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Long-Run Risk of Dynamic Asset Allocation Strategies

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This study uses empirical resampling to examine the risk of three of Perold and Sharpe's (1988) dynamic asset allocation strategies – buy and hold, constant mix, and constant proportion portfolio insurance (CPPI). Generally we find that the lower the floor percentage the greater the risk. However, which strategy has the most (or least) risk depends on how risk is measured. Finally, despite the positive floor buy and hold places on portfolio value, buy and hold is less risky than constant mix only in a few cases.

Keywords: portfolio management; risk measurement

Introduction

When viewed in a one-period context under a traditional mean-variance framework common stocks are on average much riskier than Treasury bills. However, proponents of the concept of time diversification [for example, Reichenstein (1986), Levy (1978)] argue that over long horizons common stocks as a whole may not be riskier than Treasury bills if certain shortfall risk measures are considered to be the relevant risk measures. Levy points out that over every 25-year period beginning with January 1926 or later the rate of return on the stock market as a whole has exceeded that of Treasury bills.

Butler and Domian (1991, 1992) point out that Levy's finding may be biased in favor of common stocks, since the 25-year periods overlapped, and therefore were not independent. The 25-year periods examined were January 1926 through December 1950, February 1926 through January 1951, and so on. To overcome this weakness, they use empirical resampling (also known as historical simulation or bootstrapping) to generate independent series of returns. Although their results vary somewhat depending on the sub-period used in their resampling, the benefits of time diversification are apparent, especially over long investment horizons.

The above studies focus on long-run returns of individual asset classes rather than portfolios composed of more than one asset class; thus, they do not examine the effect of diversification between asset classes. When one diversifies, one has rebalancing decisions to make—decisions that can have a considerable effect on the risk and return, especially over the long-run. Perold and Sharpe (1988) present three dynamic strategies which combine investment in stocks with investment in Treasury bills to reduce the risk of equity portfolios—buy-and-hold (BH), constant mix (CM), and constant proportion portfolio insurance (CPPI). Dichtl and Drobetz (2010) discuss the attractiveness of such strategies to both institutional and private investors and examine the performance of the CPPI strategy.

Perold and Sharpe discuss the payoffs and exposures of these strategies. For example, they point out that during a strictly rising or strictly falling market BH dominates CM while during an oscillating market CM dominates. However, they do not examine the empirical long-run performance or risk. While Dichtl and Drobetz examine the long-run performance of the CPPI strategy, they use a period of only 22 years; given the volatility of the U.S. equity market, one may consider this to be too short a period.

This study differs from the above-mentioned studies in that it (1) examines a far greater variety of risk measures than they do; and (2) focuses entirely on the risk of strategies rather than both risk and return. Although variance is a traditional measure of risk and is a key part of Markowitz portfolio theory and the capital asset pricing model, it is not clear that variance is always the most useful risk measure. Tversky (1990) points out that losses loom much larger than gains in people's minds. Also, the consequences of falling short of a given return target could determine the appropriate measure of risk. For example, if failure is defined as earning less than the Treasury bill rate of return, the probability of earning less than what one could have by investing in Treasury bills could be a useful measure of risk (Balzer (1994) and Sortino and Price (1994)). On the other hand, as Balzer (1994) points out, shortfall probability is an incomplete risk measure because it fails to consider the amount of the shortfall. If the consequences of falling short of the return that could have been earned from investing in Treasury bills is more severe the farther one's return is below that of Treasury bills, expected shortfall or lower partial variance measures which consider the frequency and magnitude of deviations below what could have been earned by investing in Treasury bills may be relevant. Finally, value at risk is a commonly-used risk management tool. If value at risk is considered to be the appropriate risk measure, specified lower percentiles of wealth relatives, such as the first or fifth percentiles, may be relevant. Still, if the conclusions as to which strategy is riskiest and the effect of floor percentage on risk are the same regardless of risk measure the choice of risk measure is unimportant.

The questions examined in this study are: (1) which strategy has the most (or the least) risk; (2) what impact the floor percentage has on the risk; (3) whether the answers to the first two questions are the same for all risk measures; and (4) what effect, if any, the choice of investment horizon has on the answers to the first three questions. Of these, the third question is the one of most interest.

This study uses empirical resampling (historical simulation) as used in Butler and Domian (1991, 1992) and Howe (1999) to examine the risk of the BH, CM, and CPPI strategies. The study uses monthly Morningstar/Ibbotson SBBI data going back to 1926 as the basis of the simulation. Thus, although this study uses fewer data points than Dichtl and Drobetz's 22 years of daily returns, the sample period is much longer.

