Linear Multi-hop Amplify-and-Forward (AF) Cooperative Relay Channels

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Abstract—Cooperative relaying of channels between base station and user equipment emerged as a great technique to enhance the reliability and throughput of cellular networks. The relay nodes normally exploit amplify-and-forward (AF) scheme to relay the connection. Although there have been many previous works on cooperative issues, none of them have considered the fundamental trade-off between the communication reliability and transmission rate. Instead of considering the performance evaluated separately on the achievable rate or the error probability, the RCEE provide insight into fundamental trade-off between the communication rate and the communication reliability in cooperative relay channels. It has been shown that multi-hop relay nodes are necessary to compensate the decreasing SNR over the distance. As a trade-off, the communication reliability in terms of RCEE is decreasing over the increase of multi-hop relay nodes with low SNR for a given data rate. Therefore, the optimization for the number of hops is preferable in the cooperative relay channels.

Index Terms—Amplify-and-forward, cooperative, linear multi-hop relay channels, random coding error exponent, signal noise ratio.

I. INTRODUCTION

In a communication system, cooperative multi-hop relay channel assists the transmission of the signal from source node to the destination node. This relay channel reduces the transmission distance to improve the communication performance in relation to channel gains and communication reliability. Hence, many researches have been conducted in this field. For instance, an evaluation of the outage probability in multi-hop wireless channel with amplify-and-forward (AF) relays over Nakagami-\(m\) fading channel had been proposed [1]. The performance bound pertained to the end-to-end signal-to-noise ratio (SNR) of multi-hop relay communication, by using the well-known inequality between harmonic and geometric means of positive random variables, was studied in [2]. The outage probability and the average bit error rate (BER) for coherent and non-coherent modulation schemes of multi-hop system with non-regenerative blind relays underwent Rice, Nakagami-\(m\) and Hoyt fading channels was studied in [3] using the moment-based approach.

Instead of evaluating the communication reliability or the transmission rate, the random coding error exponent (RCEE) can gain insight into fundamental trade-off between both evaluations. The RCEE is described as a fundamental trade-off between communication reliability and the transmission rate in linear multi-hop AF cooperative relay channels. The error exponent determined in RCEE which introduces the code-word length in a tight upper bound on the error probability [4]. The performance of a dual hop relay network using error exponent over Nakagami-\(m\) fading channel has been conducted in [5]. Most recently the error exponent was used to analyze the system performance in two-way relay communication channels [6].

The performance of one-hop and two-hop decode-and-forward relay channels was compared in [7] in which the channel model was attenuated due to path loss and Rayleigh fading. By considering the outage probability, one-hop transmission was outperforms than two-hop transmission at high spectral efficiency. The fixed infrastructure multi-hop relay system with a regenerative protocol had been analyzed by the assumptions of constant channel coefficients and same average path loss exponent for all links [8]. It showed the spectral efficiency is better in one-hop transmission than multi-hop transmission at high SNR and vice versa in low SNR. The achievable rate of the linear multi-hop relaying was analyzed in [9], and the greater number of hops showed better performance under path-loss effect at low SNR. However, the performance of those studied were measured separately among the SNR, outage probability, bit-error probability, or achievable rate which effects the number of hops. It has been shown that the SNR and RCEE is considered a trade-off between the communication reliability and the data rate. The RCEE is more adaptable for the evaluation of the number of hops in relay channels.

In this paper, a linear multi-hop AF cooperative relay channel is considered, which the all the terminals are located in
a straight line, investigate the trade-off between the communication reliability and the data rate. The performance of the SNR in multi-hop cooperative relay channel is evaluated where the relay terminals are located in fixed positions. The channel coefficients are assuming the terrain type B model for a suburban area [10]. It has been shown that multi-hop relay nodes are necessary to compensate the decreasing SNR over the distance. As a trade-off, the communication reliability in terms of RCEE is decreasing over the increase of multi-hop relay nodes with low SNR for a given data rate. Therefore, the optimization for the number of hops is preferable in the cooperative relay channels.

The organization of this paper is as follows. In Section II, the system model of the SNR, RCEE, transmission rate and capacity for multi-hop AF cooperative relay channel is developed. The performances are evaluated in Section III. Finally, the conclusion remarked on Section IV.

II. SYSTEM MODEL

The model of linear multi-hop AF cooperative relay channel as shown in Fig. 1. It consists a source terminal, a destination terminal, and relay terminals. In Fig. 1(a), the first relay, RS1 is linearly located 500m away from the base station, while the second relay, RS2 is linearly 1000m away from the base station as shown in Fig. 1(b). In Fig. 1(c), both RS1 and RS2 are respectively linearly located 500m and 1000m away from the base station. The distance of user equipment is between 600m and 1500m away from the base station.

