Concept Maps Provide a Window onto Preservice Elementary Teachers' Knowledge in the Teaching and Learning of Mathematics

Tanya Chichekian
McGill University

Bruce M. Shore
McGill University

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Abstract

This collaborative concept-mapping exercise was conducted in a second-year mathematics methods course. Teachers' visual representations of their mathematical content and pedagogical knowledge provided insight into their understanding of how students learn mathematics. We collected 28 preservice student teachers' concept maps and analyzed them by counting the number of concepts and links within and across knowledge, teaching, and learning. We also examined the nature of the links between two domains of concepts. Preservice teachers revealed limited knowledge about how students learn

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mathematics. Concept maps successfully identified areas in which preservice teachers held an unsophisticated understanding of mathematics and its connection to pedagogy.

Précis

Cet exercice collaboratif de schématisation conceptuelle a été conçu pendant la deuxième année d'un programme de formation des maîtres dans un cours de didactique en mathématiques. Les représentations visuelles du contenu mathématique des cours des enseignants, ainsi que la connaissance pédagogique ont permis de comprendre comment les étudiants apprennent les mathématiques. Nous avons recueilli les schématisations conceptuelles de 28 huit étudiants et les avons analysées en comptant le nombre de concepts et les liens avec leurs connaissances en mathématiques et en pédagogie, ainsi que l'apprentissage des mathématiques. Nous avons également étudié la nature de ces liens entre deux domaines conceptuels. Les futurs enseignants avaient une compréhension limitée de l'apprentissage des mathématiques par les étudiants. La schématisation conceptuelle a pu repérer les régions au sein desquelles les futurs enseignants disposaient d'une mauvaise compréhension des mathématiques et de ses liens avec la pédagogie.

Achieving excellence in teaching elementary-school mathematics requires overcoming many barriers. Curricular standards documents (e.g., AAMT, 2006; NCTM, 1989, 2000) have addressed some of these challenges among preservice and in-service teachers by identifying the need for (a) enhanced conceptual understanding in mathematics and (b) improved ability to plan meaningful learning sequences for students. Both needs address fundamentals of teachers' subject matter and pedagogical knowledge and their interaction as pedagogical content knowledge (Shulman, 1986, 1988). Valid means of assessment are necessary to monitor the influence of these forms of knowledge, especially in areas concerning conceptual understanding and instructional planning (Miller et al., 2009).

One way of measuring these knowledge structures is to use concept maps (Briscoe & LaMaster, 1991). At the university level, this form of assessment highlights preservice teachers' knowledge representations of pedagogical content knowledge in a meaningful fashion (Austin & Shore, 1995). Preservice teachers' conceptualizations of teaching and learning mathematics have been studied extensively; however, the interplay between content knowledge and pedagogical knowledge (Shulman, 1987), although an important and growing element in educational research, has not yet been explored with concept maps. Assuming that knowledge in mathematics is represented in a hierarchical fashion, organized around central concepts, this study used concept maps in an attempt to capture the structural representation of elementary preservice teachers' knowledge in and understanding of teaching and learning mathematics.

Literature Review

A major branch of the research about concept maps is devoted to the pioneering work of Novak (2003, 2004, 2006) and his colleagues (Novak & Cañas, 2008). Although concept maps have been widely used as tools to assess student understanding (Edmondson, 2000), their most prevalent use has been as learning aids (Hill, 2004) or instructional tools (Stoddart, Abrams, Gasper, & Canaday, 2000). Concept maps are graphical tools for organizing and representing knowledge (Novak, 2010; Novak & Gowin, 1984), and therefore can also be used as research tools for assessing knowledge and its growth (Markham & Mintzes, 1994; Van Zele, Lenaerts, & Wieme, 2004; Wallace & Mintzes, 1990). In a meta-analytical review of research studies using concept maps, Nesbit and Adesope

(2006) estimated that more than 500 peer-reviewed articles, most published since 1997, have referred substantially to the educational application of concept-mapping procedures, as either an instructional technique or a study strategy. This cognitive tool displays concepts as well as their interrelationships or crosslinks, indicated by a connecting line, often as a proposition. The strength of the relation between a pair of concepts might be represented by using one, two, or three lines (or thin, medium, and thick ones). In some disciplines (e.g., physics and mathematics), concepts can be represented in a hierarchical fashion, with the most inclusive concept appearing at the top and the less general ones arranged hierarchically lower; in other disciplines (e.g., sociology or history), the concepts are less hierarchically organized or are structured in clusters (Donald, 1983). Ausubel (2000) argued that, due to the hierarchical nature of the cognitive structures of a learner, concept maps facilitate the assimilation and retention of new knowledge; moreover, the crosslinks (i.e., links between two concepts) and accompanying labels that describe the relation in words are indicative of the type of deep understanding involved during the construction of these maps.

