

On the Deterministic Model for Planning Production Quantities in a Multi-Product, Multi-Plant, and Multi-Warehouse Environment

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In this paper, we restructure and simplify Jolayemi and Olorunniwo's model (J-O model) to make it more adoptable and applicable in industries. The New model obtained from the restructuring process has only 7 constraints while the J-O model has 11. In each of the examples, the number of iterations before reaching optimality was smaller for the new model than for the J-O model. The examples show that as both models increase in size, the J-O model becomes more and more difficult to solve than the new model. In most of the examples, the optimal solutions produced by the Excel 2007 solver are very close to those produced by LINDO.

In recent years, there have been soaring interests in the applications of supply chain (SC) management methods and techniques in industries. These have necessitated the need for the developments of more and new SC methods and techniques, particularly optimization models or techniques that integrate or solve some or many SC problems simultaneously for best decision outcomes. Unlike before, industries are now beginning to realize that modeling and solving SC problems separately lead to sub-optimal solutions and, consequently, "less-than-best" decisions outcomes.

Despite these, published works on the developments of models/techniques for integrating and solving SC problems still fall below needs. The few most integrated models in these publications integrate about three to five of these problems. Some of these models can be seen in Armtzen et al. (1995), Attaran & Attaran (2007), Jolayemi & Olorunniwo (2004), Routry et al. (2009), Sargut & Romeijn (2007), Tiwari et al. (2010), Tsiakis & Papageorgiou (2007), and You & Grossmann (2009).

Armtzen et al. (1995) developed a mixed integer linear programming (MILP) global SC model for determining: (1) the number and location of DCs, (2) customer - distribution centre assignments, (3) the number of echelons, and (4) product - plant assignments. Attaran & Attaran (2007) provide an overview of contemporary supply chain (SC) management systems through an in-depth overview and analysis of Collaborative Planning, Forecasting, and Replenishment (CPFR). CPFR helps supply chain partners to integrate demand and logistic planning, production scheduling, and new product design.

Jolayemi & Olorunniwo (2004) formulate a two-stage SC model that determines the optimal quantities of products to be produced at each plant, transported from each plant to each WH, subcontracted at each WH, and kept in inventory at each WH. The model also determines the optimal amount of extensions needed at each WH. It is one of the few most highly integrated models. However, due to the large numbers of its constraints and binary variables, its size increases rapidly as the numbers of products, plants, and WHs increase.

The optimization of a two-echelon SC in which production, inventory, transportation, backlogging, and subcontracting decisions are integrated is the focus of the article by Sargut & Romeijn (2007). Tsiakis & Papageorgiou (2007) developed a model that integrates production, facility location, and distribution alongside with other business issues like import duties, plant utilization and maintenance, and exchange rates.

A hybrid Taguchi-Immune approach is applied by Tiwari et al. (2010) to optimize an integrated supply chain design problem with multiple shipping options, distributed customer demands, and fixed lead times. Routry et al. (2009) and You & Grossmann (2009) develop multi-echelon inventory planning models with lead time and demand uncertainties. You and Grossmann's model differs from the model by Routry et al. by the addition of transportation component.

More examples of these types of integrated models can also be seen in Cokelz & Bum (1989), Cunha & Mutarelli (2007), Liang (2008), Rizk et al. (2008), Vidyathi et al. (2007), and Yung et al. (2006), to mention a few.

Due to their integrated nature, these models are usually very large and structurally complex. These can make them difficult to apply. Hence, besides the need for developing more integrated supply chain optimization models/techniques, there is also a great need for re-examining some of the existing ones - particularly the most highly integrated among them - for the possibility of modifying, simplifying, or restructuring them to make them more effective, more efficient, easier to solve and, consequently, more adoptable and applicable in industries.

As can be seen from our earlier discussions above, the model by Jolayemi & Olorunniwo (2004) is one of the few highly-integrated models. The model has many constraints and many decision variables - including many binary variables. This makes its size to increase rapidly as the numbers of products, plants, and warehouses increase.

Although, the model is applicable in industries in its present form, after a careful examination, we have observed that its adoptability and applicability will be much more enhanced if its size can be reduced much further. We believe that this can be achieved by carefully examining it and identifying where and how it can be restructured.

Therefore, the focus of this research is to restructure and simplify Jolayemi and Olorunniwo’s model (J-O model) to make it more effective, more efficient, more adoptable and more applicable. After a careful look at the model, we find that some special features and structures of the constraints can be exploited to reduce its size very substantially and to simplify it (the model) in order to make it easier to solve.

