SOME GENERAL CONSIDERATIONS ON SAILPLANE AERODYNAMIC DESIGN

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In the design of a sailplane, the most important process in order to obtain an improvement in aerodynamic performance is obviously the reduction of aerodynamic drag. The designer has two different possibilities for achieving this aim:

1) the reduction of induced drag;
2) the reduction of parasitic drag, which consists essentially of friction, form and interference drag.

The reduction of induced drag is mainly achievable through an increase of wing aspect ratio.

The reduction of parasitic drag is obtainable through a variety of architectural arrangements and choices. Sailplane design being a problem of compromise, the designer has to
Polar Equations

The relationship between lift and drag coefficients is usually given by the analytical form

\[ C_D = C_{D0} + k \frac{C_L^2}{\pi \alpha} \]  

(1)

If \( C_{D0} \) is considered a constant, equation (1) gives poor approximation at high \( C_L \), where not negligible \( C_{D0} \) increases take place.

The evaluation of \( k \) *a priori*, moreover, is subject to a rather large amount of uncertainty, \( k \) being liable to vary in a rather large interval (between 0.33 and 1.45 in special cases, and from 1.05 to 1.35 in the more common cases), as results from calculations based
on experimental data (reference 1).

In the case of sailplanes, some of the more interesting flight attitudes, such as that of maximum gliding ratio ($E_{max}$) or minimum sinking speed ($V_{ymin}$), occurring at high $C_L$, it would be desirable to have an analytical expression more suitable to represent the true polar curve in this range of high $C_L$.

A better approximation is obtainable with the simple cubic equation:

$$C_D = C_{Do} + \frac{C_L^2}{\pi A} \dot{C}_{Do} + \frac{C_L^3}{\pi A}$$

where $C_{Do}$ is to be chosen so that the cubic curve is faired to the experimental polar.

If a quadratic polar ($k = 1$) is established which is based on a certain value of $C_{Do}$, it is possible to determine $C_{Do}^*$ so that the $C_D$ relating to the quadratic and cubic polars at a certain value of $C_L$ (say $C_L^* = 0.5$), coincide:

$$C_D = C_{Do} + \frac{C_L^2}{\pi A} C_{Do}^* + \frac{C_L^3}{\pi A}$$

$$C_{Do}^* = C_{Do} + \frac{C_L^2}{\pi A} (1 - C_L^*)$$

$$C_L^* = 0.5$$

![Graph](image-url)
\[ C_{Do} = C_{Do} + 0.043/A \]  

The proper value of \( C_{Lmax} \) should be chosen in relation to the \( C_{Lmax} \) of the particular sailplane considered. The value of 0.6 is appropriated for \( C_{Lmax} = 1.3 - 1.5 \).

For most sailplanes, the lift coefficients for maximum gliding ratio and minimum sinking speed, exceed \( C_{L} = 0.6 \). It is therefore of interest to calculate the above characteristics from equation (2). The expressions derived from the cubic equation are compared in table I (see page 77) with the corresponding known expressions derived from the quadratic equation \((k = 1)\).

In order to demonstrate the suitability of the cubic equation to approximate the true polar curve at high \( C_{L} \), reliable experimental values of \( E_{max}, V_{min}, C_{Lmax}, C_{LVmin} \) relating to a number of sailplanes with known characteristics (Table II, see page 79) have been compared with the theoretical values (quadratic and cubic) calculated according to the expressions of table I (figures 1, 2, 3 and 4).

\( C_{Do} \) - A DIAGRAMS

In figure 5 \( C_{Do} \), A curves, for constant values of \( E_{max} \) and \( C_{Lmax} \) are traced, according to the following expressions derived from the quadratic and cubic equations:
quadratic: $C_{Do} = 0.785 \frac{A}{E_{max}^2}$
$C_{Do} = 0.318 \frac{C_{L_{max}}}{A}$

cubic: $C_{Do} = 0.585 \sqrt{\frac{A}{E_{max}^3}} - 0.045/A$
$C_{Do} = 0.530 \frac{C^3_{L_{max}}}{A} - 0.045/A.$

In figure 5 we have $C_{Do}$, A curves for different values of $V_{ymin}$ (W/S = 20 kg/mq) and $C_{L_{ymin}}$ according to both quadratic and cubic polar equations:

quadratic: $C_{Do} = 0.0322 \times 10^{-3} y_{ymin}^4$
$C_{Do} = 0.108 C^2_{L_{ymin}}/A$

cubic: $C_{Do} = 0.00245 \times V^2_{ymin} A - 0.046/A$
$C_{Do} = 0.313 C^3_{L_{ymin}}/A - 0.046/A.$

If we assume that the curves based on the cubic polar (full lines in figures 5 and 6)
represent the true relationship among the various parameters, some considerations can be made from figures 5 and 6 which may be of interest from the designer's point of view.

From figure 5 it is evident that at a certain value of $C_{d0}$, there practically exists a value of the wing aspect ratio $A$, above which only a small gain in $E_{max}$ is attainable.