Concept Maps as Assessment Tools in Teacher Education

According to Goldsmith, Johnson, and Acton (1991), "to be knowledgeable in some area is to understand the interrelationships among the concepts in that domain" (p. 88). Teachers, for example, must have a good conceptual understanding of their subject matter so that they can clearly explain to their students concepts—as well as the connections with other concepts in the discipline—and plan suitable instructional activities and sequences. For this reason, cognitive structures and teachers' knowledge have been major themes in teacher-education research (Wong & Lian, 1998), themes that have also been criticized for following the "Goldilocks principle." Goldilocks stumbles upon a house owned by a family of three bears—each of whom has his or her own chair, breakfast temperature, and bed—then tests the three bears' preferences and concludes that two options are always too extreme (too big or small, too hot or cold) but one is "just right." Katz and Rath (1985) proposed that research areas in teacher-education similarly fell into extreme categories, some appearing to be too specific for reasonable application, whereas others seemed too vague, general, or ambiguous to be translated into concrete terms. For example, the notion of teacher cognition is somewhat ambiguous because some concepts, such as effective teaching, are too vague to be defined simply in terms of a list of discrete tasks performed in a classroom. Finding the "just right" in understanding and applying important concepts is a complex undertaking that needs scaffolding. Therefore, cognitive tools such as concept maps have been useful in educational research when researchers are evaluating cognitive change in both students and teachers (Beyerbach, 1988; Kagan, 1990; Mason, 1992; Roehler, Duffy, Conley, Herrmann, Johnson, & Michelson, 1990; Wallace & Mintzes, 1990). These changes include teachers' structural knowledge (interrelationships between concepts) and students' conceptual understanding within a discipline.

The effectiveness of concept maps as powerful teaching and assessment tools (Afamasaga-Fuata'i, 2005; Borda, Burgess, Plog, & Luce, 2009) has been demonstrated in several studies. For example, Markham and Mintzes (1994) sampled 50 university students—25 biology majors and 25 nonmajors enrolled in a biology course—using concept mapping to provide a two-dimensional representation of the knowledge structures held by students in the domain of mammals. Biology majors represented three to four times as many valid concepts, propositions, and crosslinks as the nonmajors. The cognitive structure of a student's understanding was positively correlated with the sophistication of his or her understanding. Students with deeper understanding of content drew much more complex maps (many concepts were linked and each concept had several links; Austin & Shore, 1993) than students who had only a basic understanding of a particular subject. For concept maps to be conceived as potential assessment tools, Ruiz-Primo and Shavelson (1996) proposed a framework that included a combination of (a) a task that students must complete, (b) a specific format for the response, and (c) an accurate and consistent scoring system. Their categorization focused on the evidence used to claim conceptual knowledge in a particular domain. The present study focused on the domain of elementary mathematics education.

Concept Maps in Mathematics Education

In mathematics education, concept maps have mostly been used by teachers either as an instructional tool or as a means to identify and assess students' misconceptions (Bolte, 1999; Huerta, Galan, & Granell, 2003). Concept maps challenge teachers and students to make connections beyond discrete mathematical facts and operations. Given that many in-service and preservice teachers have had procedural experiences of learning mathematics, with little attention given to developing a conceptual understanding related to higher-order thinking, many K-to-12 mathematics teachers lack confidence in their understanding of mathematics (Hough, O'Rode, Terman, & Weissglass, 2007). Using concept maps to assess the organization of mathematics teachers' knowledge is based on the assumption that internal representations of knowledge are connected in some meaningful way (Hasemann & Mansfield, 1995). Moreover, the ability to understand and apply knowledge to a certain topic has been related to the number of concepts drawn on concept maps (Bransford, Brown, & Cocking, 2000; Liu, 2004; Markham & Mintzes, 1994; Novak & Gowin, 1984). For example, by using concept maps, Bolte (1999) depicted students' perceptions of mathematical connections in undergraduate mathematics courses, as well as their reflections on the deeper meaning of a mathematical concept. These mathematical connections highlighted inappropriate or missing linking words, as well as the omission of certain concepts and links. Concept mapping does not focus on extensive writing; rather, it focuses on exploration and discussion (Van Boxtel, Van der Linden, Roelofs, & Erkens, 2002). In science education, for example, student interactions during collaborative concept mapping have demonstrated the valuable contribution of meaningful discourse and the co-construction of key concepts (Roth & Roychoudhury, 1993, 1994; Stoyanova & Kommers, 2002; Van Boxtel et al., 2002). These cognitive tools are not only useful in identifying knowledge gaps on the part of students and teachers; they also have the potential to address learning outcomes of inquiry-related tasks, such as efficient and effective organization, the ability to consider diverse means of communication, using vocabulary appropriate to a topic, and creativity.

In the last decade, increasing attention has been given to preservice teachers' content knowledge and conceptions of teaching and learning, especially in mathematics education (Hofer, 2001; Muis, 2004). Assuming that the organization of concepts within maps is representative of conceptual understanding, the present study attempted to reflect elementary preservice teachers' knowledge of teaching and learning mathematics. We used concept mapping to seek answers to the following question: How do elementary preservice teachers' visual representations of their mathematical content and pedagogical knowledge provide insight into their understanding of the relationships between teaching and how students learn mathematics?

