THE EFFECT OF ERRORS IN INTER-THERMAL SPEED ON THE AVERAGE CROSS-COUNTRY SPEED

F. G. IRVING

Department of Aeronautics
Imperial College of Science and Technology
London, England

Presented at the 14th OSTIV Congress
Waikerie, Australia 1974

INTRODUCTION

From the familiar construction of Fig. 1, it will be seen that if the average achieved rate of climb is \( v_c \) then the optimum gliding speed between thermals will be \( U_1 \). If, however, the sailplane is actually flown at \( U_2 \), there will be a loss in average speed \( \Delta U_2 \). In principle, this loss in speed for a particular situation can be assessed by performing graphically the construction shown in the diagram. In practice, it is difficult to do so accurately: one is trying to assess a second-order error and the result obtained is very sensitive to inaccuracies in the plotted performance curve and the practical difficulty of locating the point of tangency.

It therefore seemed useful to derive a general analytical expression enabling the error to be easily assessed under any conditions, for purposes such as examining the effects of errors in variometer indications.

Throughout this note, all speeds are assumed to be "equivalent".

The Dimensionless Performance Curve

As Goodhart (Ref. 1) has noted, a good approximation to the drag coefficient of a sailplane in a given configuration is

\[ C_D = C_{D_0} + K \frac{C_L^2}{U} \]  

In this expression, the second term is not simply due to the vortex drag: it also includes a contribution due to the dependence of the profile drag on lift coefficient and wing Reynolds number. In total, this second term is often 25% or more in excess of the vortex drag contribution.

If equation (1) applies, it may be shown (Refs. 2, 3) that the performance curve corresponds to the dimensionless expression

\[ \frac{v_s}{v_s^0} = \frac{1}{2} \left( \frac{U^2}{U} + \frac{1}{U} \right), \]  

where \( U = U/U_0 \)

and \( \frac{v_s}{v_s^0} \). \( \frac{U}{U_0} \) and \( \frac{v_s}{v_s^0} \) are therefore dimensionless speeds obtained by dividing the actual forward speed and rate of descent by the values appropriate to \( U/(B/D)_{\text{max}} \).

The optimum condition of Fig. 1 corresponds to the criterion

\[ \frac{d\bar{v}_s}{d\bar{U}} = \frac{\bar{v}_c + \bar{v}_s}{\bar{U}}, \]  

where \( \bar{v}_c = \frac{v_c}{v_s^0} \) and \( \bar{U} = \frac{U}{U_1} \).

Equation (3) is the normal MacCready criterion expressed in terms of the dimensionless quantities.

In general, if the sailplane is flown at \( U \) between the thermals, the average cross-country speed will be:
\[ \bar{U}_x = \frac{\bar{U} v_c}{v_c + \bar{v}_s}, \quad (4) \]

where \( \bar{U}_x = \frac{U_x}{U_0} \).

It then follows, from equations (2), (3) and (4) that, when \( \bar{U} = \bar{U}_1 \), the corresponding dimensionless cross-country speed will be

\[ \bar{U}_{x1} = \frac{2\bar{U}_1 (\bar{U}_1^3 - 1/\bar{U}_1)}{(3\bar{U}_1^3 - 1/\bar{U}_1)}, \quad (5) \]

and \( \bar{v}_c = \bar{U}_1^3 - 1/\bar{U}_1 \). \quad (6)

**Effect of Flying at the Non-Optimum Speed**

It also follows from (2), (4) and (6) that, if the sailplane is flown at \( U_2 \) during the glide with the same average rate of climb \( v_c \), the average cross-country speed becomes

\[ \bar{U}_{x2} = \frac{2\bar{U}_2 (\bar{U}_1^3 - 1/\bar{U}_1)}{2(\bar{U}_1^3 - 1/\bar{U}_1) + (\bar{U}_2^3 + 1/\bar{U}_2)}, \quad (7) \]

If, in equation (7), we now put \( \bar{U}_{x1} = \bar{U}_{x2} + \delta \bar{U}_x \) and \( \bar{U}_2 = \bar{U}_1 + \delta \bar{U} \), we can expand this expression in the usual fashion.
After some manipulation, we compare it with equation (5) and find that

\[ \frac{\delta U}{U} = - \left[ \frac{3U^4_1 + U^4_0}{3U^3_1 - U^4_0} \right] \frac{\delta U}{U} \cdot (8) \]

Reverting to dimensional quantities, this can also be written

\[ \frac{\delta U}{U} = - \left[ \frac{3U^4_1 + U^4_0}{3U^3_1 - U^4_0} \right] \frac{\delta U}{U} \cdot (9) \]

So, knowing the optimum glide speed \( U_1 \) and the speed for \( (L/D)_{max} \) in this configuration, \( U_0 \), the effect of errors in \( U_1 \) may be found.