On the other hand, if the value of $E_{max}$ is fixed, we see from the diagram that this value may be obtained through many combinations of $C_{d0}$ and $A$. This is quite obvious, but not always the best combinations of $C_{d0}$ and $A$ have been adopted in design.

Sometimes very high wing aspect ratios have been combined with general architectures to which a $C_{d0}$ was pertinent that was far from being the lowest possible. As a result, there was not taken out of the "aerodynamic cleanliness" of the sailplane (I mean by this term the goodness of the sailplane in respect of parasitic drag) all the possible advantage, because $E_{max}$ and $V_{min}$ occur at lift coefficients at which a $C_{d0}$ increase has already taken place, as may be easily seen from figures 5 and 6.

It might be stated that, for good design, $E_{max}$ and $V_{min}$ should occur at lift coefficients of not more than 0.7. In practice $V_{min}$ can take place at lift coefficients up to 0.9, without sacrificing much in performance.

In general, for taking from a high aspect ratio all the possible advantage, it is necessary first to have attained the maximum possible reduction in parasitic drag.

The graphics of figures 5 and 6 can be used, in my opinion, for quantitative evaluation of the influence of the various parameters in sailplane performance.
REFERENCES TO LITERATURE

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(2) JAHRBUCH 1937 der D.V.L. "Flugleistungsmessungen an verschiedenen Segelflugzeugen", W. SPIELER.
(3) JAHRBUCH 1933 der D.V.L. "Weitere Flugleistungsmessungen an Segelflugzeugen", W. SPIELER.
(4) AIRCRAFT ENGINEERING, Jan. 1953 "Progress in two-seater sailplane design", B.S. SHENSTONE.
(6) INTERAVIA, March 1948 "Nouveaux principes dans la construction des planeurs", M. LANDOLT.
(7) THE AEROPLANE, July 25th, 1952 "Slingsby's international winner";
(8) INTERAVIA, Feb. 1949 "L'amélioration des performances en vol à voile par la construction en aile volante", W.M. HORTEN.
(9) AIRCRAFT ENGINEERING, April 1946 "The DFS Reiher sailplane", B.S. SHENSTONE.

<table>
<thead>
<tr>
<th>QUADRATIC POLAR</th>
<th>CUBIC POLAR</th>
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<tbody>
<tr>
<td>$E_{max} = \sqrt{\frac{\pi A}{4 C_{D0}}} - 0.886 \sqrt{\frac{A}{C_{D0}}}$</td>
<td>$E_{max} = \sqrt[3]{\frac{8 \pi}{54} \frac{3 A}{C_{D0}^2}} - 0.777 \sqrt[3]{\frac{A}{C_{D0}^2}}$</td>
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<tr>
<td>$C_{LE_{max}} = \sqrt{\frac{\pi A}{C_{D0}}} C_{D0} + 1.77 \sqrt{\frac{A}{C_{D0}}}$</td>
<td>$C_{LE_{max}} = \sqrt[3]{\frac{3 \pi A C_{D0}}{2}} - 1.15 \sqrt[3]{\frac{A}{C_{D0}}}$</td>
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<tr>
<td>$V_{y_{min}} = 13.3 \sqrt{\frac{C_{D0}}{A^2}} \sqrt{\frac{\pi A}{20}}$ (m/sec)</td>
<td>$V_{y_{min}} = 20.2 \sqrt{\frac{C_{D0}}{A}} \sqrt{\frac{\pi A}{20}}$ (m/sec)</td>
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<tr>
<td>$C_{LV_{y_{min}}} = \sqrt{\frac{\pi A C_{D0}}{C_{D0}}}$</td>
<td>$C_{LV_{y_{min}}} = \sqrt[3]{\frac{\pi A C_{D0}}{C_{D0}}}$</td>
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<table>
<thead>
<tr>
<th>Source of data (ref.)</th>
<th>1</th>
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<tr>
<td>Aspect ratio (A)</td>
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<td>14.6</td>
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<td>15.3</td>
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<td>11.5</td>
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<td>17.3</td>
<td>15.7</td>
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<td>33.4</td>
<td>44.8</td>
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<tr>
<td>Wing loading, W/S (kg/m²)</td>
<td>15.1</td>
<td>17.1</td>
<td>14.8</td>
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<td>13</td>
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<tr>
<td>E_max</td>
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<tr>
<td>V_{Emax} (km/h)</td>
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<td>67</td>
<td>68</td>
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<td>V_{y_{min}} (m/sec)</td>
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<td>V_{V_{y_{min}}} (km/h)</td>
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<tr>
<td>V_{y_{100}} (sink. speed at 100 km/h)</td>
<td>2.58</td>
<td>2.08</td>
<td>2.03</td>
<td>1.74</td>
<td>1.53</td>
<td>1.50</td>
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<tr>
<td>100 C_{D0} ***)</td>
<td>2.58</td>
<td>2.43</td>
<td>2.03</td>
<td>1.91</td>
<td>1.74</td>
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</tbody>
</table>

*) Numbers refer to points indicated in figures 1, 2, 3 and 4.

**) C_{D0} are calculated at V = 100 km/h; where V_{y_{100}} is not known, C_{D0} is calculated at V_{Emax}.