Methodology

Sample and Context

The sample included 56 third-year preservice teachers enrolled in a mathematics-methods course. Participants worked in dyads on a concept-map assignment. Therefore, if one member from the team did not give consent, their concept map was excluded from the data. Of 108 third-year elementary preservice teacher students, 56 students (*n* = 28 concept maps) agreed to participate in this study. This university hosts a four-year BEd elementary program, which includes academic and professional courses in pedagogy, methodology, and educational foundations, as well as a school-based practicum every year. In the first year, elementary preservice teachers take an Educational Psychology course (EDPE 300), in which they learn selected theories, models, and concepts relevant to planning and reflecting upon educational practice. Students experience a balance of both individual and group expectations—for example, social thinking and learning processes with high levels of dialogue. Instructors are encouraged to carry out a variety of in-class activities, including creating concept maps and discussing classroom cases. By the end of the course, students should be able to address educational questions and understand learners' and teachers' thinking and actions. The following is an excerpt from the course outline that demonstrates the manner in which preservice teachers were first exposed to concept maps in the previous Educational Psychology course.

> For this activity we will work in Staff Committees. At **the beginning** of each of these classes, every individual should come with *two copies* of a list of the 10 most important concepts that you think are presented in the assigned reading. For each concept, include a very short definition in your own words (do not directly copy any definition given in the book or anywhere else). In the Staff-Committee group you will compare notes, agree on which 10 to retain, then construct and draw a concept map. One "hard" copy of your Committee's map of 10 concepts must be submitted by the beginning of the following class by a representative of your group. Every Committee member's name must be on it, everyone will get the same grade, and it is your collective responsibility to ensure that everyone contributes a fair share of the work.

> The basic idea of a concept map is that it presents the main ideas of a reading or conversation on a sheet or screen, it shows and explains connections between the ideas, then it rates each connection as basic, strong, or extremely important (you can do this with single, double, or triple lines, or by varying the thickness or colors of the lines—be sure to have a legend or note that states what each means).
>
> In your map construction, you are encouraged to follow the same instructions. These instructions entail five steps: (a) place the main idea at the top or the centre of the map, (b) organize the words or terms from most general to most specific, (c) use a linking word (verb, preposition, or short phrase) to connect and illustrate the relationship and linkages from one idea to another, (d) use crosslinks to make connections between words in different areas of the map, and (e) when finished, take a few moments to reflect on the statements and modify them if needed.

For this educational psychology course, preservice teachers completed in total 10 concept maps (approximately one for each chapter). In the second year of the program, the students in this study registered for Elementary School Mathematics (EDEE 230). This course focuses on mathematical content teachers need to know to support students' development of mathematical understanding (the emphasis is on mathematics content, students' reasoning, and pedagogy). Finally, in the third year, preservice teachers enroll in Teaching Elementary Mathematics (EDEE 332), in which they are provided opportunities to develop increasingly sophisticated knowledge, as well as practices specific to elementary mathematics teaching. Our sample was drawn from EDEE 332.

Concept-Mapping Task

Each group was randomly assigned a mathematical concept by the instructor. The list of concepts included: Addition, Decimal Numbers, Division, Fractions, Frieze Patterns and Tessellations, Measurement, Multiplication, Natural Numbers, Plane Figures, Probability, Solids, Space, Statistics, and Subtraction. The first author (who was the teaching assistant in the course) introduced and trained students in a one-hour concept-mapping session six weeks before the final assignments were due. This was the students' first formal introduction to online concept-mapping software, but not their first introduction to concept mapping.

Students were guided through a list of three questions: (a) What is a concept map? (b) Why use a concept map? and (c) How do you develop a concept map? These questions were followed by a demonstration of *Cmap Tools* (Cañas et al., 2004). This concept-mapping software was chosen because it was free, online, and easily accessible from all the computers in the university. Optional supplementary readings were also at the students' disposal and included a list of articles addressing the theory underlying the use of concept maps (Novak & Cañas, 2008), as well as their benefits as both an instructional method and a learning aid.

Preservice teachers were instructed to expand on their knowledge about the assigned mathematical concept (e.g., fractions), compare and contrast instructional strategies as recommended by their required textbook for the course versus the mandated curricular guidelines, and identify the challenges and misconceptions associated with learning the assigned mathematical concept. To have a consistent framework for the organization of knowledge structures, all concept maps were designed to have an identical starting point. The mathematical concept was to appear at the top of the map and branch out to Knowledge, Teaching, and Learning (see Figures 1, 2, and 3). Students were specifically instructed to make links within and across Knowledge, Teaching, and Learning. Data were collected during class time in the last week of the semester.

Results

We counted the total numbers of concepts, links, and crosslinks represented in each map. The numbers of concepts and links were taken as indications of the extent of applied knowledge in the domain of elementary mathematics, whereas crosslinks reflected the extent of knowledge integration—essentially, student-teachers' ability to synthesize information. In addition, we examined how propositions in a concept map represented specificity of knowledge, by extracting linking words and assessing whether they expressed a static or dynamic relationship between concepts. Static relationships tend be more descriptive and serve to define or organize knowledge, whereas dynamic relationships are more explanatory and describe how one concept affects another. In general, both static and dynamic propositions are needed to capture the changing relationships in a concept map (Derbentseva, Safayeni, & Cañas, 2004).