The restructuring of the model will not be focused only on its constraints. We will also redefine and restructure some terms of the model’s objective function. In the way the objective function is presently formulated, it is easy to commit errors in the expansions of some of the terms that have multiple summations. We will clearly illustrate the procedures involved in the restructuring process.

Numerical examples will be given to illustrate the restructured model and to compare it with the old model. The comparisons will be done with respect to model size, solution results, and computational efficiency. The LINDO and Excel 2007 solvers will be used for this purpose.

One of the major assumptions underlying the developments of the J-O model is that a producer will like to use his capacities to the maximum before subcontracting. While we believe that this is a good assumption that will suit the operations strategies of many producers, we also believe that there are many producers whose decision to produce or subcontract will be based solely on the cost associated with each. We will show how this model can be applied quite easily when the decision to produce or subcontract is mainly based on cost consideration.

The J-O Model and Its Restructuring Process

In order for the restructuring process to be clearly or well-illustrated, we first present the J-O model and its interpretations before embarking on the process. The J-O model, as developed by Jolayemi & Olorunniwo (2004), is as follows:

$$\begin{aligned} \text{Maximize } & \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N s_{ijt} (y_{ipjt} - v_{ipt}) + \sum_{t=1}^T \sum_{j=1}^J \sum_{i=1}^N (s_{ijt} - h_{ijt}) v_{ijt-1} + \sum_{t=1}^T \sum_{j=1}^J \sum_{i=1}^N (s_{ijt} - g_{ijt}) q_{ijt} \\ & - \sum_{t=1}^T \sum_{p=1}^P \sum_{i=1}^N (f_{ipt} z_{ipt} + c_{ipt} x_{ipt}) - \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N k_{ipjt} y_{ipjt} - \sum_{t=1}^T \sum_{j=1}^J e_{jt} w_{jit} \end{aligned}$$

subject to

$$\sum_{i=1}^N a_{ript} x_{ipt} \leq b_{ript}, \quad r = 1, 2, \dots, R; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \quad \dots (1.1)$$

$$x_{ipt} \leq u_{ipt}, \quad i = 1, 2, \dots, N; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \quad \dots (1.2)$$

$$\sum_{p=1}^P y_{ipjt} + v_{ijt-1} + q_{ijt} - v_{ijt} = d_{ijt}, \quad i = 1, 2, \dots, N; j = 1, 2, \dots, J; t = 1, 2, \dots, T. \quad \dots (1.3)$$

$$L_1 z_{ipt} - x_{ipt} \geq 0, \quad i = 1, 2, \dots, N; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \quad \dots (1.4)$$

$$\sum_{j=1}^J \sum_{i=1}^N v_{ijt} = 0, \quad t = T. \quad \dots (1.5)$$

$$\sum_{j=1}^J \sum_{i=1}^N v_{ijt-1} = 0, \quad t = 1. \quad \dots (1.6)$$

$$\left(\sum_{j=1}^J d_{ijt} - \sum_{p=1}^P u_{ipt} \right) \geq 0, \quad t = 1, 2, \dots, N; t = 1, 2, \dots, T. \quad \dots (1.7)$$

$$L_2 \quad it - \sum_{j=1}^J q_{ijt} \geq 0, i=1, 2, \dots, N; t=1, 2, \dots, T. \quad \dots (1.8)$$

$$\sum_{j=1}^J q_{ijt} \leq it \left(\sum_{j=1}^J d_{ijt} - \sum_{p=1}^P u_{ipt} \right), i=1, 2, \dots, N; t=1, 2, \dots, T. \quad \dots (1.9)$$

$$\sum_{j=1}^J y_{ipjt} - x_{ipt} = 0, i=1, 2, \dots, N; p=1, 2, \dots, P; t=1, 2, \dots, T \quad \dots (1.10)$$

$$\sum_{p=1}^P \sum_{i=1}^N i y_{ipjt} + \sum_{i=1}^N i v_{ijt-1} + \sum_{i=1}^N i q_{ijt} - \sum_{m=1}^t w_{jm} \leq w_{ojt} \quad j=1, 2, \dots, J, t=1, 2, \dots, T. \quad \dots (1.11)$$

$$q_{ijt}, v_{ijt}, w_{jt}, x_{ipt}, y_{ipjt} \geq 0; z_{ipt} = 0 \text{ or } 1 \text{ and } it = 0 \text{ or } 1 \quad \forall i, j, p, t$$

where:

- a_{ript} : amount of resource r required to produce a unit of product i in plant p in period t .
- b_{ipt} : the total amount of resource r available in plant p in period t .
- i : conversion factor in m^3 per ton of product i .
- c_{ipt} : the production cost per unit of product i in plant p in period t .
- d_{ijt} : demand for product i in warehouse j in period t .
- e_{jt} : the cost of construction/extension per m^3 of warehouse j in period t .
- f_{ipt} : the setup cost with respect to product i in plant p in period t .
- g_{ijt} : the unit cost of subcontracting product i in warehouse j in period t .
- h_{ijt} : the holding cost per unit of product i in warehouse j in period t .
- k_{ijpt} : the cost of transporting a unit of product i from plant p to warehouse j in period t .
- L_1 : a very large number.
- L_2 : a very large number.
- it : a binary variable which is 1 if the quantities of product i produced in all P plants in period t cannot meet customers' demands and zero otherwise.
- q_{ijt} : the quantity of product i subcontracted in warehouse j in period t .
- s_{ijt} : the selling price of product i in warehouse j in period t .
- u_{ipt} : the maximum possible capacity of plant p in period t with respect to product i .
- v_{ijt} : the amount of inventory of product i in warehouse j in period t . It is the portion of y_{ipjt} that is set aside to be kept in warehouse j in period t .
- w_{jt} : the amount of extension in m^3 that is needed in warehouse j in period t .
- w_{ojt} : the initial capacity of warehouse j in period t .
- x_{ipt} : the quantity of product i produced in plant p in period t .
- y_{ipjt} : the quantity of product i (in tons) shipped from plant p to warehouse j in period t .
- z_{ipt} : a binary variable which is 1 if product i is produced in plant p in period t and zero otherwise.

Model Interpretation

The first term of the objective function is the total revenue from the sales of all products shipped to the warehouses (WHs) (excluding the quantities kept in inventory at the beginning of each period) from all plants in all the T periods. The second term is the total net revenue from the sales of inventory kept in all WHs over the T periods after all the total inventory costs have been subtracted. The third term is the net revenue realized from all products subcontracted during the planning horizon after deducting the total cost of subcontracting. The fourth

is the total cost of production, including the setup costs, over the T periods. The fifth term is the total cost of transporting all products from all plants to all WHs over the T periods. The last term is the total cost of extension needed in all the J WHs in all the T periods. Thus, the objective function maximizes the total profit obtained after subtracting the production, transportation, inventory, WH constructions, and setup costs from total revenue.

Constraint (1.1) ensures that a manufacturer does not plan beyond the resources that are available at each plant in each period. Constraint (1.2) expresses the fact that no plant can produce beyond its maximum capacity. Constraint (1.3) enables a manufacturer to satisfy demands for each product. Constraint (1.4) ensures that if product i is produced in plant p in any period t, there will be a setup cost charged against him with respect to i. Constraint (1.5) ensures that no inventory remains in stock at the end of the planning horizon. Constraint (1.6) expresses the condition that inventory should not be carried from one planning horizon to the next. In many situations, a producer would like to operate his plants to maximum capacity before subcontracting. Constraints (1.7) and (1.8) make this possible. Constraint (1.9) ensures that the total quantity of item i subcontracted in all the J WHs in period t is not greater than the difference between the total demand for i from all the J WHs and the total capacity of all the P plants with respect to item i in period t. This ensures that subcontracting is done only when demand cannot be met after all plant capacities have been fully utilized.

Every product produced at every plant has to be shipped to the WHs. Constraint (1.10) makes this possible. A manufacturer would not like WH space to be a constraint to his operations. Constraint (1.11) takes care of this. The constraint ensures that extension is made at any WH at any period whenever necessary.

MODEL RESTRUCTURING PROCESS

The restructuring of the objective function

In the first term of the objective function, v_{ijt} is supposed to be the portion of y_{ipjt} kept in inventory in warehouse j in plant p. However, the exclusion of subscript p among the subscripts of v_{ijt} does not show that v_{ijt} is a portion of or that there is any relationship or connection between y_{ipjt} and v_{ijt} .

Additionally, since both y_{ipjt} and v_{ijt} are being summed over i, p, j, and t, the omission of p among the subscripts of v_{ijt} may lead to errors in the expansion of the multiple summation term, if great care is not taken.