If equation (9) is written

\[ \frac{\delta U}{U} = - E \left[ \frac{\delta U}{U} \right]^2 \cdot (10) \]

then \( E \) is easily found for various values of \( U_1/U_0 \), i.e. of \( U_1 \). It will be seen from Fig. 2 that, for likely values of \( U_1 \), it lies between 1.1 and 1.3. Under typical circumstances, a 10% error in \( U_1 \) will lead to a loss of about 1.2% in average cross-country speed.

Example A Standard-Class glider has a speed for \( (L/D)_{max} \) of 46 knots (85 km/h). It is being flown under conditions such that the optimum glide speed is 69 knots (128 km/h). The maximum average cross-country speed would then be 39.5 knots (73 km/h). Since \( U_1 = 1.5 \), \( E \) is 1.14.

If the glide speed differs from the optimum by 10% (i.e. 7 knots, 13 km/h), the loss in average speed will be 1.14% (i.e. 0.45 knots, 0.84 km/h).

Conclusions Since we are considering departures from an optimum condition, it follows that errors in gliding speed will have a second-order effect on the loss in average speed, as shown by the foregoing analysis. The example indicates that quite noticeable errors in gliding speed have only a small effect. However, championships are won or lost by quite small margins so one concludes that, whilst it is worth making reasonable efforts to optimise the gliding speed, it is hardly worth going to the trouble to achieve precision. In any case, the available data (e.g., on average rate of climb) is not usually sufficiently accurate to enable one to do so.

It is much more profitable to cultivate one's skill and judgment in the pursuit of improved rate of climb, since this quantity has a first-order effect on the average speed.

It may be shown from the foregoing equations that, under optimised conditions, an increase in rate of climb \( \delta v_c \) improves the average speed by \( \delta U \) where

\[ \frac{\delta U}{U} = - \left[ \frac{3U^4_1 + U^4_0}{3U^3_1 - U^4_0} \right] \frac{\delta v_c}{v_c}, \]

or

\[ \frac{\delta U}{U} = F \left[ \frac{\delta v_c}{v_c} \right] \cdot (11) \]

\( F \) varies from 1 to 1/3, depending on \( U_1 \), as shown in Fig. 3. In the case considered in the example above, it would have a value of 0.43. An increase in average rate of climb of 3%, i.e., from 3.73 knots (1.92 m/s) to 3.84 knots (1.98 m/s), would more than offset the loss in average speed due to errors in gliding speed.
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Figure 2.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Figure 3}
\end{figure}
REFERENCES

1. Goodhart, H. C. N. "A Note on the Measurement of the Induced Drag Factor (k) of a Glider". OSTIV Publication XI.

2. Irving, F. G. "An Analysis of Goodhart's Figure of Merit". Aero Revue, May 1958.

3. Galvao, F. L. "A Universal Table for Gliding". OSTIV Publication XI.

SYMBOLS

All speeds are 'equivalent'.

\( \bar{U} \) forward speed
\( U_0 \) the value of \( U \) at \( (L/D)_{\text{max}} \)
\( U_1 \) optimum gliding speed for a given \( v_c \)
\( U_2 \) a non-optimum gliding speed
\( \delta U \) \( U_2 - U_1 \)
\( \bar{U}_x \) average cross-country speed
\( U_{x_1} \) \( U_x \) corresponding to \( U = U_1 \)
\( U_{x_2} \) \( U_x \) corresponding to \( U = U_2 \) \( U_{x_2} = U_{x_1} \) for the same \( v_c \).
\( \delta U_x \) \( U_{x_2} - U_{x_1} \)
\( \bar{U} \) \( U/U_0 \). Similarly, all of the above quantities are rendered dimensionless, e.g. \( \bar{U}_x = U_x/U_0 \), etc.
\( v_s \) rate of descent in still air
\( v_{s_0} \) \( v_s \) at \( (L/D)_{\text{max}} \)
\( \bar{v}_s \) \( v_s/v_{s_0} \)
\( v_c \) average achieved rate of climb
\( \bar{v}_c \) \( v_c/v_{s_0} \)
\( E, F \) functions of \( \bar{U}_1 \).