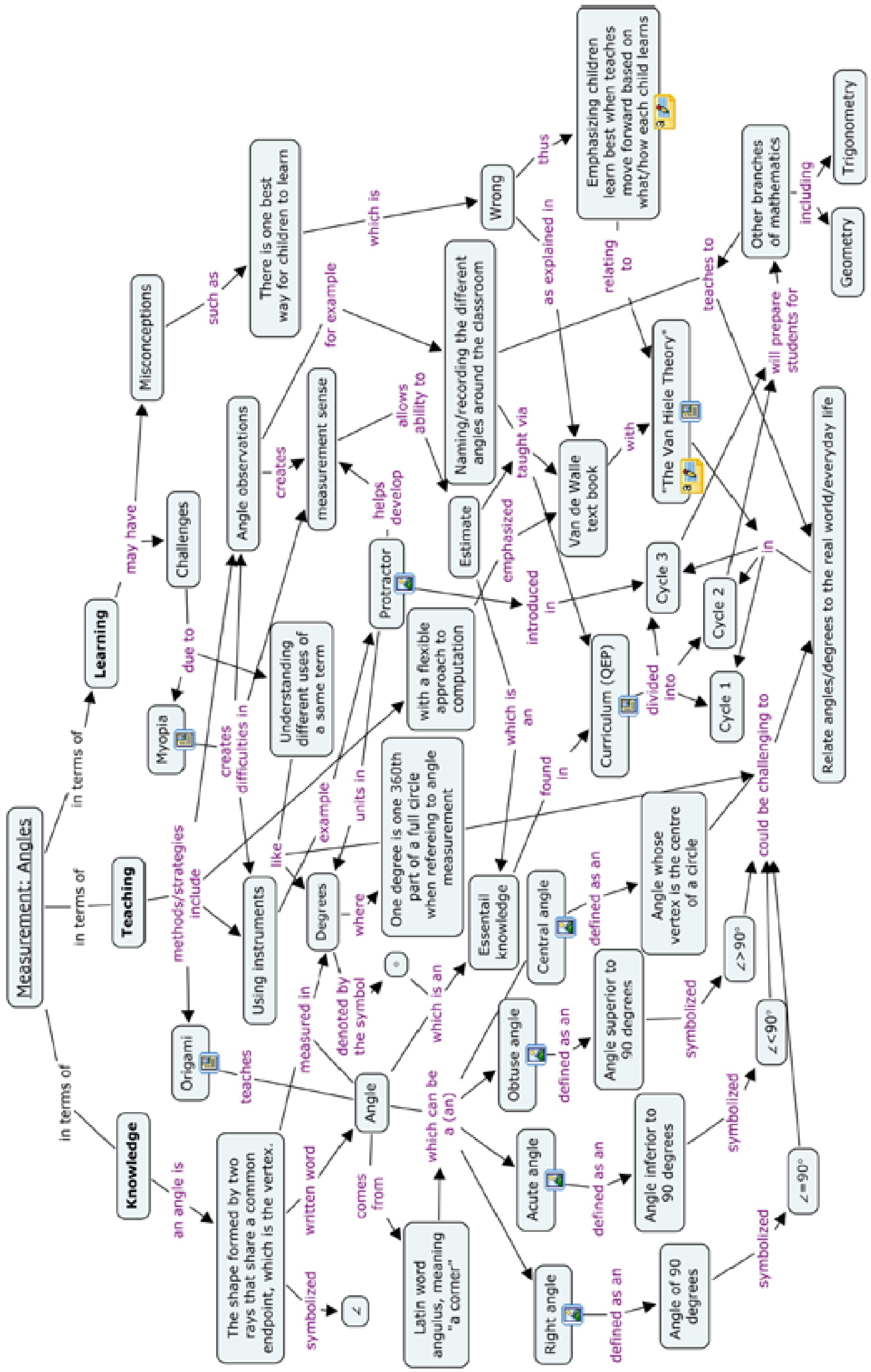

Figure 1: Example of Concept Map, "Angles" (CMA)