To correct for these, we replace v_{ijt} with v_{ipjt} in the first and second terms of the objective function and sum the second term over i, p, j, and t, like the first term, to obtain:

$$\begin{aligned} & \text{Maximize } \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N s_{ijt}(y_{ipjt} - v_{ipjt}) + \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N (s_{ijt} - h_{ijt})v_{ipjt-1} + \sum_{t=1}^T \sum_{j=1}^J \sum_{i=1}^N (s_{ijt} - g_{ijt})q_{ijt} \\ & - \sum_{t=1}^T \sum_{p=1}^P \sum_{i=1}^N (f_{ipt}z_{ipt} + c_{ipt}x_{ipt}) - \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N k_{ipjt}y_{ipjt} - \sum_{t=1}^T \sum_{j=1}^J e_{jt}w_{jt} \end{aligned}$$

where v_{ipjt} is the amount of product i from plant p kept in inventory in warehouse j in period t and v_{ipjt} is a portion of y_{ipjt}

Restructuring of the model's constraints

In restructuring the model's constraints, we identify and eliminate variables and constraints that are redundant. We also reduce, merge, and/or reformulate some constraints. The constraint-restructuring process is as presented below.

Constraints (1.5) and (1.6) are nothing more than bound constraints. Therefore, they can be written as bound statements at the end of the model as follows:

$v_{ijt} = 0$ for $t = T$ and $v_{ijt-1} = 0$ for $T = 1$ and for all i, and j ($i = 1, 2, \dots, N; j = 1, 2, \dots, J$) (i). This reduces the number of the constraints of the model.

In constraint (1.7), given that $\sum_{j=1}^J d_{ijt} > \sum_{p=1}^P u_{ipt}$ and that the values of $\sum_{j=1}^J d_{ijt}$ and $\sum_{p=1}^P u_{ipt}$ are known, on substituting these known values into the constraint, it simplifies to: $i_t \geq 0$ for all i and t ($i = 1, 2, \dots, N; t = 1, 2, \dots,$

T), (ii). Similarly, if $\sum_{j=1}^J d_{ijt} < \sum_{p=1}^P u_{ipt}$, constraint (1.7) will simplify to: $x_{it} \leq 0$ for all i and t ($i = 1, 2, \dots, N; t = 1, 2, \dots, T$), (iii). By definition, x_{it} can only be 0 or 1. Therefore, $x_{it} \geq 0 \Rightarrow x_{it}$ is 0 or 1, and $x_{it} \leq 0 \Rightarrow x_{it} = 0$.

Therefore, if necessary, the constraint can be written as a bound constraint at the end of the model as: $x_{it} = 0$ or 1. for all i and all t ($i = 1, 2, \dots, N; t = 1, 2, \dots, T$), (iv). Otherwise, the constraint can be eliminated completely. Either way, the number of the model's constraints is reduced by NT. In constraint (1.8),

$$L_2 x_{it} - \sum_{j=1}^J q_{ijt} \geq 0 \Rightarrow L_2 x_{it} \geq \sum_{j=1}^J q_{ijt} \Rightarrow \sum_{j=1}^J q_{ijt} \leq L_2 x_{it} \dots (v). \text{ In constraint (1.9), it is given that:}$$

$$\sum_{j=1}^J q_{ijt} \leq x_{it} \left(\sum_{j=1}^J d_{ijt} - \sum_{p=1}^P u_{ipt} \right) \dots (vi). \text{ Since } L_2 \text{ is defined to be a very large number, } L_2 x_{it} \text{ in (v) must be, at}$$

least, equal to or greater than $x_{it} \left(\sum_{j=1}^J d_{ijt} - \sum_{p=1}^P u_{ipt} \right)$ in (vi) above. Hence, if constraint (vi) is satisfied, constraint (v) must be satisfied. Therefore, constraint (1.8) is redundant and must be dropped. This further reduces the number of the model's constraints by another NT.

Recall from the definition of x_{it} that $x_{it} = 0$ for $\sum_{j=1}^J d_{ijt} \leq \sum_{p=1}^P u_{ipt}$. This implies that, in constraint (1.9),

$$\sum_{j=1}^J q_{ijt} \leq 0 \text{ whenever } \sum_{j=1}^J d_{ijt} \leq \sum_{p=1}^P u_{ipt}. \text{ Since } q_{ijt} \text{ must be equal to or greater than zero for all } i, j, \text{ and } t, \sum_{j=1}^J q_{ijt} \leq$$

$0 \Rightarrow q_{ijt} = 0$ for all i, j , and t for which $\sum_{j=1}^J d_{ijt} \leq \sum_{p=1}^P u_{ipt}$. Thus, in any application of this model, all constraints in

(1.9) for which $\sum_{j=1}^J d_{ijt} \leq \sum_{p=1}^P u_{ipt}$ should be dropped from the model together with their associated variables q_{ijt} 's

This will reduce the size of the model much further.