This study focuses on an investment horizon of 10 years. However, to examine whether the conclusions regarding the risks of the strategies relative to each other and the relationship between risk and floor percentage depend on the investment horizon it also examines investment horizons of 20 and 30 years. This study uses CPPI floor percentages ranging from 10 percent to 95 percent. For comparison, this study also examines the long-run risk and ending wealth of the BH and CM strategies using equity positions of 10 through 95 percent. In addition to variance, this study uses a number of downside risk measures: value at risk, conditional value at risk, and lower partial moments. This study examines three types of lower partial moments: shortfall probability, expected shortfall, and lower partial variance. Lower partial moments require that the minimum acceptable wealth relative be specified. This study uses three different minimum acceptable wealth relatives: the mean wealth relative, the wealth relative one would have earned by investing entirely in Treasury bills, and a wealth relative of 1.0, which corresponds to a return of zero.

This study finds that generally the lower the floor percentage the greater the risk. However, which strategy is the riskiest (or the least risky) depends on how risk is measured. For risk measured based on deviations from the mean or deviations from the risk-free rate the CPPI strategy is the riskiest and the constant mix strategy is the least risky, with the buy and hold strategy being only slightly riskier than the constant mix strategy. For the other risk measures examined in this study the findings as to how the risks of the strategies compare with each other are very mixed. Finally, while the buy and hold strategy places a positive floor on value of the portfolio while the constant mix strategy does not, this does not make the buy and hold strategy less risky than the constant mix strategy unless one views risk as a particular low percentile of the ending wealth distribution, and even then only if the investment horizon is sufficiently short.

Survey of Relevant Literature

The relevant literature addresses long-run assets class returns, portfolio insurance, and risk measurement.

Long-Run Asset Class Return Studies

One type of study examining long-run returns is a Monte Carlo simulation study, in which the return distributions (along with the autocorrelations and cross-correlations) of the various asset classes are assumed. Examples of such studies are Leibowitz and Langetieg (1989) and Shoven and Sialm (1998). Monte Carlo simulation studies involve sampling from assumed distributions, often normal or lognormal distributions with specified means and variances. In addition to avoiding the problem of a small number of independent multi-year periods, simulation studies allow one to vary the expected returns and standard deviations (and other parameters if the assumed distribution is non-normal). This enables one to examine the sensitivity of the results to the expected returns, variances, and so on. A disadvantage of the Monte Carlo simulation study is its sensitivity to the choice of return-generating process. Given the finding that return distributions tend to have fatter tails than a normal or lognormal distribution, using a normal or lognormal distribution is likely to underestimate the frequency and magnitude of extremely low returns, thus making common stocks look less risky than they really are (Lucas and Klaassen, 1998).

A second type of study involves assuming return-generating distributions and analytically deriving the future wealth or return distributions. Examples include Reichenstein (1986), Ho, Milevsky, and Robinson (1994) and McCabe (1999). While there is less of a problem of sampling error than in the Monte Carlo simulations, the results of this type of study are still sensitive to the choice of return-generating process.

A third type of long-run investment performance study is the empirical resampling (also known as bootstrapping or historical simulation) study. Depending on the extent to which past stock and Treasury bill market performances repeat themselves, this is potentially more realistic than the Monte Carlo simulation study. Empirical resampling differs from Monte Carlo simulation by using the observed distribution of monthly returns rather than an assumed distribution. Butler and Domian's initial study (1991) uses empirical resampling to examine the performance of lump-sum investments in common stocks and Treasury bonds. Their second study (1992) examines Treasury bills, corporate bonds, and low-capitalization common stocks as well. In addition, their second study considers the case of making deposits into a retirement fund, where an equal amount (in real terms) is invested each month. Both of their studies find that in the vast majority of cases common stocks earn more than bonds over periods of 20 years or longer.

Portfolio Insurance Studies

The major motive behind portfolio insurance is to minimize the chance of the large losses possible from investing in common stocks. Perold and Sharpe (1988) present four portfolio insurance strategies: buy-and-hold, constant mix, constant proportion portfolio insurance (CPPI), and option-based portfolio insurance. Two decades later, Dichtl and Drobetz (2010) point out the continuing popularity of CPPI strategies, despite criticism from the academic community.

Risk Measurement

Variance and standard deviation have long been presented in textbooks as the most commonly-used measures of total risk. However, they are not necessarily the best measures. Balzer (1994) lists several undesirable events that could imply risk measures other than variance are relevant. This list, which he points out is non-exhaustive, is dominated by shortfall situations such as negative returns or returns less than the risk-free rate. As further support for shortfall-based risk measures, Tversky (1990) points out that losses loom much larger than gains in people's minds.

If investors are more concerned with downside variability than upside volatility, this would suggest that one should consider skewness when examining risk. Models which consider skewness include a three-moment CAPM (Kraus and Litzenberger 1976) and the mean-lower partial moment CAPM (Bawa and Lindenberg 1977).

Methodology

This section describes the historical simulation methodology used in this study as well as dynamic asset allocation strategies and expectations.