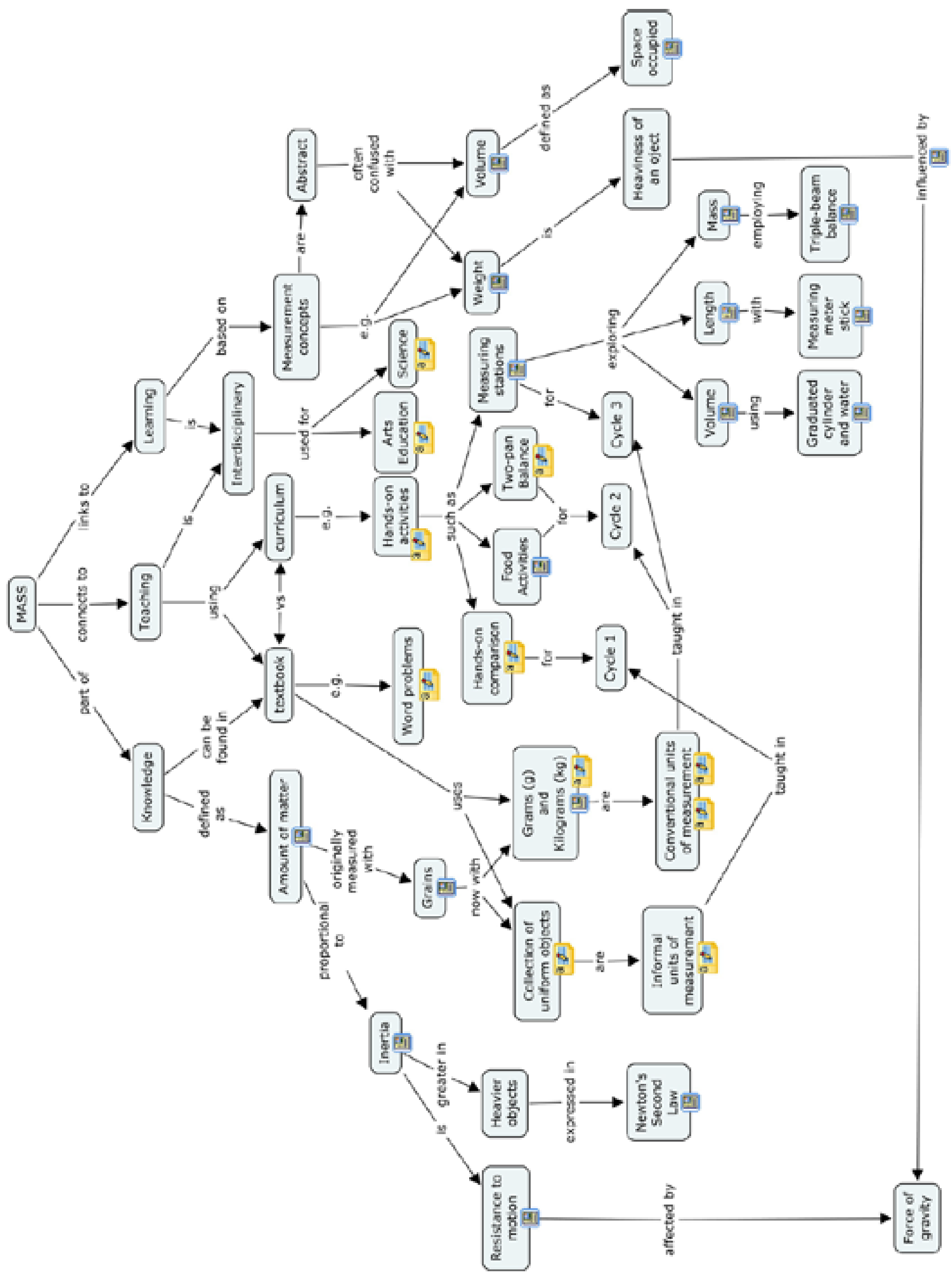

Figure 2: Example of Concept Map, "Mass" (CMM)