As defined earlier in the first sub-section, $x_{it} = 1$ for $\sum_{j=1}^J d_{ijt} \geq \sum_{p=1}^P u_{ipt}$. Hence, 1 can be substituted for any x_{it}

for which $\sum_{j=1}^J d_{ijt} \geq \sum_{p=1}^P u_{ipt}$ in constraint (1.9) during the model's application

Therefore, with 1 substituted for x_{it} whenever $\sum_{j=1}^J d_{ijt} \geq \sum_{p=1}^P u_{ipt}$ and with all constraints for which $\sum_{j=1}^J d_{ijt} \leq$

$\sum_{p=1}^P u_{ipt}$ completely eliminated in constraint (1.9), all the binary variables x_{it} 's will be eliminated from constraint

(1.9) during the model's application. Furthermore, since constraints (1.7) and (1.8) have been deleted from the model, all the binary variables x_{it} 's will be completely eliminated from the model during its application. This will reduce the model's size drastically. Putting everything together, the restructured or the New model is given as:

$$\begin{aligned} \text{Maximize } & \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N s_{ijt} (y_{ipjt} - v_{ipjt}) + \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N (s_{ijt} - h_{ijt}) v_{ipjt-1} + \sum_{t=1}^T \sum_{j=1}^J \sum_{i=1}^N (s_{ijt} - g_{ijt}) q_{ijt} \\ & - \sum_{t=1}^T \sum_{p=1}^P \sum_{i=1}^N (f_{ipt} z_{ipt} + c_{ipt} x_{ipt}) - \sum_{t=1}^T \sum_{j=1}^J \sum_{p=1}^P \sum_{i=1}^N k_{ipjt} y_{ipjt} - \sum_{t=1}^T \sum_{j=1}^J e_{jt} w_{jt} \end{aligned}$$

subject to

$$\sum_{i=1}^N a_{ript} x_{ipt} \leq b_{ipt}, r = 1, 2, \dots, R; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \dots (2.1)$$

$$x_{ipt} \leq u_{ipt}, i = 1, 2, \dots, N; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \dots (2.2)$$

$$\sum_{p=1}^P y_{ipjt} + \sum_{p=1}^P v_{ijt-1} + q_{ijt} - \sum_{p=1}^P v_{ijt} = d_{ijt}, i = 1, 2, \dots, N; j = 1, 2, \dots, J; t = 1, 2, \dots, T. \dots (2.3)$$

$$\sum_{j=1}^J q_{ijt} \leq i_t \left(\sum_{j=1}^J d_{ijt} - \sum_{p=1}^P u_{ipt} \right), i = 1, 2, \dots, N; t = 1, 2, \dots, T. \dots (2.4)$$

$$L_1 z_{ipt} - x_{ipt} \geq 0, i = 1, 2, \dots, N; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \dots (2.5)$$

$$\sum_{j=1}^J y_{ipjt} - x_{ipt} = 0, i = 1, 2, \dots, N; p = 1, 2, \dots, P; t = 1, 2, \dots, T. \dots (2.6)$$

$$\sum_{p=1}^P \sum_{i=1}^N i y_{ipjt} + \sum_{p=1}^P \sum_{i=1}^N i v_{ijt-1} + \sum_{i=1}^N i q_{ijt} - \sum_{m=1}^t w_{jm} \leq w_{ojt}, j = 1, 2, \dots, J; t = 1, 2, \dots, T. \dots (2.7)$$

Where $i_t = 1$ for $\sum_{j=1}^J d_{ijt} > \sum_{p=1}^P u_{ipt}$ and 0 otherwise, $q_{ijt} = 0$ for $\sum_{j=1}^J d_{ijt} \leq \sum_{p=1}^P u_{ipt}$, $v_{ijt} = 0$ for $t = T$, $v_{ijt-1} = 0$ for $t = 1$, and $q_{ijt}, v_{ijt}, w_{jt}, x_{ipt}, y_{ipjt} \geq 0$ for all i, p, j , and $t; z_{ipt} = 0$ or 1.

As can be seen from the above results, the restructuring of the model has reduced the number of its main constraints from 11 to 7 through the elimination of four major constraints. This will amount to a very big reduction in any industry application in which many products, many plants, many WHs, and many periods are involved.