Historical Simulation

This study uses the empirical resampling methodology of Butler and Domian (1991, 1992) to simulate Treasury bill and common stock returns. The steps in the empirical resampling procedure are as follows:

1. Randomly select, with replacement, 600 months from the sample period, in this study January 1926 through December 2009.
2. Obtain the return for each of these months for each asset category
3. Use the returns from step 2 to calculate wealth relatives for 600 months.
4. Repeat the first three steps 9,999 more times to generate distributions of the wealth relatives.

Thus, the simulation uses 10,000 series of 600 simulated monthly returns. To better identify the effects of the choice of strategy, floor percentage, and risk measure on the results, the same 10,000 series are used for each strategy, floor percentage, and risk measure.

Although this procedure generates wealth relatives for periods as long as 50 years, the study will focus on the results for 10-, 20-, and 30-year periods. The common stock and Treasury bill returns used in this study are the monthly large-company common stock and 30-day Treasury bill returns from Morningstar/Ibbotson Associates.

Dynamic Asset Allocation Strategies

The constant mix strategy specifies an initial common stock/Treasury bill allocation and rebalances the portfolio each period that the common stock and Treasury bill returns were not equal. Thus, the portfolio return in any given month i, R_{ip}, is:

$$R_{ip} = x_{CS} R_{iCS} + (1 - x_{CS}) R_{iTB} \qquad (1)$$

where
x_{CS} = weight of the portfolio invested in the common stock index
R_{iCS} = return on the common stock index in period i
R_{iTB} = return on Treasury bills in period i

This study implements the constant mix strategy by creating a file of returns for each month from

January 1926 through December 2009 for each weight of common stocks used in this study. In addition to examining the performance of 100 percent stock and 100 percent Treasury bill portfolios, this study examines the performance of portfolios containing the following proportions of common stock: 0.05, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, and 0.90, which imply floor proportions of 0.95, 0.90, 0.80, 0.70, 0.60, 0.50, 0.40, 0.30, 0.20, and 0.10, respectively. Thus, this study creates ten such return files to sample from in the simulation.

The buy and hold strategy assumes a specified initial weight of common stocks but no rebalancing. Therefore, the wealth relative, that is, the ending wealth per dollar initially invested, after n periods, W_{np}, is:

$$W_{np} = x_{CS} W_{nCS} + (1 - x_{CS}) W_{nTB} \qquad (2)$$

where
x_{CS} = weight of the portfolio initially invested in the common stock index
W_{nCS} = wealth relative on the common stock index in period n
W_{nTB} = wealth relative on Treasury bills in period n.

This study uses the same initial weights for the buy and hold strategy as for the constant mix strategy.

For the CPPI strategy, the dollar amount allocated to common stocks as of period n, D_{nCS}, is calculated as

$$D_{nCS} = m[W_{np} - F_0 \prod_{i=1}^{n} (1 + R_{iTB})] \qquad (3)$$

where
m = CPPI multiplier
F_0 = initial floor value of the portfolio

The multiplier for a typical CPPI strategy is greater than 1. The greater the multiplier or the lower the floor the more aggressive the strategy. If the common stock index drops by a proportion greater than 1/m in one period the strategy fails in that the value of the portfolio drops below the floor. In such a case, this study assumes the entire portfolio is invested in Treasury bills. In this study leveraging is not allowed; therefore, in this study, D_{nCS} is the lesser of W_{np} or the result from equation (3).

The implementation of CPPI strategies in this study involves a simulation with the following steps for each period t:

1. As of the beginning of period t calculate W_{t-1p} and D_{tCS}.
2. Calculate the weights of common stocks and Treasury bills as D_{tCS}/W_{t-1p} and $1-D_{tCS}/W_{t-1p}$, respectively.
3. Applying these weights to the returns on common stocks (x_{tcs}) and Treasury bills ($1-x_{tCS}$), respectively, for period t, calculate the return on the portfolio, R_{tp}, as
   $R_{tp} = x_{tcs}R_{tCS} + (1-x_{tCS})R_{tTB}$
4. Calculate the wealth relative for the end of period t as W_{t-1p} $(1+R_{tp})$.

This study uses CPPI multipliers of 1.5 and 2.0 and floor proportions of 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, and 0.95.

This study applies various risk measures to the cross sectional distribution of 10-, 20-, and 30-year wealth relatives. Other than mean absolute deviation and value at risk, all of the risk measures this study examines are generalizations of:

$$S = \sum_{j=1}^{10000} L(W_B - W_j)^K / N \quad (4)$$

where

L = a dummy variable equal to 1 for all observations included in calculating the risk measure and equal to zero for all other observations. For observations below the benchmark L equals 1. For downside risk measures, L equals 0 for all observations above the benchmark while for risk measures which consider all observations L equals 1 for all observations above the benchmark.

W_B = benchmark wealth relative. For calculations of the variance and standard deviation W_B is the cross-sectional mean of the wealth relatives.In addition to the mean, this study uses benchmark wealth relatives equal to 1.0 (representing a geometric mean return of zero) and that earned by a 100 percent investment in T-bills.