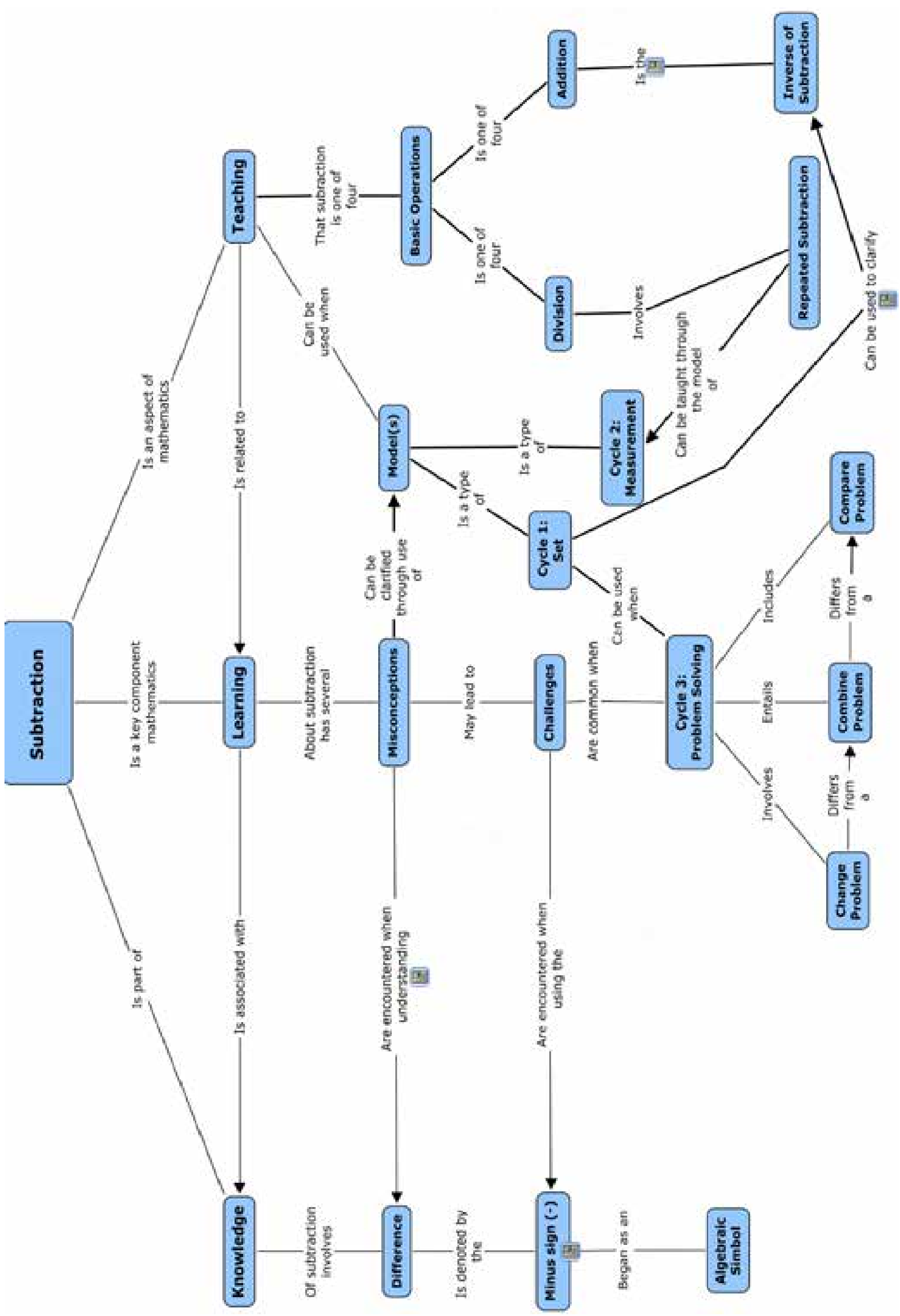

Figure 3: Example of Concept Map, "Subtraction" (CMS)

We used CL to denote crosslinks, K for Knowledge in mathematics, T for Teaching mathematics, and L for Learning mathematics. Crosslinks were identified when, for example, a concept under Teaching was branched out to make a link with another concept, under either Knowledge or Learning. The most frequent crosslinks were drawn from Teaching (TCL; $M = 5.79$, $SD = 3.61$), followed by Knowledge (KCL; $M = 4.14$, $SD = 2.48$), then Learning (LCL; $M = 3.82$, $SD = 2.35$). Correspondingly, the largest number of concepts appeared under Teaching (T; $M = 23.50$, $SD = 13.39$), followed by Knowledge (K; $M = 16.96$, $SD = 8.58$), then Learning (L; $M = 9.75$, $SD = 6.58$) (see Figure 4). A multiple regression analysis indicated that the number of concepts was not a significant predictor of the number of crosslinks. There was no significant correlation between the numbers of concepts and crosslinks ($r = 0.091$). The model fit was also not significant, confirming the existence of a nonlinear relation between the number of crosslinks and the number of concepts.

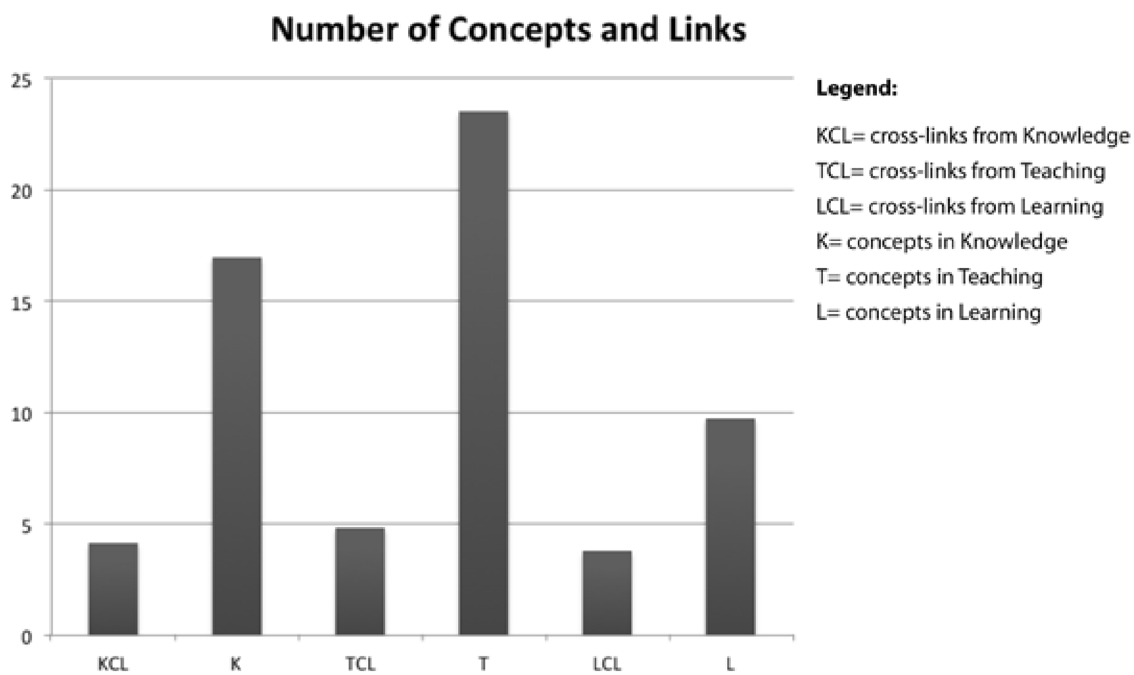

Figure 4: Number of concepts and links in Knowledge, Teaching, and Learning

Examples of concept maps (CM), as illustrated in this study, included one on "Angles" (CMA) with a total of 44 concepts (see Figure 1), another on "Mass" (CMM) with 37 concepts (see Figure 2), and one on "Subtraction" (CMS) with 17 concepts (see

Table 1: Examples of static, dynamic, and ill-fitting propositions

Figure	Static Proposition	Dynamic Proposition	Ill-fitting Proposition
3			"Using Instruments" example "Protractor"
3		"Angle Observations" creates "Measurement Sense"	
3	"Right Angle" defined as an "Angle of 90 Degrees"		
4			"Measurement Concepts," e.g., "Weight"
4	"Grams and Kilograms" are "Conventional Units of Measurement"		
5	"Addition" is one of four "Basic Operations"		
5		"Misconceptions" can be clarified through the use of "Models"	