![Fig. 1. Linear multi-hop cooperative relay channel, (a) first relay, (b) second relay and (c) both relays.](image)

The terminals assume that the node cannot transmit and receive signals simultaneously, which is referred as half duplex. In addition, the transmission is performed through time-division where the transmission from the source to the destination is divided into K time slots to ensure that there is no interference between the different hops. In the kth time slot, the received signal is amplified by the kth intermediate terminal with the relaying gain of Gk and then forwarded to the (k-1)th terminal in the (k-1)th time slots.

A. Channel Model

The channel propagation model parameters are listed in Table 1.

### Table 1. Propagation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Frequency (GHz)</td>
<td>2.5</td>
</tr>
<tr>
<td>Channel bandwidth (MHz)</td>
<td>10</td>
</tr>
<tr>
<td>Distance (m)</td>
<td>1500</td>
</tr>
<tr>
<td>Antenna model</td>
<td>Omni-directional</td>
</tr>
<tr>
<td>BS height (m)</td>
<td>32</td>
</tr>
<tr>
<td>RS height (m)</td>
<td>10</td>
</tr>
<tr>
<td>SS height (m)</td>
<td>2</td>
</tr>
<tr>
<td>BS transmit power Ps (dBm)</td>
<td>50</td>
</tr>
<tr>
<td>RS transmit power Pr (dBm)</td>
<td>38</td>
</tr>
<tr>
<td>Noise power (dBm)</td>
<td>-132.3951</td>
</tr>
</tbody>
</table>

The path loss propagation scenarios A, B, and C [10] are categorized as

- Terrain Type A: Hilly terrain with moderate-to-heavy tree densities
- Terrain Type B: Intermediate path-loss condition
- Terrain Type C: Flat terrain with light tree densities

The considered propagation scenario in our simulation is the intermediate path-loss condition, also known as the terrain type B model for a suburban area. Its path loss model can be formulated as

\[ l(d) = 10 \log_{10} \left( \frac{4\pi d}{\lambda} \right) \quad \text{for} \quad d < d_0 \]  
\[ l(d) = A + 10 \gamma \log_{10} \left( \frac{d}{d_0} \right) + \Delta P_L + \Delta P_h \quad \text{for} \quad d > d_0 \]  

where \( d_0 = 100 \) m and \( d \) is the distance between each transmitter and receiver antennas in meters. The other parameters are

\[ A = 20 \log_{10} \left( \frac{4\pi d_0}{\lambda} \right) \]  
\[ \gamma = ((a-b)h_b + c/h_b) \lambda \]  

where \( \lambda \) is the wavelength in meters,

\[ \Delta P_L = 6 \log_{10}(f(MHz)/2000) \]  
\[ \Delta P_h = -10 \log_{10}(h_r/3) \quad \text{for} \quad h_r \leq 3m \]  

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Terrain Type A</th>
<th>Terrain Type B</th>
<th>Terrain Type C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>4.6</td>
<td>4</td>
<td>3.6</td>
</tr>
<tr>
<td>( b )</td>
<td>0.0075</td>
<td>0.0065</td>
<td>0.005</td>
</tr>
<tr>
<td>( c )</td>
<td>12.6</td>
<td>17.1</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ \Delta P_L = 6 \log_{10}(f(MHz)/2000) \]  
\[ \Delta P_h = -10 \log_{10}(h_r/3) \quad \text{for} \quad h_r \leq 3m \]
\[ \Delta PL_h = -20 \log_{10}(h_j/3) \quad \text{for } h_j > 3m \]  
where \( h_j \) is the height of relay station or user equipment below roof top (BRT) antenna, \( \Delta PL_f \) and \( \Delta PL_h \) are correction factor for the carrier frequency and correction factor for the received antenna height respectively.

