discussed were not philosophical, pupils nevertheless questioned and thought much more critically about their prejudices with respect to “math-brains.” In fact, the question discussed, while itself trivial, becomes more complex with the search for the origins and consequences of our prejudices and when such searching leads to pangs of conscience and the deconstruction of experience. On the other hand, in parallel with philosophical dialogue, analysis of the second transcript indicates that later in the year pupils used several complex thinking skills (nuances and criteria, criticism, self-correction, etc.), that their language was more articulate, that their interventions were more focused on the common goal, and that dialectical argumentation developed among them.

Can we thus conclude that these young people have become “philosophers”? Nothing concrete supports such a conclusion, as what is philosophical is a question of degree (philosophical compared to what or to whom?). We can nevertheless assert that most of the pupils began learning to philosophize, showed complex thinking skills, and experienced dialogical argumentation.

Did the pupils philosophize about mathematics? The philosophical, as we present it, concerns form rather than content, whereas philosophizing about what mathematics is brings us to the discipline of mathematics. In the two transcripts, we see more spontaneous movement among pupils towards philosophical exchange about everyday experience (prejudice against “math-brains”) than about a mathematical idea (for example, “Does a perfect cube exist?”). This class had difficulty centring their philosophical questions on mathematics; this type of question did not arise in either of the two recordings made. In other classes, however, pupils did ask questions such as “Does a perfect cube exist?” (Daniel & Pallascio, 1997). This type of discussion arises regularly, but it does require that the teacher guide the pupils further in this direction — which not all teachers can do.

This observation highlights the role of the mediator. Within a community of philosophical enquiry, the mediator must stimulate pupils’ curiosity about mathematical matters and develop their desire to engage in critical thinking and dialogical communication.

From an initial role essentially centred on management of both the class and the activity, the mediator successfully transformed a conventional pedagogical role into a “Socratic role” by basing activities primarily on questioning. The state of doubt constructed by this questioning provokes a cognitive conflict in the pupil, which is the starting point in reflection. And to the extent that this individual questioning is shared among a few or even many members of the group, another process is set into motion — communication. The pupils are then likely to enter into authentic dialogue.

When the mediator expects homogeneous, even ready-made, answers from pupils, arising from a textbook or from an idea in the mediator’s world, pupils lose part of their intrinsic motivation. However, when they feel that their creativity, judgment, and life experiences are not only respected but required, they gain interest and strive to surpass themselves (Dewey, 1916/1983). In this state, their thinking and their communication capacities are activated and actualized.

CONCLUSION

We do not consider the teaching of mathematics as an end in itself but as a means to the overall education of pupils. The introduction of PD in mathematics courses is aimed at stimulating pupils' curiosity and at developing their critical sense and their sense of social responsibility.

Adapting the approach of Lipman and Sharp to mathematics in a Grade 6 class, we studied two sets of exchanges among pupils, one at the beginning of a school year, the other at the end of the same school year. Pupils in this class evolved both cognitively and in how they exchanged ideas, from simple responses of one or a few words at the beginning of the year (responses directed mainly at the mediator) to a dynamic in which pupils' interventions at the end of the year occurred in a dialogical context and involved higher-order thinking skills.

To become dialectical and foster pupil evolution, the exchange requires the mediator to provide systematic and guided learning. Our analysis highlighted the fact that the quality of philosophical leadership by the mediator constitutes a prime condition for pupils' cognitive and dialogical evolution: such leadership is a fundamental requirement for this type of evolution.

This project did not include a comparative test to measure whether pupils' scholastic results improved after their community of enquiry in P4CM. Neither did it study the impact of this approach on the development of positive attitudes to mathematics. Both these questions are part of a separate research project currently underway.[3]

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NOTES

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2 A description of the overall training model is beyond the scope of this article; for further details about the model, see Daniel (1992), Lebuis (1987, 1990), and Schleifer, Lebuis, and Daniel (1990).

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REFERENCES

Baruk, S. (1992). *Dictionnaire de mathématiques élémentaires.* Paris: Seuil.

Bauersfeld, H. (1980). Hidden dimensions in the so called reality of a mathematics classroom. *Educational Studies in Mathematics, 11,* 23–41.