K = a parameter equal to 2 for variance-based measures, 1 for expected shortfall measures, and 0 for shortfall probabilities

N = the number of degrees of freedom. If K equals 0 or 1, N equals the number of observations. If K equals 2, N equals the number of observations minus 1.

Normally x% value at risk is defined as the loss which is exceeded (100-x)% of the time, and a positive number for value at risk corresponds to a loss. In this study, however, the wealth relative that corresponds to this implies a positive return rather than a negative return in many cases. To avoid the possible confusion from negative value at risk numbers, this study reports the $(100\text{-}x)^{th}$ percentiles. As presented by Jorion (2011), the (100-x)% conditional value at risk equals the expected loss below the x^{th} percentile of ending wealth conditional on ending wealth being below the x^{th} percentile of ending wealth. Thus, (100- x)% conditional value at risk can be calculated from equation (4) where: L equals 1 if the wealth relative is less than the x^{th} percentile wealth relative and 0 otherwise; W_B is x^{th} percentile wealth relative; K equals 1; and N equals the number of observations less than the x^{th} percentile wealth relative.

Expectations

One would not expect the ending wealth relatives to be normally distributed. For one thing, the distribution of common stock returns has been found to be fatter-tailed than the normal distribution. Also, even if one-period returns were normally distributed, the ending wealth relatives would not be normally distributed, since the wealth relatives are products of weighted sums of lognormal random variables. Even if the ending wealth relatives could be assumed to be normally distributed, including only the observations below the threshold leaves a distribution that is non-normal; for these reasons, standard significance tests such as the F-test for equality of variances are not appropriate. In addition, the same 10,000 series of stock and Treasury bill returns are used for each strategy, floor percentage, and risk measure. Therefore, the results for the different strategies, floor percentages, and risk measures are not independent of each other; this would also invalidate the standard statistical tests.

Even if valid statistical tests were available, the method used to calculate the wealth relatives would make the tests largely meaningless. Every set of 10,000 calculated wealth relatives is based on the same 10,000 sets of common stock and Treasury bill returns; the only things that differ when the strategy or floor percentage is changed are the weights of common stock and Treasury bills. Thus, this study is best viewed as a demonstration rather than a traditional hypothesis-based empirical study. In addition, historical simulation, which this study employs, assumes that the distributions of Treasury bill and common stock returns over the 1926-2009 period are typical of the Treasury bill and common stock markets.

Even though the simulated results generated in this study do not lend themselves to statistical testing, we can develop *a priori* expectations of some of the results. Because on average common stocks earn a higher rate of return than Treasury bills, one would expect the weight of common stocks in a buy and hold portfolio to generally increase over time. Because common stocks have a higher variance of return than Treasury bills, this implies that the variance and mean absolute deviation of the buy and hold strategy are higher than those of the constant mix strategy for any given initial weight of common stocks in the portfolio greater than 0 but less than 1. However, because the constant mix strategy involves buying stock as the value of the portfolio drops, the value of the constant mix portfolio could theoretically drop to practically 0.

On the other hand, the initial investment in T-bills places a nonzero lower limit on the possible value of the portfolio in the buy and hold strategy. Therefore, it is conceivable that some of the lower percentiles of the ending wealth distribution could be higher using the buy and hold strategy than the constant mix strategy, given the same initial common stock / Treasury bill weights for the buy and hold strategy as for the constant mix strategy. Because the greater the percentage of stock the greater the variation in wealth relative, the greater the percentage invested in common stocks the greater the conditional value at risk. This implies that the conditional value at risk for the buy and hold strategy is higher than that of the constant mix strategy

The buy and hold strategy is a special case of the CPPI strategy, in which the multiplier equals 1 and the floor equals the initial investment in T-bills. Because for any given floor a higher CPPI multiplier implies a higher proportion invested in common stocks, one would expect the average variance of ending wealth for the CPPI strategy with a multiplier of 1.5 to be higher than those for the buy and hold strategy but lower than those for a CPPI strategy with a multiplier of 2.0.

Results

Table 1 presents the mean-based risk measures, in this study the mean absolute deviation, variance, and semi-variance of the wealth relatives, for each strategy for the 120-month investment horizon. Throughout this study the results for the 240- and 360-month investment horizons are presented only if they differ qualitatively from those for the 120-month investment horizon. Also, the results for the CPPI strategy with a multiplier of 2.0 are qualitatively identical but more pronounced than those for the CPPI strategy with a multiplier of 1.5 and are therefore not presented. The floor is the percentage invested in Treasury bills for the constant mix and buy and hold strategies and is the floor initially calculated by equation (3) for the CPPI strategies. Initial floors of 0 and 100 percent correspond to 100 percent allocations in common stocks and Treasury bills, respectively.

As expected, for the most part, as the floor decreases the mean absolute deviation and variance of the wealth relative increase. There appears to a minor exception to this for CPPI strategies with a floor percentage of approximately $1-1/\text{CPPI}$ multiplier. The reason for this is unclear.