The channel gain is given by

\[ |H_k|^2 = G_f G_d (d) \sigma \eta \]  
where \( \sigma \) = shadowing, and \( \eta \) = multipath fading. The multipath fading value, \( \eta = 10 \text{ dB} \) has been considered. The standard deviation, \( \sigma \) of the log normal shadowing, dependent on the particular frequency needs to be taken into account. The expression for this situation as given by Okumara is given as [11]

\[ \sigma = 0.65 |\log_{10}(f)|^2 - 1.30 \log(f) + X \]  
where \( f \) is in MHz, \( X = 6.6 \text{ dB} \) as for the suburban Type B Model. The thermal noise is calculated as

\[ NT = KT B \]  
where \( K = \text{Boltzmann's Constant}, \) \( T = \text{Ambient Temperature in degrees of Kelvin}, \) \( KT = -174 \text{ dBm/Hz} \) and \( B = 10 \log(10MHz). \)

### B. SNR Analysis

The AF relaying can be categorized into two types. The first one is the relaying gain is fixed for all fading states which the protocol reduces the implementation complexity in terms of channel state information (CSI) [2], [12]. Another one is CSI-assisted relaying where the gain is calculated based on the preceding channel state which is much interest due to less complexity, and it has better performance compared to fixed-gain relaying.

For CSI-assisted relaying, the AF relaying gain is given as [13]

\[ G^2 = P_s \frac{1}{(n_{k-1}) |H_{k-1}|^2 + N_0} \]  
where \( P_s \) is the transmitted power.

The end-to-end equivalent SNR of cooperative multi-hop channels is given by [14]

\[ y = y_{sd} + \sum_{k=0}^{K} (1/y_k)^{-1} \]  
where \( y_{sd} = P_{sd} |H_{sd}|^2 / N_0 \) and \( y_k = P_k |H_k|^2 / N_0 \) are the instantaneous SNR of link which \( sd \) denotes the link from source to destination.

### C. RCEE, Transmission Rate and Capacity of Linear Multi-hop Cooperative Relay Channels

The end-to-end SNR in the above is intractable that problem in the derivation of the exact RCEE due to it is dominated by the most noisy hop. Thus, Cauchy-Schwarz inequality is used to provide more precise end-to-end SNR as

\[ y = y_{sd} + \sum_{k=0}^{K} (1/y_k)^{-1} \leq y_b \Delta \min(y_{b,1}, y_{b,2}) \]  
where \( y_{b,1} = \min(y_1, y_2, \ldots, y_k) \) using the most noisy hop, and \( y_{b,2} = (y_1, y_2, \ldots, y_k)/K^2 \) using the Cauchy-Schwarz inequality. The derivation of the cumulative density function (CDF) and probability density function (PDF) of \( y_b \) is used to further analyze on the RCEE [15].

\[ p_{y_b}(y) = \sum_{k=0}^{K-2} \left( \frac{(K^2 - K)^K}{k!y^k} e^{-K^2y/K^2} + \frac{(K^2 - K)^{K-1}}{(K-1)!y^k} e^{-K^2y/K^2} \right) \]  
\[ F_{y_b}(y) = 1 - \sum_{k=0}^{K-2} \left( \frac{(K^2 - K)^{k}}{k!y^k} e^{-K^2y/K^2} \right) \]

The RCEE of linear multi-hop cooperative relay channel is given by [15]

\[ \tilde{E}_s(R) = \max_{0 \leq \rho \leq 1} \left\{ \tilde{E}_q(\rho) - \rho KR \right\} \]  
where \( \rho \in [0,1] \) is an arbitrary number, \( \tilde{E}_q(0) \) for \( \rho = 0 \) and

\[ \tilde{E}_q(\rho) = -\ln \left( 1 + \frac{1}{1 + \rho} y_{b} \right) \rho \]  
\[ \tilde{E}_q(\rho) = \sum_{k=0}^{K-2} \left( \frac{(K-1)^{k}}{k!K^{k+1} \Gamma(\rho)} G_{2,1}^{1,2} \left( \frac{K^2 - y_b}{1 + \rho} \right)^{k+1} \right) + \left( \frac{(K-1)^{K-1}}{(K-1)!K^{K+1} \Gamma(\rho)} G_{2,1}^{1,2} \left( \frac{K^2 - y_b}{1 + \rho} \right)^{K-1} \right) \]  
where \( \Gamma(\cdot) \) is Euler’s gamma function, and \( G_{m,n}^{p,q}(\cdot) \) is the Meijer’s G-function [16]. The source and destination communicate each code-word in \( KN \) symbol-time intervals using K-hop transmission, the transmission rate \( R \) in nats/Hz [15] is

\[ R = \frac{\ln M}{KN} \]  
where \( M \) is the number of code-words of length \( N \).