The restructuring has lead to the complete elimination of the binary variables from the model, leaving only the binary variables This will prevent the model’s size from increasing rapidly with increases in the numbers of products and plants. We believe that this new model will be much easier to solve than the old model, due to its smaller size. Data preparations and entry will also be much easier with it than with the old model. Furthermore, its solution will require less computer memory to process.

NUMERICAL EXAMPLES AND COMPARISONS

In the first subsection of this section, we give four numerical examples to illustrate the restructured model and to compare it with the J-O model. In the second subsection, we present the results of further examples given to illustrate the new model and compare it with the J-O model. The LINDO and Excel 2007 solutions to the examples are also compared in the subsection (i.e. the second subsection).

Numerical Examples, Model Illustrations, and Comparisons

The first example (see example 1) in table 1 is on a multi-period and multi-product production-distribution and warehouse capacity problem involving 2 products, 2 plants, 2 warehouses, and 2 periods.

In the numerical example, the total demands for products 1 and 2 in period 2 are each less than plant capacity for each product. This made two constraints to be dropped from each of the model’s subcontract constraints (see constraints (1.8) and (1.9) of the J-O model and constraint (2.4) of the New model in the previous section) in the example. This reduced the number of constraints of the new model in the example from 56 to 54 and of the J-O model from 60 to 56. It also reduced the number of the J-O model’s binary variables from 12 to 10. The application of the results in the last subsection of the previous section relating to constraint (2.4) reduced the number of the new model’s binary variables from 10 to 8.

As shown in the table, the numbers of the New and the J-O models’ continuous variables in the example are 40 and 36 respectively. This is after dropping all subcontract variables for which total demand is less than total plant

capacities for the two products. The number of the new model's continuous variables is greater than that of the J-O model because (as can be seen in the first and last subsections of the previous section) the inventory component of the former has more variables than those of the latter.

The LINDO solver was used to obtain optimal solution to each problem. The solver was run on a HP personal computer with a Pentium (R) 1.993MH₃ processor under Windows XP. The solutions to the two models showed good results.

The solutions to the New and J-O models were obtained after 111 and 141 iterations respectively. This amounts to a difference of 30 iterations. This shows that, with smaller number of binary variables, the new model is easier to solve.

The optimal value of each of the model's objective function is \$3,536,800.00. This shows that the imperfections in the structures of the J-O model do not affect its ability to produce good results. They only make it more difficult to solve.

The numbers of products, plants, warehouses, and periods involved in example 2 is the same with those in example 1 (see table 1). However, unlike in example 1, none of the subcontract variables and constraints of the J-O model is dropped in example 2. This is because total demand for each product in each period is greater than plant capacity in the example. As a result, the number of its constraints is 60 and the numbers of its continuous and binary variables are 40 and 12 respectively.

Also, since total demand for each product in each period is greater than plant capacity, none of the New model's subcontract constraints and variables is dropped. Hence, the number of the model's constraints is 56 while the number of its continuous variables is now 44 (see table 2). However, the application of the results in the last subsection of the previous section relating to constraint (2.4) makes the number of the new model's binary variables to be 8 – just as in example 1.

The outputs of the LINDO solutions to the two models in the example show that the number of iterations before obtaining optimal solutions to the new and J-O models are 120 and 144 respectively, resulting in a difference of 24 iterations. This result corroborates our conclusion in example 1 that the new model is easier to solve. Again, the two models produce the same optimal value of the objective function.

In example 3, the number of products is 3 and the total demand for each product in period 2 is less than plant capacity for each product. Like in example 1, this made three constraints to be dropped from each of the model's subcontract constraints (i.e. from constraints (1.8) and (1.9) of the J-O model and (2.4) of the New model). This reduced the number of constraints of the J-O model from 80 to 74 and that of the new model from 74 to 71 respectively. The number of J-O model's binary variables is also reduced from 18 to 14. The applications of the results in the last subsection of the previous section relating to constraint (2.4) reduced the number of the new model's binary variables from 15 to 12.

Table 1 shows that the LINDO solutions to the two models were obtained after 185 and 215 iterations respectively, which is a difference of 30 iterations. This supports the results in the first two examples which show that the new model is easier to solve.

Unlike in example 3, none of the two plants has enough capacity to satisfy demand for each product in each period in example 4. Therefore, due to the reasons explained for the similar case of example 2, the numbers of the new and J-O models' constraints are 74 and 80 respectively and the numbers of their binary variables are respectively 12 and 18. The number of each model's continuous variables is now 65 and 60 respectively.