As one would expect given the difference in the proportion invested in common stocks and the risk of common stocks relative to Treasury bills, the constant mix strategy consistently has the lowest mean absolute deviation and variance of wealth relative and the CPPI strategy with a multiplier greater than 1 has the highest mean absolute deviation and variance of wealth relative.

Although variance is a traditional measure of risk and is a key part of Markowitz portfolio theory and the capital asset pricing model, it is not clear that variance is always the most useful risk measure. Given evidence such as Tversky (1990) that losses loom much larger than gains in people's

Table 1.
Mean-Based Risk Measures — 120 Month Investment Horizon

Floor %	MAD	Variance	Semivariance
		Buy and Hold	
0	1.4603	4.1618	1.1241
10	1.3143	3.3708	0.9105
20	1.1682	2.6632	0.7195
30	1.0222	2.0389	0.5509
40	0.8763	1.4979	0.4049
50	0.7303	1.0403	0.2813
60	0.5844	0.6659	0.1803
70	0.4386	0.3748	0.1017
80	0.2931	0.1671	0.0456
90	0.1485	0.0427	0.0120
95	0.0785	0.0117	0.0036
100	0.0312	0.0015	0.0007
		Constant Mix	
0	1.4603	4.1618	1.1241
10	1.2219	2.7974	0.8131
20	1.0097	1.8415	0.5742
30	0.8212	1.1794	0.3933
40	0.6541	0.7277	0.2588
50	0.5067	0.4262	0.1612
60	0.3769	0.2311	0.0927
70	0.2632	0.1110	0.0471
80	0.1644	0.0428	0.0192
90	0.0806	0.0103	0.0049
95	0.0471	0.0035	0.0017
100	0.0312	0.0015	0.0007
		CPPI 1.5	
0	1.4603	4.1618	1.1241
10	1.4603	4.1618	1.1241
20	1.4605	4.1631	1.1249
30	1.4627	4.1732	1.1307
40	1.4699	4.2006	1.1404
50	1.4691	4.1804	1.1170
60	1.4032	3.8140	0.9791
70	1.2476	3.0740	0.7307
80	0.9789	2.0083	0.4187
90	0.5662	0.8079	0.1333
95	0.3077	0.2848	0.0393
100	0.0312	0.0015	0.0007

minds, downside risk measures may be more useful that variance or mean absolute deviation.

Table 1 presents one such measure–the semi-variance of the wealth relatives. Although semi-variance considers only variations below the mean while variance and mean absolute deviation consider all variations from the mean, whether below or above the mean, the semi-variances show exactly the same pattern as the mean absolute deviation and the variance. This suggests that even if downside deviations weigh considerably more on people's minds than upside deviations do, this does not qualitatively change the risk of the strategies relative to each other or the relationship between floor percentage and risk. The finding that the semi-variances are less than half of the corresponding variances, especially for the CPPI strategy, is consistent with positive skewness in the distributions of the wealth relatives.

Table 2 presents three shortfall risk measures for which the benchmark is the wealth relative of a 100 percent investment in Treasury bills. The measures are the shortfall probability, the expected shortfall, and the lower partial variance relative to Treasury bills.

For the buy and hold strategy it can be shown that if the initial investment in common stocks is greater than zero the probability of the wealth relative being less than that from a 100 percent investment in Treasury bills is independent of the initial investment in common stocks and, therefore, the floor percentage. When the magnitude of the shortfall in considered, as in the expected shortfall and the lower partial variance, the risk increases as the floor percentage decreases. For the constant mix strategy, the increase in risk as the floor percentage decreases is apparent for all three risk measures.

For the CPPI strategy, the relationship between floor percentage and risk measure varies somewhat depending on the risk measure. For floor percentages of greater than 90 percent the probability of a wealth relative less than that from Treasury bills increases as the floor percentage decreases. However, for floor percentages less than 90 percent the probability of a wealth relative less than that from Treasury bills declines as the floor percentage decreases. This reversal is less pronounced and occurs at a lower floor percentage for the expected shortfall than for the shortfall probability. While there is a slight reversal in the lower partial variance, it is much less pronounced and occurs at a lower floor percentage than the reversal in expected shortfall. This finding is not surprising, considering that the amount of the shortfall affects the lower partial variance proportionally more than it affects the expected shortfall, and the amount of the shortfall has no effect on the shortfall probability.

For all three risk measures, the constant mix strategy is the least risky and the CPPI strategy is the riskiest for all floor percentages between 10 and 95 percent. However, the buy and hold strategy is only slightly riskier than the constant mix strategy.