Furthermore, the capacity can derive based on the random coding exponent [15]. The ergodic capacity is given by

\[ \langle C \rangle = \sum_{k=0}^{K-2} \left( \frac{(K-1)^{k}}{k!K^{k+1} \Gamma(1)} G_{3,2}^{1,3} \left( \frac{y_b}{K^2} \right)^{k+1} \right) + \left( \frac{(K-1)^{K-1}}{(K-1)!K^{K+1} \Gamma(1)} G_{3,2}^{1,3} \left( \frac{y_b}{K^2} \right)^{K-1} \right) \]  

### III. PERFORMANCE EVALUATIONS

The performance evaluations of transmission signal through first relay, second relay, and both relays are presented in this section. These evaluations are based on the mathematical expressions of end-to-end SNR, RCEE, transmission rate, and...
capacity presented in the previous sections. The simulations are done using MATLAB® software which supports basic toolbox of communication system operations. The simulation is initiated by presenting the SNR

A. SNR Analysis

The performance of SNR versus distance for the linear multi-hop AF cooperative relay channel is shown in Fig. 2. As expected, the SNR of cooperative transmission using both relays (three hops) outperforms the transmissions of using only the first relay (two hops) and only the second relay (two hops) throughout the distance. The transmission by using the first relay outperforms the transmission by using the second relay when the user equipment in the distance 700m away from the base station. In contrast, when user equipment is located at a distance between 700m and 1500m away from the base station, the transmission by using the second relay outperforms the transmission by using the first relay. This figure shows that the shorter the signal transmission distance, which the number of hops is increased, have better SNR performances.

The results show that the overall performances of transmission signal through RS2 outperform transmission signal through both RSs and through RS1. It can be seen from this figure that user equipment who is located 700m away from the base station, the performance of signal transmission through RS1 outperforms transmission signal through both RSs and through RS2. In contrast, user equipment that located between 700m and 1500m away from base station, the performance of transmission signal through RS2 outperforms transmission signal through both RSs and through RS1. From Figs. 2 and 3, it can be shown that the higher the SNR with lesser number of hops offers better RCEE performance.

Furthermore, the error exponents at a distance of 1200m are 0.76, 0.30 and 0.16 for transmissions over only the second relay, both RSs and only the first relay respectively. These results indicate that the transmission over both RSs and only the first relay must exploit more than 2.53 and 4.75 times the required code-word length respectively compared to the transmission over only the second relay.

The SNR values of RCEE versus Rate for Figs. 4 and 5 are referring to Fig. 2. The SNR value of the transmission signal through relays at a distance of 600m away from the base station is used as simulation values for RCEE versus Rate in Fig. 4. The SNR value of the transmission signal through relays at distance 1100m away from the base station is used as simulation values for RCEE versus Rate in Fig. 5. From Figs. 4 and 5, it can be seen that the RCEE will be decreased with the R increase. Figure 6 demonstrated the capacity versus distance for linear multi-hop AF cooperative relay channels. Here, the simulation results show that the capacity is increased with the increase of SNR.

B. Random coding error exponent, Transmission Rate and Capacity

The simulation results of RCEE versus distance by using $\gamma_b$ case for linear multi-hop AF cooperative relay channels is shown in Fig. 3.
Cooperative relaying of channels is used to enhance the signal transmission between base station and user equipment, especially when user equipment at the cell edge. The AF relay channels is studied due to less complexity to implement. Thus, the studied in SNR and RCEE of linear multi-hop AF cooperative relay channels is considered due to it provide trade-off between the communication reliability and transmission rate. It has been shown that the cooperative multi-hop relay channels will enhance the transmission of signal throughout the distances. From this studied, the signal transmission through RS2 outperform the signal transmission through RS1 and both RSs. This is because the signal transmission through RS2 provide high SNR throughout the distance with lesser number of hops. The RCEE provides better performances in low transmission rate. Therefore, cooperative multi-hop relaying with smaller of the number of hops and high SNR is preferable in high transmission rate.

References

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Nor Kamariah Noordin received her BSc in Electrical Engineering majoring in Telecommunications from University of Alabama, USA, in 1987. She became a tutor at the Department of Computer and Electronics Engineering, Universiti Putra Malaysia, and pursued her Masters Degree at Universiti Teknologi Malaysia and PhD at Universiti Putra Malaysia. She then became a lecturer in 1991 at the same department where she was later appointed as the Head from year 2000 to 2002. She is currently the Deputy Dean (Academic, Student Affairs and Alumni) of the Faculty. During her more than 15 years at the department she has been actively involved in teaching, research and administrative activities. She has supervised a number of undergraduate students as well as postgraduate students in the area of wireless communications, which led to receiving some national and UPM research awards. Her research work also led her to publish more than 100 papers in journals and in conferences.


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