Table 1: Results of the Numerical Examples on the New and the J-O Models.

Model details and values of solution parameters	Examples							
	1		2		3		4	
	New	J-O	New	J-O	New	J-O	New	J-O
Number of products	2	2	2	2	3	3	3	3
Number of plants	2	2	2	2	2	2	2	2
Number of warehouses	2	2	2	2	2	2	2	2
Number of periods	2	2	2	2	2	2	2	2
Number of constraints	54	56	56	60	71	74	74	80
Number of continuous variables	40	36	44	40	57	52	65	60
Number of binary variables	8	10	8	12	12	14	12	18
Number of iterations	111	141	120	144	185	215	179	285
Objective function value	35360800	35360800	35661800	35661800	25020320	25020320	30580720	30580720

As can be seen in table 1 above, the LINDO solver produced optimal solutions to the two models (the new and the J-O models) after 179 and 285 iterations respectively. This amounts to a difference of 149 iterations, which is a very

big difference. This remarkable result could be due to the fact that the difference between the new and J-O models' binary variables is larger in this example than in example 2.

We find the LINDO solver to be very good for solving the two models. After entering the input data for each model, the solver produced optimal solution instantaneously at a click on the Solve command. The solver's reports/statistics on each solution showed the elapse time to be zero. This shows that the CPU time for each solution is virtually zero.

Further Examples and Comparisons

In this subsection, we use the Excel solver to solve each of the four numerical examples in the previous subsection (the previous subsection of this section). The reasons for these are to enable us: (1) determine the solution time for each of the numerical examples, (2) compare the Excel solutions and solution times for the new model with those of the J-O model in each numerical example, and (3) compare the LINDO and Excel 2007 solutions.

Additionally, we use a much bigger numerical example than any of the four numerical examples in the previous subsection to compare the two models. The purpose of this is to give the illustrations and comparisons of the two models some measure of practical reality. Table 2 shows the results of the five numerical examples for each of the two models.

In example 1 (see table 2) the optimal objective-function values produced by the excel solver for the two models are the same. These optimal objective function values are also the same with the objective-function values produced for the two models by the LINDO solver. This shows that the Excel 2007 and the LINDO solver are equally good for solving LP problems of this size. The Excel solver produced optimal solution to each model in 2 seconds under the example. This means that the two models are equally easy to solve under this example.

Like in example 1, the optimal objective-function values produced by the Excel solver for the two models in example 2 are the same. However, each of the values deviates from the corresponding value produced by the LINDO solver by 0.8% (see table 2). Since this is a very small deviation, it can be inferred that the Excel solver is still as good as the LINDO solver in solving LP problems of these sizes. As can be seen in the table, the sizes of the LP problems for the New and the J-O models in this example are slightly larger than their respective sizes in example 1. The numbers of their binary variables are also slightly larger.

The Excel-solution times for the New and the J-O models are one and two seconds respectively. This shows that the new model is relatively much easier to solve with Excel than the J-O model.

Again, the Excel solver produced the same objective-function values for the two models in example 3. Each of the objective-function values deviates from the corresponding value produced by LINDO by 0.00012%, which is practically zero. This shows that the Excel solver is still as good as LINDO in solving the problems, even though they are larger than the problems in the first two examples.

The Excel solution times for the new and J-O models are 3 and 4 seconds respectively. This again shows that the new model is easier to solve than the J-O model.

The results in example 4 are similar to those in the three earlier examples. The Excel solutions to the new and the J-O models are the same. The solutions deviate from the LINDO solutions by only 0.0001%. The Excel solution times to the new and the J-O models are 4 and 7 seconds respectively, showing that the new model is relatively much easier to solve than the J-O model.

The examples (examples 1 to 4) show that as each model's size and number of binary variables increase, it becomes more and more difficult to solve; and that the J-O model becomes much more difficult to solve than the new model.

To inject some practical realities into the illustrations and comparisons of the two models and of the Excel and LINDO solutions, we increase the sizes of the problems in example 5. The LP problem for the new model has 150 constraints and 144 variables, 27 of which are binary variables. The one for the J-O model has 159 constraints and 153 variables, 36 of which are binary.

As can be seen in the table (table 2), the Excel solver produced the same solution to the two models. The optimal objective-function value for each of the two models deviates from the corresponding value produced by LINDO by 1.82%. This is a small deviation for LP problems of these sizes. This shows that Excel 2007 LP solver is almost as good as the LINDO solver in solving big new and J-O models of these sizes.