Table 3 presents the same three risk measures as Table 2 except that the benchmark return is zero. For floor percentages of 80 percent or higher all three risk measures were zero for all strategies; therefore, results for floor percentages greater than 80 percent are not presented. In general the relationships between floor percentage and risk measure are consistent with those in Table 2, although the decline in the shortfall probability for the CPPI strategy as the floor percentage decreases is much less pronounced and begins at a much lower floor percentage than it does in Table 2. Unlike the cases shown in Tables 1 and 2, the buy and hold strategy generally appears less risky than the constant mix. This is consistent with the hypothesis that the buy and hold strategy places a positive lower limit on the value of the portfolio while the constant mix strategy implies that one sells T-bills and buys stocks as the value of the portfolio drops and therefore has a theoretical lower limit of zero. However, as Table 4 shows, this effect disappears when the investment horizon is lengthened to 240 months.

Table 2.
Shortfall Risk Relative to Risk-Free Rate — 120 Month Investment Horizon

Floor %	Prob. < RF	Exp. Shortfall vs RF	LPV vs RF
		Buy and Hold	
0	0.1757	0.0635	0.0345
10	0.1757	0.0571	0.0280
20	0.1757	0.0508	0.0221
30	0.1757	0.0444	0.0169
40	0.1757	0.0381	0.0124
50	0.1757	0.0317	0.0086
60	0.1757	0.0254	0.0055
70	0.1757	0.0190	0.0031
80	0.1757	0.0127	0.0014
90	0.1757	0.0063	0.0003
95	0.1757	0.0032	0.0001
		Constant Mix	
0	0.1757	0.0635	0.0345
10	0.1681	0.0550	0.0276
20	0.1588	0.0471	0.0216
30	0.1513	0.0397	0.0164
40	0.1443	0.0328	0.0119
50	0.1362	0.0263	0.0082
60	0.1295	0.0203	0.0052
70	0.1226	0.0146	0.0029
80	0.1164	0.0094	0.0013
90	0.1100	0.0045	0.0003
95	0.1078	0.0022	0.0001
		CPPI 1.5	
0	0.1757	0.0635	0.0345
10	0.1757	0.0635	0.0346
20	0.1757	0.0637	0.0354
30	0.1773	0.0659	0.0349
40	0.1882	0.0709	0.0289
50	0.2171	0.0737	0.0206
60	0.2498	0.0674	0.0133
70	0.2658	0.0535	0.0075
80	0.2717	0.0365	0.0034
90	0.2731	0.0186	0.0009
95	0.2729	0.0096	0.0002

Table 3.
Shortfall Risk Relative to Return of Zero — 120 Month Investment Horizon

Floor %	Prob. (Loss)	Expected Loss	LPV vs 0
		Buy and Hold	
0	0.0625	0.0137	0.0046
10	0.0534	0.0099	0.0028
20	0.0444	0.0064	0.0015
30	0.0310	0.0036	0.0007
40	0.0196	0.0016	0.0002
50	0.0079	0.0004	0.0000
60	0.0008	0.0000	0.0000
70	0.0000	0.0000	0.0000
80	0.0000	0.0000	0.0000
		Constant Mix	
0	0.0625	0.0137	0.0046
10	0.0519	0.0101	0.0031
20	0.0415	0.0069	0.0019
30	0.0297	0.0044	0.0010
40	0.0205	0.0024	0.0005
50	0.0117	0.0011	0.0002
60	0.0045	0.0003	0.0000
70	0.0007	0.0000	0.0000
80	0.0000	0.0000	0.0000
		CPPI 1.5	
0	0.0625	0.0137	0.0046
10	0.0625	0.0137	0.0046
20	0.0629	0.0140	0.0048
30	0.0674	0.0150	0.0047
40	0.0788	0.0142	0.0034
50	0.0778	0.0090	0.0014
60	0.0438	0.0025	0.0002
70	0.0019	0.0000	0.0000
80	0.0000	0.0000	0.0000

Table 4.
Shortfall Risk Relative to Return of Zero — 240 Month Investment Horizon

Floor %	Prob (Loss)	Expected Loss	LPV vs 0
		Buy and Hold	
0	0.0155	0.0035	0.0013
10	0.0102	0.0019	0.0006
20	0.0055	0.0008	0.0002
30	0.0027	0.0002	0.0000
40	0.0002	0.0000	0.0000
50	0.0000	0.0000	0.0000
60	0.0000	0.0000	0.0000
70	0.0000	0.0000	0.0000
		Constant Mix	
0	0.0126	0.0028	0.0010
10	0.0101	0.0021	0.0008
20	0.0059	0.0013	0.0004
30	0.0036	0.0007	0.0002
40	0.0019	0.0003	0.0001
50	0.0008	0.0001	0.0000
60	0.0001	0.0000	0.0000
70	0.0000	0.0000	0.0000
		CPPI 1.5	
0	0.0155	0.0035	0.0013
10	0.0155	0.0035	0.0014
20	0.0175	0.0041	0.0014
30	0.0208	0.0037	0.0009
40	0.0164	0.0014	0.0002
50	0.0006	0.0000	0.0000
60	0.0000	0.0000	0.0000
70	0.0000	0.0000	0.0000

Table 5 presents the fifth and first percentiles of ending wealth and the conditional value at risk associated with them. As expected based on the proportion of common stock in the portfolio, the lower the floor percentage the lower the first and fifth percentiles of ending wealth for the buy and hold and constant mix strategies. For the CPPI strategy this relationship reverses slightly for low floor percentages, a finding consistent with the CPPI findings shown on Tables 1 through 3.