Borasi, R. (1992). *Learning mathematics through inquiry.* Portsmouth, NH: Heinemann.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29,* 573–604.

Daniel, M.-F. (1992). Reflections on teacher formation: When school and university enter together in a process of continuous thinking. *Analytic Teaching, 12*(2), 39–45.

Daniel, M.-F., Lafortune, L., Pallascio, R., & Sykes, P. (1994). A primary school curriculum to foster thinking about mathematics. *Analytic Teaching, 15*(1), 29–41.

Daniel, M.-F., Lafortune, L., Pallascio, R., & Sykes, P. (1996a). *Les aventures mathématiques de Mathilde et David.* Quebec: Le Loup de gouttière.

Daniel, M.-F., Lafortune, L., Pallascio, R., & Sykes, P. (1996b). *Philosopher sur les mathématiques et les sciences.* Quebec: Le Loup de gouttière.

Daniel, M.-F., Lafortune, L., Pallascio, R., & Sykes, P. (1996c). *Rencontre avec le monde des sciences.* Quebec: Le Loup de gouttière.

Daniel, M.-F., & Pallascio, R. (1997). Community of inquiry and community of philosophical inquiry. *Inquiry: Critical Thinking Across the Disciplines, 17*(1), 51–67.

Dewey, J. (1933). *How we think.* Lexington, MA: D.C. Heath.

Dewey, J. (1960). *The quest for certainty.* New York: Capricorn Books. (Original work published 1929)

Dewey, J. (1983). *Democracy and education.* New York: MacMillan. (Original work published 1916)

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12–22.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30,* 393–425.

Hoyles, C. (1985). What is the point of group discussion in mathematics? *Educational Studies in Mathematics, 16,* 205–214.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27,* 29–63.

Lebuis, P. (1987). Training teachers: An experimentation in primary schools in Quebec. *Analytic Teaching, 8*(1), 62–70.

Lebuis, P. (1990). La formation philosophique des enseignants et des enseignantes. In A. Caron (Ed.), *Philosophie et pensée chez l'enfant* (pp. 207–223). Montreal: Agence d'Arc.

Lipman, M. (1991). *Thinking in education.* Cambridge: Cambridge University Press.

Lipman, M. (1995). Good thinking. *Inquiry: Critical Thinking Across the Disciplines, 15*(2), 37–42.

Lipman, M., Sharp, A. M., & Oscanyan, F. (1980). *Philosophy in the classroom.* Philadelphia, PA: Temple University Press.

Prawat, R. S. (1991). The value of ideas: The immersion approach to the development of thinking. *Educational Researcher, 20*(2), 3–10.

Richards, J. (1991). Mathematical discussions. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 13–51). Dordrecht, Netherlands: Kluwer Academic Publishing.

Schleifer, M., Lebuis, P., & Daniel, M.-F. (1990). Training teachers for Philosophy for Children: Beyond coaching. *Analytic Teaching, 11*(1), 9–12.

Sfard, A., Nesher, P., Streefland, L., Cobb, P., & Mason, J. (1998). Learning mathematics through conversation: Is it as good as they say? *For the Learning of Mathematics, 18*(1), 41–51.

Siegel, M., & Borasi, R. (1996). Demystifying mathematics education through inquiry. In P. Ernest (Ed.), *Constructing mathematical knowledge* (pp. 201–215). Washington, DC: The Falmer Press.

Smith, G. (1995). Critical thinking, a philosophical community of inquiry and the science/maths teacher. *Analytic Teaching, 15*(2), 43–53.

Splitter, L., & Sharp, A. M. (1995). *Teaching for better thinking.* Melbourne: Australian Center for Educational Research.

Van der Maren, J.-M. (1995). *Méthodes de recherche pour l'éducation.* Montreal: Les Presses de l'Université de Montréal.

Vygotsky, L. (1984). *Thought and language* (E. Hanfmann & G. Vakar, Trans.). Cambridge: MIT Press.

Wood, T., & Sellers, P. (1996). Assesment of a problem-centered mathematics program: Third grade. *Journal for Research in Mathematics Education, 27,* 337–353.

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