The Excel-solution times for the new and the J-O models in this example are 26 and 42 seconds respectively. This strongly validates our observation in example 4 that as each model's size and number of binary variables increase, it becomes more and more difficult to solve; and that the J-O model becomes much more difficult to solve than the new model. This means that the adoptions and applications of the new model in industries will result in much bigger time and cost savings than the applications of the J-O model. This can make the new model to be more appealing, more applicable, and more adoptable than the J-O model in industries.

Table 2: The results of the excel and LINDO solutions to the New and J-O models

Examples	Model	No. of binary variables	No. of continuous variables	Total No. of variables	No. of constraints	Objective function-value (Excel)	Objective function-value (LINDO)	Deviation from the LINDO value	Excel-solution time
Example 1	New Model	8	40	48	54	35360800	35360800	0.0%	2 seconds
	J-O Model	10	36	46	56	35360800	35360800	0.0%	2 seconds
Example 2	New Model	8	44	52	56	35360800	35661800	0.8%	1 second
	J-O Model	12	40	52	60	35360800	35661800	0.8%	2 seconds
Example 3	New Model	12	57	69	70	25020288	25020320	0.00012%	3 seconds
	J-O Model	14	52	66	72	25020288	25020320	0.00012%	4 seconds
Example 4	New Model	12	65	77	74	30580688	30580720	0.00010%	4 seconds
	J-O Model	18	60	78	80	30580688	30580720	0.00010%	7 seconds
Example 5	New Model	27	117	144	150	74676055	76060300	1.82%	26 seconds
	J-O Model	36	117	153	159	74676055	76060300	1.82%	42 seconds

Application in Produce-or-Subcontract Decisions

One of the major assumptions underlining the development of our model is that a producer will like to use his plants to maximum capacities before subcontracting. However, there are situations in which produce-or-subcontract decision are based mainly on cost considerations. This model can be very easily modified to make it applicable under such situations.

Dropping constraint (2.4) from the New model allows the producer to subcontract if it is cheaper than to produce and vice versa, irrespective of whether he has used his plant to maximum capacity or not. The fact that the model can be easily applied under this type of situations further enhances its adoptability and applicability in industries.

We would like to recommend here that in applying the model to make produce-or-subcontract decision, the cost that may be incurred by the producer due to unused capacities must be factored in.

Concluding Remarks

We have examined and restructured the J-O model to correct some imperfections in its formulation. The new model obtained from the restructuring process has only 7 constraints while the original model (the J-O model) has 11. The restructuring process leads to the complete elimination of some binary variables from the new model.

We used numerical examples to illustrate and compare the two models. The examples were solved with LINDO and Excel 2007 solvers. In each example, the number of iterations before obtaining optimal solution with LINDO was smaller for the new model than for the J-O model. In nearly all the examples, the Excel-solution times for the new model were less than those for the J-O model. In some of the examples, the differences between the numbers of iterations for the two models were large.

As the two models' sizes increase: (1) the number of the J-O model's binary variables becomes much larger than that of the new model, (2) the Excel-solution time increases, and (3) the number of iterations before optimal solutions were obtained (with LINDO) increases. The obvious conclusion from these is that the J-O model becomes more and more difficult to solve than the new model as the two models' sizes increase.

The Excel solutions to the two models are the same in each of the numerical examples. The LINDO solutions to the two models are also the same in each example. These show that the imperfections in the structures of the J-O model do not actually affect its ability to produce good solutions. They only affect its processing and computational time, and this can be a big problem if it (the J-O model) is large.

It is very obvious from the results of the numerical examples that the applications of the New model will lead to significant time and cost savings over the application of the J-O model. Apart from the fact that the new model requires less solution/computational time and cost, data preparations and entries will be much easier with it than with the J-O model. Additionally, with the new model's smaller size, less computer memory will be required for data storage and processing.

A very good advantage of the model is that it can be easily modified for applications in situations where produce-or-subcontract decisions can be based mainly on cost considerations, irrespective of whether there are enough capacities or not.

The examples show that Excel 2007 solver produces good solutions to the two models. In most of the examples, the optimal value of the objective function produced by the Excel solver are very close to those produced by LINDO. The only obvious drawback is that the former is much slower than the latter. After entering the in-put data in each example, optimal solution was instantaneously produced by LINDO at a click on the Solve command. It took some couple of seconds to solve each example with Excel 2007 solver.

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