The fifth percentiles of ending wealth are only slightly lower for the buy and hold strategy than for the constant mix strategy while the first percentiles of ending wealth are in almost all cases higher for the buy and hold strategy than for the constant mix strategy. Furthermore, both the 95% and 99% conditional values at risk are lower for the buy and hold strategy than for the constant mix strategy, with the 95% conditional value at risk enough lower for the buy and hold strategy to more than make

Table 5.
Lower Percentiles and Conditional Value at Risk — 120 Month Investment Horizon

Floor %	5th Percentile	95% CVAR	1st Percentile	99% CVAR
		Buy and Hold		
0	0.9257	0.1918	0.6002	0.0989
10	0.9767	0.1827	0.6818	0.0879
20	1.0289	0.1733	0.7639	0.0773
30	1.0807	0.1560	0.8465	0.0674
40	1.1304	0.1385	0.9296	0.0582
50	1.1809	0.1189	1.0120	0.0490
60	1.2306	0.0815	1.0978	0.0439
70	1.2805	0.0637	1.1762	0.0329
80	1.3281	0.0456	1.2542	0.0247
90	1.3694	0.0277	1.3254	0.0195
95	1.3852	0.0221	1.3491	0.0143
100	1.3701	0.0015	1.3457	0.0012
		Constant Mix		
0	0.9257	0.1918	0.6002	0.0989
10	0.9842	0.1856	0.6676	0.0995
20	1.0439	0.1786	0.7390	0.0984
30	1.1015	0.1676	0.8137	0.0947
40	1.1589	0.1553	0.8918	0.0885
50	1.2120	0.1378	0.9752	0.0821
60	1.2629	0.1178	1.0603	0.0719
70	1.3106	0.0950	1.1497	0.0608
80	1.3515	0.0672	1.2383	0.0451
90	1.3841	0.0371	1.3218	0.0260
95	1.3919	0.0233	1.3538	0.0173
100	1.3701	0.0015	1.3457	0.0012
		CPPI 1.5		
0	0.9257	0.1918	0.6002	0.0989
10	0.9257	0.1918	0.6002	0.0990
20	0.9229	0.1932	0.5968	0.1037
30	0.8935	0.1765	0.6011	0.0585
40	0.8694	0.1142	0.6803	0.0370
50	0.9223	0.0804	0.7915	0.0284
60	1.0128	0.0622	0.9146	0.0247
70	1.1142	0.0487	1.0362	0.0201
80	1.2168	0.0370	1.1577	0.0185
90	1.3140	0.0255	1.2725	0.0164
95	1.3584	0.0221	1.3222	0.0143
100	1.3701	0.0015	1.3457	0.0012

up for the fifth percentile of ending wealth being slightly lower for the buy and hold strategy than for the constant mix strategy. Combined, this implies that the average ending wealth of the bottom 1% and 5% of the wealth relatives is higher for the buy and hold strategy than for the constant mix strategy for almost all floor percentages examined in this study. This is consistent with the hypothesis that the buy and hold strategy places a positive lower limit on the value of the portfolio while the constant mix strategy does not. However, as Table 6 shows, this does not show up in the 240-month results. This suggests that floor on the portfolio value provided by the buy and hold strategy provides less downside protection the longer the investment horizon.

Summary and Conclusion

This study has focused on the following questions: (1) of the buy and hold, constant mix, and CPPI strategies, which strategy has the most (or the least) risk; (2) what impact the floor percentage has on the risk; (3) whether the answers to the first two questions are the same for all risk measures. In addition, this study has examined what effect, if any, the choice of investment horizon has on the answers to the first three questions. The study makes no attempt to determine which strategy is optimal; this would depend on not only on how the investor perceives risk, but also on the investor's risk tolerance and return target.

This study generally finds the CPPI strategy to be the riskiest and the constant mix strategy to be least risky of the three strategies. Consistent with expectations, the strategies which involve the greatest investment in common stocks showed the greatest variance of ending wealth. Also generally consistent with expectations, as the floor percentage decreases the mean absolute deviation and variance of ending wealth increased in most cases, the exception being the constant proportion portfolio insurance strategy with a floor percentage below approximately 1-1/CPPI Multiplier.

Even though variance is the most traditional risk measure used in this study, the other risk measures based on deviations from the mean—semi-variance and mean absolute deviation—yield the same conclusions regarding the risk of the strategies relative to each other and regarding the relationship between risk and the floor percentage. In addition, the risk measures based on deviations below the risk-free rate yielded the same conclusions.