Figure 3). CMA included the highest number of total (within and across) links (n = 50), followed by CMM with a total of 49 links, and CMS with 29 links. Of the total number of links, ten (20%) were crosslinks for CMA, eight (16%) for CMM, and six (21%) for CMS. In addition to counting the numbers of concepts and crosslinks represented on the maps, we examined the nature of the links between two concepts by extracting the linking labels forming the propositions. Ill-fitting propositions did not convey any meaning because they lacked a verb and, therefore, failed to describe the relationship between corresponding concepts. The concept map on "Angles" (CMA) included 24 static, 22 dynamic, and 22 ill-fitting propositions; the one on "Mass" (CMM) included 20 static, five dynamic, and 19 ill-fitting propositions, and the concept map on "Subtraction" (CMS) included 18 static propositions, 12 dynamic propositions, and no ill-fitting linking labels. Table 1 illustrates examples of static, dynamic, and ill-fitting propositions extracted from Figures 1, 2, and 3, with concepts in quotations and the linking labels in italic.

In addition to CMS, including the greatest number of crosslinks despite having the lowest number of concepts, this concept map included a somewhat balanced number of static and dynamic relationships and did not contain any ill-fitting propositions. CMM portrayed the smallest number of dynamic relationships between concepts and an almost equal number of static and ill-fitting propositions. CMA, on the other hand, included an almost equal number and the highest number of static, dynamic, and ill-fitting propositions.

Discussion

Basic terms and how they are related to each other constitute the fundamental building blocks of a discipline (Amundsen, Saroyan, & Donald, 2004); however, the basic terms are not enough because the creative potential lies in how ideas interact. Concept maps highlight the importance of the connections and interactions, and allow for creative exploration of these links. As students become more expert in a domain, their maps become more complex and fewer concepts are stranded without strong links to others (Austin & Shore, 1995). Simply counting the number of well-described links in concept maps can be used to measure progress in student or class learning (Austin & Shore, 1993).

Our results provided important but unanticipated information regarding cognitive differences that may exist among elementary preservice teachers with similar academic backgrounds, differences that extended beyond content knowledge to also include pedagogical content. Comparing concept maps drawn by elementary preservice teachers, as opposed to comparing the products of novices with those of experts (Koponen & Mäntylä, 2006), provided the insight that a more complex map (in terms of the numbers of concepts and links) does not necessarily imply expertise in subject-matter knowledge or pedagogy. Although it is well established that complex concept maps can well reflect the quality of subject-matter knowledge, our results suggest that they do not as clearly reflect pedagogical or pedagogical content knowledge (Shulman, 1987). The number of concepts was not necessarily an indicator of one's knowledge integration of teaching and learning mathematics. Even though the concept map about "Angles" (CMA) had the greatest number of concepts and links (see Figure 1), crosslinks represented only 20% of all links on this map. The dilemma in our findings is that the crosslinks within the concept map "Subtraction" (CMS), which included the least number of concepts (see Figure 3), also represented 21% of all links on the map (the remaining 79% of the links did not connect concepts across domains). Moreover, the concept map on "Mass" (CMM, see Figure 2), which contained a similar number of links as CMA but with a lower number of concepts, included crosslinks that represented 16% of all links on the map. Therefore, even though the literature claims that crosslinks reflect concept integration, in the present data, the number of crosslinks appeared unrelated to the total number of concepts.

By counting the number of concepts and the number of crosslinks on the concept maps, we further illustrated how the maps revealed more teaching knowledge (T) than either content knowledge (K) or knowledge about how pupils learned mathematics (L). This would be an indication of the preeminence of teacher-directedness in preservice teachers' mental representations of teaching mathematics. Without the use of this cognitive tool, it would have been very difficult to observe such a nuance. This occurrence may have also been due to the influence of the program's teaching philosophy, the conditions under which field experiences occurred (e.g., agreements of teaching methods between preservice teacher and supervisor), or past educational experiences (e.g., teacher-centered classrooms).

This study supports the contention that content knowledge alone does not provide the expertise sufficient to teach the discipline. It is not enough to know how to advance knowledge in a discipline; one must also be able to teach others how to do so (Austin & Shore, 1995). We observed the beginnings of this role when we realized that 46% (13 of 28) of the concept maps included crosslinks among Knowledge, Teaching, and Learning. This could be of particular interest to teacher-educators and educational researchers who are interested in using this cognitive tool as a means to assess pedagogical content knowledge, given that concept maps provide a visual representation of teachers' knowledge. One explanation for the wide range between the number of crosslinks from Teaching (TCL) and Learning (LCL) is that elementary preservice teachers' limited knowledge of the mathematical concepts may have played a mediating role between Teaching and Learning, thus influencing the overall structure of the concept map.