If this were the case for all risk measures, the question of what risk measure to use would be largely moot. However, it is not the case. For the other risk measures examined in this study no strategy was consistently the most—or the least—risky. This raises the question of which risk measure is the best to use. There is no clear answer to this question. Traditional finance theory suggests that variance is the best risk measure. However, evidence that investors prefer positive skewness (Kraus and Litzenberger 1976) and that deviations below the mean weigh two or more times as heavily on investors' minds than deviations of equal magnitude above the mean (Tversky 1990) suggests that downside risk measures are more appropriate than risk measures which consider upside and downside deviations equally.

Although shortfall probabilities and value at risk, which are measures of the mere probability of a sufficiently poor outcome, have been criticized for being too simplistic in that they ignore the severity of the shortfall, they have their uses. For example, using the probability of loss as a risk measure is consistent with an individual viewing risk as the possibility of losing money (Kaiser 1990). Also, value at risk is commonly used in financial institution risk management. Finally, investment policy statements may specify that the mere fact that a portfolio manager earns less than a specified rate of return over a specified period may be grounds for the portfolio manager being replaced (Trone, Allbright, and Taylor 1996).

Even if the magnitude of the shortfall is considered, there is no theoretical reason that

Table 6.
Lower Percentiles and Conditional Value at Risk — 240 Month Investment Horizon

Floor %	5th Percentile	95% CVAR	1st Percentile	99% CVAR
		Buy and Hold		
0	1.5768	0.4172	0.8782	0.1989
10	1.6244	0.3758	0.9924	0.1769
20	1.6717	0.3343	1.1163	0.1651
30	1.7190	0.2930	1.2284	0.1421
40	1.7708	0.2564	1.3489	0.1282
50	1.8153	0.2131	1.4617	0.1077
60	1.8599	0.1708	1.5752	0.0894
70	1.9077	0.1332	1.6863	0.0714
80	1.9502	0.0942	1.7940	0.0573
90	1.9863	0.0616	1.8851	0.0422
95	1.9899	0.0468	1.9101	0.0277
100	1.9232	0.0301	1.8746	0.0222
		Constant Mix		
0	1.5768	0.4172	0.8782	0.1989
10	1.6831	0.4137	0.9979	0.2118
20	1.7788	0.4005	1.1220	0.2203
30	1.8607	0.3757	1.2435	0.2182
40	1.9327	0.3450	1.3658	0.2098
50	1.9903	0.3055	1.4943	0.2010
60	2.0264	0.2520	1.6114	0.1759
70	2.0509	0.1964	1.7322	0.1512
80	2.0593	0.1372	1.8394	0.1145
90	2.0432	0.0757	1.9188	0.0634
95	2.0150	0.0484	1.9355	0.0358
100	1.9232	0.0301	1.8746	0.0222
		CPPI 1.5		
0	1.5768	0.4172	0.8782	0.1989
10	1.5768	0.4177	0.8749	0.1976
20	1.5713	0.4404	0.8226	0.1711
30	1.4872	0.4147	0.8244	0.0905
40	1.3280	0.2483	0.9395	0.0551
50	1.3364	0.1461	1.0996	0.0430
60	1.4484	0.1056	1.2768	0.0415
70	1.5901	0.0831	1.4541	0.0402
80	1.7396	0.0685	1.6272	0.0386
90	1.8790	0.0545	1.7880	0.0320
95	1.9366	0.0468	1.8586	0.0265
100	1.9232	0.0301	1.8746	0.0222

investors' utility is inversely proportional to the square, or any other given power, of the deviations below the target return. The square of these deviations may be easier to work with statistically, but this does not imply that investors really perceive risk that way.

Finally, although the buy and hold strategy guarantees a higher floor value than the constant mix strategy, for floor percentages of 90 percent or less this led to the buy and hold strategy having a higher first percentile of wealth relative over relatively short investment horizons such as ten years, but not over investment horizons of twenty or more years. Also, for floor percentages between 40 and 70 percent the buy and hold strategy had a lower shortfall probability relative to a benchmark return of zero that the constant mix strategy did. All other downside risk measures were higher for the buy and hold strategy than for the constant mix strategy. Thus, while the buy and hold strategy theoretically has a floor greater than zero while the constant mix strategy does not, the effect of this floor appears to be beneficial only over relatively short investment horizons in the most extreme cases of poor performance.

Because the study is a historical simulation, it assumes that the distributions of Treasury bill and common stock returns will continue to be similar to what they were over the sample period, 1926-2009. This assumption limits the extent to which one can generalize the results. An extension to this study would be to change the average return or volatility of Treasury bills or common stocks. Finally, this study assumes the first order serial correlation is zero for both Treasury bills and common stocks. However, Butler and Domian (1991) suggest that this assumption has little quantitative and no qualitative effect on the results.

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