Our study raises the question of whether there might be specialized content knowledge that is unique to teachers' professional knowledge and whether it, in turn, is related to the creative thinking involved when establishing relations between concepts. A concept-map study by Borda et al. (2009) also provided insight into how the structural changes in participants' overall conceptions of science as a field of study influenced their content knowledge. Having observed an increase in the number of valid links over the number of concepts, Borda et al. (2009) inferred that the largest change in participants' epistemologies was to form new propositions using ideas that were already part of their epistemologies. One way of encouraging the formation of valid links is by changing the structure of the map (Derbentseva, Safayeni, & Cañas, 2007, 2008). By specifying an identical starting point, we modified the structure of the concept maps that might have been generated unprompted. For example, the concept map on "Subtraction" (CMS) modified the three domains (Knowledge, Teaching, and Learning) at the head of the map by drawing links among them and creating interdependence between Knowledge, Teaching, and Learning from the very beginning. This changed the view of the map to a more cyclical one compared to the ones on "Angles" (CMA) and "Mass" (CMM). The overall structural representation of the CMS concept map may also have been conducive to the absence of ill-fitting propositions, thus the formation of more valid links. On the other hand, CMA and CMM did not contain links among Knowledge, Teaching, and Learning from the beginning; the integration happened in the lower half of the hierarchy, after static relationships were established within each domain. Student teachers who established a valid link among the three domains in the top half of the hierarchy represented concepts in a more concise manner and included fewer ill-fitting propositions. Could this knowledge integration be an indication of teacher effectiveness? This is a question for future exploration.

Using concept maps created a unique visualization of preservice teachers' perceptions regarding the interconnectedness between Knowledge, Teaching, and Learning mathematics. This cognitive tool can provide teacher-educators with a new means of assessing the extent to which preservice teachers understand pedagogical content knowledge in a given discipline. One advantage of using concept maps in teacher education is their ability to identify areas of difficulties, challenges, and weaknesses encountered in preservice teachers' cognitive structures. For example, some maps appeared more elaborate than others due to the number of concepts and links illustrated; however, the nature of the linking labels did not always describe or explain the relationship between connected concepts. This study suggests the possibility of tracking changes in preservice teachers' knowledge of teaching and learning mathematics as they progress through the teacher-education program. By designing identical starting points in each concept map, we might be able to observe the progressive differentiation in preservice teachers' knowledge of teaching and learning mathematics.

Limitations and Future Research Directions

One limitation of this study was the timing of data collection. The assignments were due in the last class of the semester and, therefore, the number of absences was higher than usual. Although all 144 students submitted their assignments, 36 were absent the day of data collection and 52 did not give consent for us to use their work for this study. Another limitation was the use of an online concept-mapping tool. Although students were familiar with the notion of concept maps through their educational psychology course, due to time constraints, not enough class time was devoted to practise using this particular online tool. Also, concept maps were not used as an instructional tool in this course, which could have enhanced students' familiarity and skill with concept mapping, but might otherwise have hampered interpretability of the outcomes, depending on the content addressed. Limitations posed by class time and size also increased the difficulty in scaffolding the process. Only informal feedback was given to those students who had completed a draft of their assignment a week before the deadline. Perhaps building in some class time for peer feedback, and maybe even time to interview some students about their concept maps—asking them to explain the logic behind the links they had drawn—would have yielded more accurate interpretations.

The limited number of concepts associated with learning mathematics may be explained by the sequence of course offerings in the teacher-education program. Preservice teachers were introduced to an educational psychology course in the first semester of their four-year program, to the didactics of elementary school mathematics in the second year, and to teaching and learning in the elementary classroom in the third year. Ideally, the latter would have fit in the second year of the program, which was when most didactics courses were offered and field placements were occurring. Moreover, other than the educational psychology course, preservice teachers were not repeatedly exposed to related topics in psychology, such as cognition, intelligence, and memory, directly addressing issues applicable to learning in the classroom.

One important finding from this study was that the number of crosslinks among Knowledge, Teaching, and Learning were not related to the number of concepts. Ultimately, these cognitive tools could be useful in methods courses for probing preservice teachers about their mental representations of pedagogical content knowledge regarding particular subject matter (e.g., mathematics, science, social studies) or even a specific topic. For a more precise interpretation, interviews could encourage preservice teachers to discuss and elaborate on the nature of the links. Concept maps are also useful in assessing the degree and type of cognitive reorganization (Borda et al., 2009) and are, therefore, helpful in addressing questions related to the processes by which knowledge of mathematics changes. As far as teaching future educators, why elementary preservice teachers' knowledge representation of Learning was not as elaborate as that of Knowledge and Teaching remains of interest. It is quite possible that this elaboration could only be completed through the acquisition of authentic teaching experiences during professional practice. A comparative study between preservice and practicing teachers would be able to address this issue.

Concept maps also provide a unique form of assessing detailed structural information when compared to written responses in a questionnaire or an exam. Although this cognitive tool did not give us detailed information about the types of structural changes in preservice teachers' knowledge of teaching and learning mathematics, it did provide us with more insight about the nature of the links (by examining the linking labels) as perceived by preservice teachers. Techniques for analyzing knowledge representations and interactions between knowledge and learning have already permeated the field of human cognition, and a tool such as the concept map may be a contributing element to this field, especially when used to compare findings from network representations. Finally, it would be interesting to explore whether and how preservice teachers who create early connections between mathematical Knowledge, Teaching, and Learning and who represent the concepts more concisely and without ill-fitting or erroneous propositions are also more effective in actual instructional situations, as student teachers, and in their later professional practice.

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